Stochastic User Equilibrium with Equilibrated Choice Sets

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The aim of this paper is to remove the known limitations of Deterministic and Stochastic User Equilibrium (DUE and SUE), namely that only routes with the minimum cost are used in DUE, and that all permitted routes are used in SUE regardless of their costs. We achieve this by combining the advantages of the two principles, namely the definition of unused routes in DUE and mis-perception in SUE, such that the resulting choice sets of used routes are equilibrated. A family of models is formulated to address this issue, which allows the combination of the unused routes with the use of well-known random utility models for used routes, without the need to pre-specify the choice set. We explore properties of different model specifications and present a corresponding generic path-based solution algorithm. Numerical results are also reported of the application of alternative specifications to two real-life cases, in which we explore convergence patterns and choice set composition and size.

Introduction

The Stochastic User Equilibrium (SUE, Daganzo and Sheffi, 1977) in connection with the most commonly applied Random Utility Models (RUM) induces (as remarked by Damberg et al., 1996) 'that of every route receiving a positive flow in the equilibrium state, regardless of its travel cost'. While only considering actual minimum cost routes as in the (deterministic) User Equilibrium (DUE, Wardrop, 1952) seems difficult to justify, moving to a case where all possible routes are used seems equally questionable. The need for full route enumeration has also a computational consequence for practical applications, as at least in principle any SUE algorithm has the *aim* to consider all such routes. This is not feasible even for medium-sized networks, and as a consequence, in practical implementations, an SUE using all available paths is typically not found. Instead, the issue of deciding which routes to consider, whether generated implicitly while iterating towards network equilibrium or explicitly prior to this, is left for the solution algorithm without the aid of any explicit criterion within the model formulation. As a consequence, the algorithmic heuristics implicitly determine the choice set and hence influence the choice probabilities in a way that it not explicitly stated in the base model or even fully understood by the modeller. An example is the solution of a multinomial probit SUE (Sheffi, 1985) with sampling, where paths are added with low probabilities, however, where new paths are added with some probabilities even when the solution algorithm is run with a number of iterations approaching infinity (or a number that is beyond practical applications).

The paper proposes a model which tackle the challenges of removing the known limitations of the DUE and SUE, namely that only routes with the minimum cost are used in DUE, and that all permitted routes are used in SUE regardless of their costs. The paper then moves on to propose consistent and applicable solution methods to the new model. Lastly, the applicability and reasonability of the solutions are demonstrated through applications to the Sioux Falls as well as the large-scale Zealand networks.

Model formulation

A model family is presented that combines the advantages of the two principles of DUE and SUE, namely the definition of unused routes in DUE and mis-perception in SUE, such that the resulting choice sets of used routes are equilibrated: a Restricted SUE model with an additional constraint that must be satisfied for unused paths (Watling et al., 2015). The overall advantage of this consists in its ability to combine the unused routes with the use of well-known random utility models for used routes, without the need to pre-specify the choice set.

Consider a network as a directed graph consisting of A links, indexed a=1, 2, …, A with flow f_a and generalised cost function $t_a(\mathbf{f})$ on link a. $\mathbf{f}$ denotes the A dimensional vector of link flows. Let there be M origin-destination (OD) pairs, indexed m=1, 2, …, M. Let d_m refer to the demand for OD-pair m. Let R_m denote the index set of all simple paths (without cycles) between the origin and destination of OD-pair m, and let x_{mr} and $c_{mr}(\mathbf{x})$ be the flow and cost on route r for OD-pair m. $\mathbf{x}$ and $\mathbf{c(x)}$ denotes vector of route flows and generalised route travel cost functions. Let the random utility U_{mr} for each route:

$$U_{mr} = -\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \qquad \left(r \in R_m; m = 1,2,...,M\right) \tag{1}$$

where $\xi = \left\{\xi_{mr} : r \in R_m, m = 1,2,...,M\right\}$ are random variables following some given joint probability distribution, and θ > 0 is a given parameter.

Let $\tilde{R}_m$ be any non-empty subset of R_m and we define the 'conditional' probability (assuming continuous, unbounded error distributions $\left\{\xi_{mr} : r \in R_m, m = 1,2,...,M\right\}$):

$$P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) = \Pr\left(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \in \tilde{R}_m\right) \quad \left(r \in \tilde{R}_m \subseteq R_m; m = 1,2,...,M\right) \tag{2}$$

We define the Φ-Restricted Stochastic User Equilibrium model:

Definition 1: Φ-Restricted Stochastic User Equilibrium (RSUE(Φ))

Suppose that we are given a collection of continuous, unbounded random variables $\left\{\xi_{mr} : r \in R_m, m = 1,2,...,M\right\}$ defined over the whole choice set R_m, and that for any non-empty *subsets $\tilde{R}_m$ of R_m (m = 1,2,…,M), probability relations $P_{mr}(\mathbf{c} | \tilde{R}_m)$ are given $\tilde{R}_m$ by considering the relevant marginal joint distributions from $\left\{\xi_{mr} : r \in R_m, m = 1,2,...,M\right\}$. The route flow* $\mathbf{x} \in G$ *is a RSUE(*Φ*) if and only if for all* $r \in R_m$ *and m = 1,2,…,M:*

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad x_{mr} = d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) \tag{3}$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}\right) \tag{4}$$

The RSUE(Φ) consists of two conditions, one concerning the utilised paths and the distribution of flow among these and one concerning non-utilised paths. The first condition allows the distribution of flow according to RUM among the paths in the restricted choice set. Thereby one of the limitations of the DUE is avoided by allowing flow to be distributed to non-minimum cost paths. In comparison with the SUE model, condition (3) in the above overcomes one of the main limitations, in that the flow allocation mechanism may only apply to a sub-network of the paths available. This is also true of SUE models applied to a pre-defined Master Choice set, but the key difference in Definition 1 is that, at equilibrium, condition (4) must be simultaneously satisfied alongside condition (3). That is, given that perceived utility is an affine function of travel cost plus random errors, the two conditions above must be *consistently* satisfied, at the *same* travel cost levels. Thus, they do indeed yield an alternative mechanism for defining *equilibrated*, non-universal choice sets in an SUE framework. The function Φ specifies a reference cost to be fulfilled by the cost on unused paths and thereby poses the distinction between used and unused paths. Since there exist several alternative, plausible ways for defining the reference costs, Definition 1 defines a *class* of conditions that is as wide as the ways in which the relationship Φ may be defined.

It should be noted that, in the RSUE definition, we consider only RUM models with continuous and unbounded error distributions. Under such an assumption all alternatives in the RUM (in this case, those in $\tilde{R}_m$) will have a non-zero probability of being chosen. Thus, condition (4) will never be relevant for a path that is subject to the RUM, i.e. in $\tilde{R}_m$, since such a path will always attract a positive flow. This makes the separation of used/unused paths coincide with the separation of those paths subject to the RUM and not subject to it.

A final remark is on the relation of the RSUE(Φ) model to conventional notions of equilibrium in networks. The RSUE(Φ) model does not contain DUE as a special case, in spite of the similarities in the specification of RSUE(Φ) and DUE. This is due to the fact that we restrict the attention in RSUE(Φ) to choice models which have *continuous* random utilities on the used paths, and thus the probability of two paths being exactly equal in terms of perceived utility is zero, whatever continuous distribution is adopted for the error terms. RSUE(Φ) does, however, contain SUE as a special case (regardless of the specification of Φ). This may be seen by setting $\tilde{R}_m = R_m$ in the RSUE definition, meaning that there are no paths for which condition (4) is tested, and condition (3) is simply an SUE condition on the universal choice set. This is true for any problem, and therefore we can guarantee existence of at least one RSUE(Φ) solution by exactly the same conditions as those that guarantee existence of a SUE solution (e.g., Cantarella, 1997).

Instances of RSUE(Φ)models

A key question that appears is the definition of Φ. Since in condition (4) the actual travel cost on an *unused* alternative must be compared with the actual travel costs on *used* alternatives, and since these unused alternatives are not subject to the random utility specification, it seems reasonable that Φ must map to something that makes sense in terms of the actual travel costs (rather than the randomly perceived utilities). Thus, while it might seem a possibility, it is not so sensible that Φ is a satisfaction function (expected maximum perceived utility, such as logsum for multinomial logit) over the used alternatives, as then we are in the 'scale' of perceived utility as opposed to actual travel cost. An alternative, then, might be to define Φ as the average or median travel cost of the used alternatives, but there are surely many possibilities that might be explored. In our case, we focus on two example possibilities (without wishing to rule out others), each seemingly having its own attractive features.

The two particular examples are the RSUE(min) model, obtained by defining for any non-empty set *B*:

$$\Phi(B) = \min\{b : b \in B\} \tag{5}$$

and the RSUE(max) model, obtained by defining:

$$\Phi(B) = \max\{b : b \in B\} \ . \tag{6}$$

An attractive property of the RSUE(min) model is that it leads in the direction of a computationally tractable method: a candidate flow pattern can easily be verified using some standard shortest path algorithm (for each OD movement) to identify the minimum cost path of any kind on the network. If the cost on this is (strictly) less than the cost on the currently minimum cost used route (for the corresponding OD-pair), then condition (4) is not satisfied. However, the RSUE(min) has a disadvantage in that it allows for traffic to be assigned to paths with actual travel costs greater than the actual travel costs of unused paths. From a behavioural point of view, one might question the plausibility of this, and in this respect the RSUE(max) model has an advantage.

The RSUE(max) model requires that no path is unutilised if it has an actual travel cost that is lower than or equal to the actual travel cost on the longest utilised path. While this seems behaviourally more defensible, it may lead to a less tractable computational model. Certainly, condition (4) is more difficult to verify from a computational perspective for the RSUE(max) model than it is for RSUE(min), yet still there are standard network analysis tools for doing so. In particular, given some path flow allocation and the resulting network link costs, a standard tool can be used (for each OD movement) to identify the current *k* shortest paths (where *k* is the number of used paths). If there among these exists any currently unused path on which the cost is (strictly) less than the cost on currently maximum cost used route (for the corresponding OD movement), then condition (4) is violated. Clearly, the computational effort involved in solving *k*-shortest path problems and identifying any unused paths among these is significantly greater than that required for solving standard shortest path problems, and so verifying that the RSUE(max) conditions are satisfied is much more demanding than the verification of the RSUE(min) conditions.

Proposition 1

Any RSUE(max) solution is also a RSUE(min) solution. An RSUE(min) solution may not, however, necessarily fulfil the RSUE(max) conditions.

Proof

Suppose a flow allocation satisfies the RSUE(max) conditions. Then from condition (4) when Φ is the max operator, any unused path must have a travel cost greater than or equal to the maximum cost used path. By definition, the maximum cost used path must have cost at least as great as the minimum cost used path, and so condition (4) is also satisfied when instead Φ is the min operator. Condition (4) of RSUE(max) is the same regardless of the choice of Φ, and so we have shown that the flow allocation must also satisfy the RSUE(min) conditions. For the converse situation, suppose that a flow allocation satisfies the RSUE(min) conditions, and in addition has an unused path which has a cost less than the maximum cost of any used path. Then the RSUE(max) conditions are violated as illustrated in the following Example 1. □

Example 1

In this example we explore the multiplicity of solutions in a simple example in which we can exhaustively check the conditions for all non-empty subsets $\tilde{R}_m$ of the universal choice set R_m. We illustrate that RSUE solutions do indeed exist with an equilibrated but non-universal choice set and that RSUE(min) solutions may violate the RSUE(max) conditions.

Consider a network serving an OD demand of d_1=100 and consisting of three parallel links/paths, with link cost functions $t_1(\mathbf{f}) = 8 + f_1/10; \quad t_2(\mathbf{f}) = 13 + f_2/15; \quad t_3(\mathbf{f}) = 15 + f_3/50$,

Suppose that the choice model for used routes is a multinomial logit model with θ = 1. With such a small network, it is possible to identify all 7 possible choice sets, and for each choice set to find an SUE solution by some traditional path-based solution method. We may then subsequently check each of these 7 possibilities with respect to the RSUE conditions. Clearly such an exhaustive search of possible choice sets would be infeasible for large-scale networks, but this example allows investigating the existence and multiplicity of RSUE solutions. The solutions are shown in Table 1.

Table 1 SUE solutions for all seven possible choice sets.

	Choice set (included paths)						
	{1,2,3}	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}
	cost/flow	cost/flow	cost/flow	cost/flow	cost/flow	cost/flow	cost/flow
Path 1	13.9/59.1	18.0/100.0	8.0/0	8.0/0	14.6/66.0	14.9/68.5	8.0/0
Path 2	14.7/26.0	13.0/0	19.7/100.0	13.0/0	15.3/34.0	13.0/0	16.2/47.4
Path 3	15.5/14.8	15.0/0	15.0/0	17.0/100.0	15.0/0	15.6/31.5	16.1/52.6
RSUE(min)	YES (=SUE)	NO	NO	NO	YES	NO	NO
RSUE(max)	YES (=SUE)	NO	NO	NO	NO	NO	NO

For all cases, SUE has been found among utilised paths. This means that the first condition (3) of the RSUE(min) as well as the RSUE(max) definition is fulfilled in all cases. The second condition is fulfilled if the *actual* travel cost of paths not in the choice set is not shorter than the *actual* travel cost on the shortest (longest) utilised path for the RSUE(min) (RSUE(max)). Note that this is always fulfilled in the case where all paths are in the choice set, and the traditional SUE will always be a RSUE solution. From Table 1 we see that there exist unused paths which are shorter than the shortest (longest) used path for the choice sets {1}, {2}, {3}, {1,3} and {2,3} and these do thus not fulfil the second RSUE(min) (RSUE(max)) condition and do therefore not constitute RSUE(min) (nor RSUE(max)) solutions. The violation of the RSUE(max) conditions could have also been realised by using Proposition 1 and the knowledge that the RSUE(min) conditions are violated.

However, since $\min\left\{c_{1s}(\mathbf{x}): s \in \tilde{R}_1\right\} = \min(14.6, 15.3) = 14.6$ for the configuration with paths 1 and 2 used (i.e. $\tilde{R}_1$={1,2}), and since 15.0≥14.6 then the second RSUE(min) condition is fulfilled, consequently giving a RSUE(min) solution. Assuming instead a max operator for Φ, then the second condition (4) requires that any unused paths have cost at least as great as the *maximum* cost of a used path (= 15.3 in this case), and since 15.0 < 15.3 the flow solution where paths 1 and 2 are used is not a RSUE(max) solution.

From this example we can see that RSUE solutions exist with equilibrated but non-universal choice sets, and that solutions that satisfy RSUE(min) may not satisfy RSUE(max) for a given problem. In the example we did not find any RSUE(max) solutions using a non-universal choice set. We could however imagine such a solution by retaining the flow allocation in Table 1 for the choice set {1,2,3} and adding a fourth non-overlapping route with free-flow travel cost of e.g. 20.

Solution methods

The RSUE seems straightforward to apply, as an extension of existing, calibrated SUE models, especially if supplemented with some information on routes actually chosen to aid in the determination of the Φ operator. We propose a new class of path-based solution algorithms to solve the RSUE, which allows a flexible specification of how the choice sets are 'systematically' grown by considering congestion effects and how the flows are allocated among routes (Rasmussen et al., 2015).

<table>
<tr><th>Algorithm</th><th colspan="3"></th></tr>
<tr><td>Step 0</td><td colspan="3">Initialisation. Iteration n=1. Perform deterministic all-or-nothing assignment for all $m \in M$ OD-pairs and obtain the flow vector for all utilised paths $\mathbf{X}_n$.
Perform network loading, compute link travel costs $t_a\left(\mathbf{f}_n\right)$ on all network links $a \in A$, and compute generalised path travel costs $c_{mr}(\mathbf{X}_n)$. Set n=2.</td></tr>
<tr><td rowspan="2">Step 1</td><td colspan="3">Column generation phase. Let $k_{m,n-1}$ denote the current number of distinct paths in the choice set of used paths for OD-pair m=1...M in iteration n-1.</td></tr>
<tr><td>For RSUE(min):
For each origin, perform a shortest path search to all destinations based on actual link travel costs $t_a(\mathbf{f}_{n-1})$. If for any OD-pair m=1...M a new distinct path i is generated, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$ =0.</td><td>For RSUE(Φ):
For each OD-pair mϵM, based on actual link travel costs $t_a(\mathbf{f}_{n-1})$, check for a new route to add to the choice set $\tilde{R}_{m,n}$ by applying some path generation method which supports the fulfilment of the Φ operator. If for any OD- pair m=1...M a new distinct path i is generated, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$ =0; if several routes possible, add only the shortest one.</td><td>For RSUE(max):
Perform a $k_{m,n-1}$-shortest path search for each OD-pair (m=1...M) based on actual link travel costs $t_a(\mathbf{f}_{n-1})$.
If for any OD-pair m=1...M a new distinct path i is generated among the $k_{m,n-1}$ generated paths, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$ =0; if several new distinct paths are generated, add only the shortest one.</td></tr>
<tr><td>Step 2</td><td colspan="3">Restricted master problem phase. Given the choice sets $\tilde{R}_{m,n}$ for all m=1...M, apply the selected inner assignment component and averaging scheme to find the new flow solution $\mathbf{X}_n$.</td></tr>
<tr><td>Step 3</td><td colspan="3">Network loading phase. Perform the network loading to obtain $\mathbf{f}_n$ from $\mathbf{X}_n$, compute the link travel costs $t_a(\mathbf{f}_n)$, the generalised path travel costs $\mathbf{C}(\mathbf{X}_n)$ and (if relevant/included) the Path Size factors.</td></tr>
</table>

The *Step 2* allow the use of a wide range of flow allocation methods, here among SUE methods designed for pre-defined choice sets. We also propose a cost transformation function and show that by using this we can, for certain Logit-type choice models, modify existing path-based DUE solution methods to fit the RSUE solution algorithm. The function transforms the original flow dependent cost $c_{mr}(\mathbf{x})$ into an equivalent cost measure $\tilde{c}_{mr}(\mathbf{x})$, which is used in the equilibration of the DUE solution algorithms:

$$\tilde{c}_{mr}(\mathbf{x}) = x_{mr} \cdot \exp\left(\theta \cdot c_{mr}(\mathbf{x})\right) \tag{7}$$

where θ corresponds to the scale parameter used in the logit-formula.

The transformation function also leads to the proposal of a new convergence measure that is applicable to *any* RSUE solution algorithm for logit-type choice models:

$$\text{Rel.gap}_n^{Used} = \frac{\sum_{\forall m \in M} \sum_{r \in \tilde{R}_m} x_{mr,n} \cdot \left(\tilde{c}_{mr}(\mathbf{x}_n) - \tilde{c}_{m,\min}(\mathbf{x}_n)\right)}{\sum_{\forall m \in M} \sum_{r \in \tilde{R}_m} x_{mr,n} \cdot \tilde{c}_{mr}(\mathbf{x}_n)} \tag{8}$$

where $\tilde{c}_{m,\min}(\mathbf{x}_n)$ refers to the minimum transformed cost on routes utilised for OD-pair *m*. This measure is supported by an additional measure capturing how close the costs on the unused route are to fulfilling the second RSUE condition. For the RSUE(min):

$$Rel.gap_n^{Unused} = \frac{\sum_{\forall m \in M} d_m \cdot \left(\min_{\forall r \in R_m, x_{mr}>0}\left(c_{mr}(\mathbf{x}_n)\right) - \min_{\forall r \in R_m}\left(c_{mr}(\mathbf{x}_n)\right)\right)}{\sum_{\forall m \in M} d_m \cdot \min_{\forall r \in R_m, x_{mr}>0}\left(c_{mr}(\mathbf{x}_n)\right)} \tag{9}$$

Numerical tests

We have performed numerical tests on the Sioux Falls network as well as the large-scale Zealand network (5.4 million daily trips), in which we explore convergence patterns and choice set composition and size, for alternative specifications of the RSUE solution algorithm.

The Sioux Falls application compared several different approaches to allocate flow between routes, and found that the algorithms, in general, reliably to an RSUE solution within a few iterations. The algorithms were analysed based on the number of iterations they required, but also importantly (given their different per-iterations requirements) on computation time. It was seen that the strategy for step-size determination highly influences the convergence speed, as would be anticipated.

Two promising algorithms were tested for the RSUE(min) on the Zealand network using a multinomial logit choice model: one utilising the cost transformation function to apply a modified DUE algorithm based on pairwise path swapping proposed by Carey and Ge (2012) (Path Swap); one utilising the closed-form choice probabilities directly to find the auxiliary solution (Inner Logit). The algorithms have been implemented in the Traffic Analyst traffic assignment module for ArcGIS (Rapidis, 2013), and both accommodated the use of the MNL and the PSL models. In the application we specified θ=0.2 and the step-size constant *d*=2 (an initial test comparing *d*=0 to *d*=2 found best performance in terms of convergence speed when *d*=2). The generalised travel costs were constituted by a weighted sum of free flow travel time, congested travel time, travel distance, and travel (monetary) cost. Figure 11 illustrates the convergence pattern as a function of the iteration number for the MNL choice model. Both algorithms converged extremely fast to fulfil the underlying RSUE conditions and were efficient in generating the choice sets within the first few iterations (Figure 1).

The converged solution generated was not the same for the two algorithms, as the composition of the choice sets varied between them (Table 2). In both cases the equilibrated non-universal choice sets were however reasonable and computationally attractive in size. A high share of the OD-pairs only contain one or two paths, and an average choice set size of 2.5-3 routes may seem small. However, this should be seen in light of the network composition; the case-study area includes, in addition to urban areas, large rural areas in which there is no congestion and only one or two relevant alternatives. An analysis of the spatial distribution of the choice set size showed that the choice sets generated for trips conducted in rural areas were considerably smaller than those generated for urban trips.

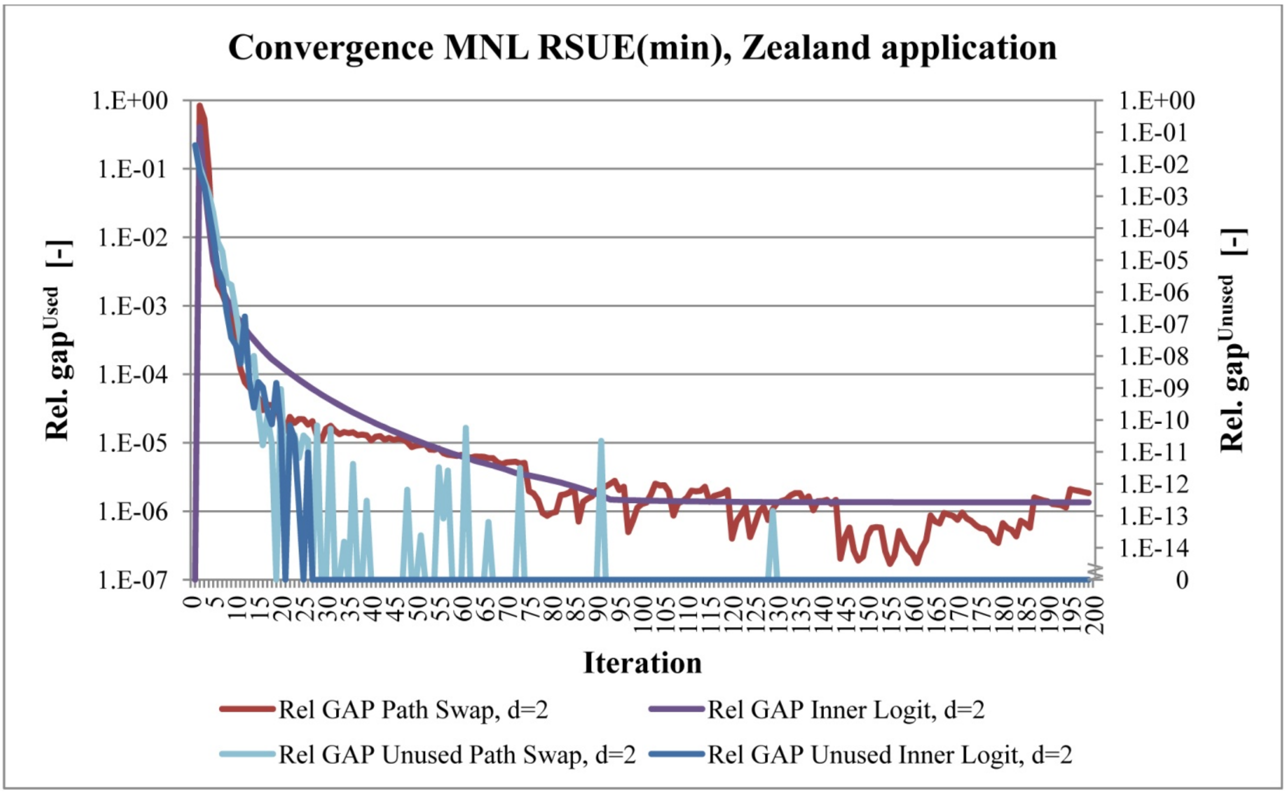

Figure 1. Relative gap measures, Zealand network application using the method of successive weighted averages with step-size parameter *d*=2 (MSWA, Liu et al., 2009). Notice the log-scale on the vertical axis.

Table 2. Average choice set size and distribution of number of paths in the choice sets, Zealand application.

		Choice set size										
	Avg. size	1	2	3	4	5	6	7	8	9	10	11
Inner Logit d=2	2.45	479,286	452,449	357,576	200,260	92,046	30,175	8,019	1,131	256	0	2
Path Swap d=2	2.71	444,808	396,939	327,525	222,882	132,667	68,354	21,366	5,502	749	400	8

The above analysis has focused on the multinomial logit (MNL) model, but we also tested the path-size logit (PSL) choice model and found similar results.

In the PSL application we tested different values of β_{PS} (ranging from –25 to 0) and evaluated the results by comparing the link flows obtained with corresponding real life observed link flow counts (for 1169 links distributed across the network). The evaluation was done using the coefficient of determination (R^2) obtained from a linear regression of the modelled flows as a function of the observed flows (using the Path Swap algorithm). In general very high correspondence was found, with R^2=0.9444 when β_{PS} =0 (MNL case), with slightly declining R^2 with increasing negative value of β_{PS} until R^2=0.9404 when β_{PS} =-25.

Last, we performed a qualitative, disaggregate evaluation of the choice set composition and flow distribution for one OD-pair within the study area. Both algorithms generated the same five distinct routes shown in Figure 2 for the MNL as well as the PSL choice models. The trip was a commuting trip and the size and composition of the choice sets seems plausible from our local knowledge of the network; one alternative (Path 3) used motorways as far as possible, one alternative avoided motorways but rather used uncongested minor local roads (Path 2) and three alternatives were versions of the lowest cost route using a combination of motorways and minor local roads.

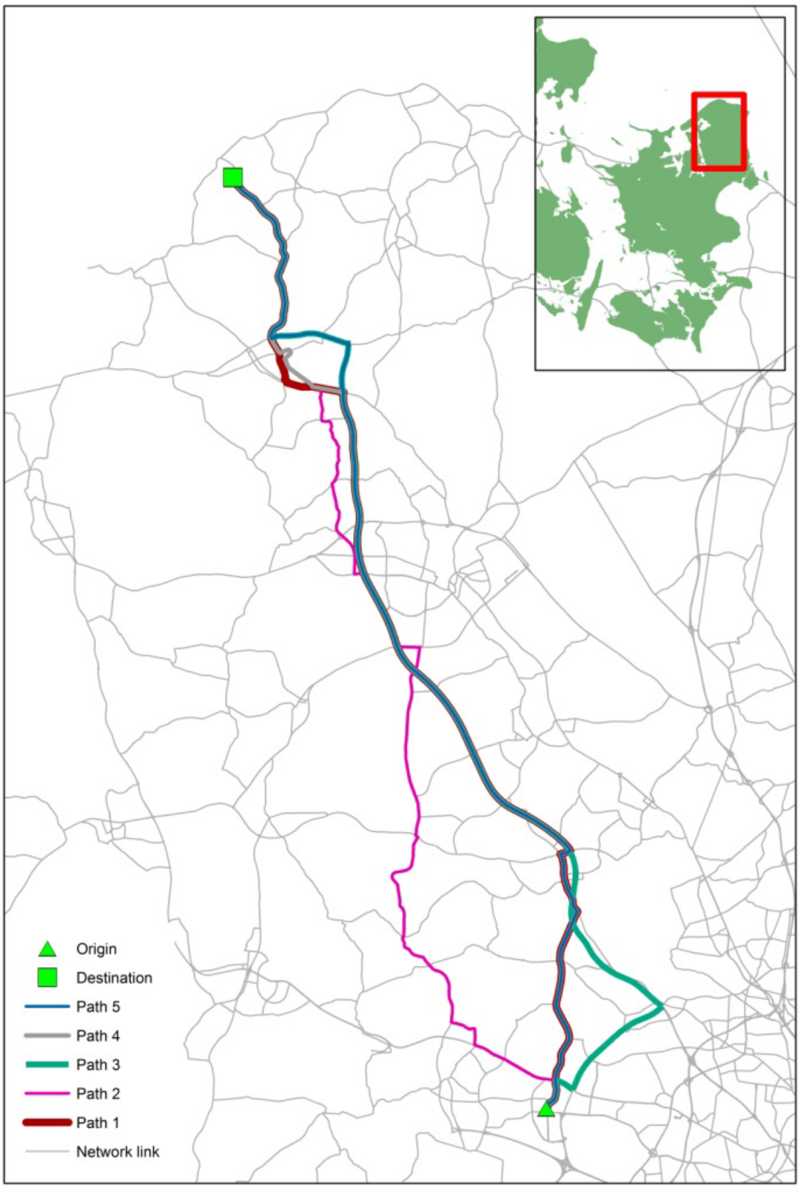

Figure 2. Illustration of generated choice set, 1 OD relation Zealand application.

The generalised route costs and flow shares for the MNL and the PSL choice models can be seen in Table 3 (results from two different β_{PS} values reported).

Table 3. Generalised costs and flow distribution for various choice models, 1 OD relation Zealand application.

	MNL		PSL β_{PS} =-3			PSL β_{PS} =-8.5		
PathID	Gen. Cost [-]	Flow share [%]	Gen. Cost [-]	PS term [-]	Flow share [%]	Gen. Cost [-]	PS term [-]	Flow share [%]
1	126.16	22.7	126.07	0.30	21.0	125.90	0.30	16.2
2	130.40	9.7	130.49	0.78	15.2	130.65	0.78	30.9
3	129.19	12.4	129.18	0.48	14.9	129.18	0.48	18.4
4	125.28	27.1	125.20	0.28	23.7	125.05	0.28	16.6
5	125.08	28.1	125.00	0.29	25.1	124.87	0.29	18.0

In the MNL case, the three routes with the lowest generalised costs (paths 1,4 and 5) attracted 78% of the traffic, whereas path 2 (which has almost no overlap with other used paths) only attracted 9.7% of the flow, despite being only 4% more expensive than the cheapest path. In the PSL case, accounting for path overlap changed the distribution of flow between the path as well as the path costs (through redistribution of flows for all OD-pairs). While paths 1, 4 and 5 highly overlap, path 2 is the most distinct path, and thus was the one that attracted flow from the other paths when the PSL results are compared to the MNL case. The share on path 2 ranged from 15.2% when β_{PS} =-3 to as much as 30.9% when β_{PS} =-8.5. This highlights the need for aggregate as well as disaggregate analysis when evaluating the models; while on an aggregate level it was difficult to choose between the models (and in fact the MNL performed a little better in reproducing link counts), accounting for path overlapping (by choice of a suitably-calibrated β_{PS} value) can have a major influence on the distribution of flow among paths. Such a disaggregate-level calibration would, however, require more informative data than link counts alone.

Conclusions

The paper shows how we might overcome the limitations of the DUE and SUE by *consistently integrating* the problem of distinguishing used and unused paths within the concept of SUE. We set out the RSUE(Φ) model as a new methodological approach to address this problem. This model defines not only an equilibrated flow solution but also an equilibrated choice set in which the equilibrium conditions (and not the solution algorithm adopted) specify that some available routes could be unused at perfect equilibrium. The potential benefits of such an approach are greatest, it would seem, in large-scale regional and trans-national studies, meaning that we no longer have the choice only between DUE (which will tend to assign all-or-nothing at non-congested parts of such networks) and SUE (which can be computationally demanding and rather implausible, in attempting to assign some traffic to all routes).

The paper also addresses the issue of applying the RSUE(Φ) to large-scale cases. A generic solution algorithm is proposed, and applications have demonstrated the applicability, convergence and scalability of different variants of the RSUE(min) and RSUE(max) solution algorithms on the well-known Sioux Falls network as well as the large-scale Zealand network. The results are very promising as the algorithms converges extremely fast to fulfil the underlying RSUE conditions and are efficient in generating the choice sets within the first few iterations.

References

Cantarella, G.E., 1997. A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand. *Transportation Science*, 31 (2), 107-128.

Carey, M., Ge, Y.E., 2012. Comparison of methods for path flow reassignment for Dynamic User Equilibrium. *Networks and Spatial Economics*, 12 (3), 337-376.

Daganzo, C.F., Sheffi, Y., 1977. On stochastic models of traffic assignment. *Transportation Science*, 11, 351-372.

Damberg, O., Lundgren, J.T., Patriksson, M., 1996. An algorithm for the stochastic user equilibrium problem. *Transportation Research Part B: Methodological*, 30 (2), 115-131.

Liu, H. X., He, X., He, B., 2009. Method of Successive Weighted Averages (MSWA) and Self Regulated Averaging Schemes for Solving Stochastic User Equilibrium Problem. *Networks and Spatial Economics*, 9 (4), 485-503.

Rapidis, 2013. A transportation Planning Extension for ArcGIS. *Version 3.6.0Beta*. Accessed Juli 1, 2013 at the web address http://www.rapidis.com/products/traffic-analyst/.

Rasmussen, T.K., Watling, D.P., Prato, C.G., Nielsen, O.A., in press. Stochastic User Equilibrium with equilibrated choice sets: Part II – Solving the Restricted SUE for the logit family. *Transportation Research Part B: Methodological*, doi: 10.1016/j.trb.2015.03.009.

Sheffi, Y., 1985. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice-Hall, Englewood Cliffs, NJ.

Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. *Proceedings of Institution of Civil Engineers*, Part II 1, 325–378.

Watling, D.P., Rasmussen, T.K., Prato, C.G., Nielsen, O.A., in press. Stochastic User Equilibrium with equilibrated choice sets: Part I – Model formulations under alternative distributions and restrictions. *Transportation Research Part B: Methodological*, doi: 10.1016/j.trb.2015.03.008.