Adaptive Max Pressure Control of Network of Signalized Intersections

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A network of signalized intersections is modeled as a store-and-forward network of controlled queues, with one queue per movement or phase. At any time, a control policy operates a *stage* – a set of simultaneous movements – for a duration of time. We study an adaptive version of the max pressure control policy described in Varaiya (2013) using the '.Q' discrete-event simulator introduced in Lioris et al. (2014). The study network of Fig. 1 has nodes or intersections like 37612 at the top and directed links entering and leaving the nodes. A queue is associated with each incoming and outgoing link pair: thus $q(741,737)$ is the number of vehicles at this intersection on link 741 waiting to make the turn $(741 \to 737)$. When the stage that includes this turn is operated, vehicles will leave this queue at a rate given by the saturation flow rate $C(741,737)$, travel along link 737 for a specified travel time, arrive at the downstream intersection, namely 37610, and join a queue for one of the four outgoing links, 733, 738, 2351, or 20471, with a specified turn probability.

Vehicles enter the network at each link l in a Poisson stream with specified demand rate d_l. Let $R = \{R(l,m)\}$, with $R(l,m)$ equal to the probability that a vehicle on link l turns into link m. The vector $f = \{f_l\}$ of average link flows satisfies the conservation equation $f = R'f + d$ (d is the vector of demand rates); hence $f = [I - R']^{-1}d$ (R' is the transpose of R), and $R(l,m)f_l$ is the average rate of turns from link l to m. If a control policy can support demand d, it must operate movement (l,m) at rate at least $R(l,m)f_l$. If this condition holds, the queues are stable, i.e. (**E** denotes expectation)

$$\sup_T T^{-1} \sum_{l,m} \sum_{t=1}^{T} \mathbf{E} q(l,m)(t) < \infty. \tag{1}$$

At each intersection and at each time a control operates a stage. So a control is represented by a matrix U with $U(l,m) = 1$ or 0, accordingly as turn (l,m) is or is not operated. Let $C = \{C(l,m) \geq 0\}$ be the matrix of saturation flow rates (in, say, vehicles/hour). Then $\{C(l,m)U(l,m)\}$ is the matrix of service rates at which queues at this intersection are served when control U is invoked. The control for the entire network at any given time consists in selecting for each intersection n a control $U(n)$ from the finite set of controls $\mathcal{U}(n)$.

The max pressure or MP control works as follows. Time is divided into cycles (typically 60-120s) and each cycle is divided into say 2,4, ... , 10 periods. Suppose at intersection n the queue measurements are $q(t) = \{q(l,m)(t)\}$. Compute the 'pressure' $\pi(U, q(t))$ exerted by each control $U \in \mathcal{U}(n)$:

$$\pi(U,q(t)) = \sum_{l,m} C(l,m)U(l,m)W(l,m) \tag{2}$$

$$W(l,m) = q(l,m) - \sum_{p} R(m,p)q(m,p)]. \tag{3}$$

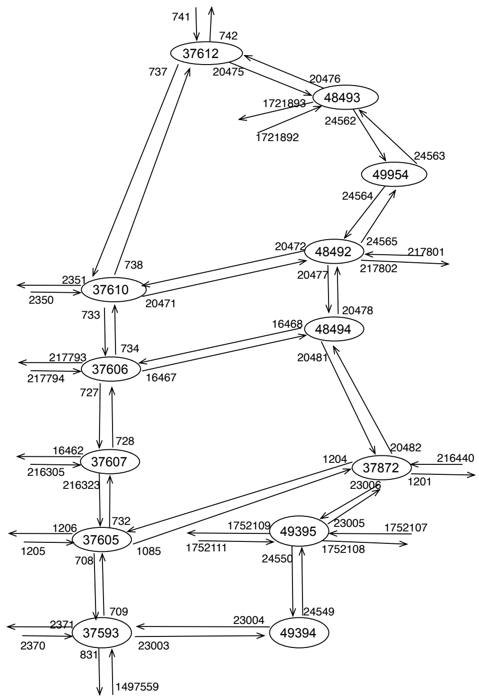

Figure 1: Study network near I-15 San Diego, CA

The MP policy selects the control with maximum pressure at state $q(t)$,

$$U^*(q(t)) = \arg\max\{\pi(U, q(t)) \mid U \in \mathcal{U}(n)\}. \tag{4}$$

$W(l,m)$ is the upstream queue minus the (average) downstream queue for movement (l,m); it is called the weight of the phase (l,m). So MP selects the stage that maximizes the instantaneous rate at which W decreases. The stage U^* is operated until the end of the period, when the queue is measured again and a new MP is computed. Observe that U^* does not depend on the demand rate d, but it does depend on the turn probabilities R.

Varaiya (2013) shows that if the turn ratios used in calculating the pressure are correct, MP will achieve stability (1), provided there exists a stabilizing control. Moreover, the average queue size is proportional to $1/K$ where K is the number of times per cycle that a fresh MP decision is made. Thus, by increasing K one can keep the queue lengths arbitrarily short. Unfortunately, this attractive result is impractical because each time a stage is switched one must allow 3-4s of 'lost time' (called red clearance) for reasons of safety. Thus for example with a 60s cycle time, 10 stage switches per cycle will give a lost time of 30-40s, so the intersection can be effectively used only for 20s. By contrast a fixed-time or FT control limits the number of switched to (say) 4, with a lost time of 12-16s.

We present three modifications of MP that make it practical without any loss in performance. Result 1 was presented in Lioris et al. (2014).

Result 1 We present a practical modification of MP called MP-pract that surprisingly keeps the number of switches to be 4 (comparable with FT) while achieving a queue size comparable to MP with 10 switches per cycle. MP-pract computes (4) 10 times each cycle (as in MP) but implements the new decision only if it significantly increases the pressure:

$$\max_U \pi(U,q(t)) \geq (1+\eta)\pi(U^*,q(t)). \tag{5}$$

Here U^* is the previously selected MP stage, and the threshold $\eta > 0$ is chosen to prevent excessive stage switching. In the results below, $\eta = 1.2$. The box plots in Fig. 2 show that using MP-pract instead of MP does not noticeably increase the queue size. Moreover, for 10 decisions/cycle, the number of *evaluations* of (5) at intersection 37593 over 3 hours or 180 cycles is 2007, but the number of stage *switches* is only 764. Thus MP-pract works much better than an FT controller with four stages per cycle which makes 960 switches with average sum of queue lengths equal to 150, which is three times as large.

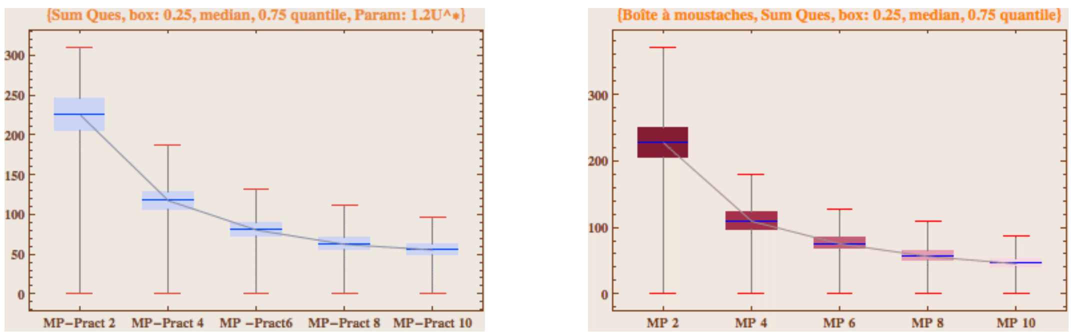

Figure 2: Box plot of sum of queue lengths for MP-pract (left) and MP with 2, ..., 10 decisions/cycle

Result 2 Varaiya (2013) also shows that if the true turn ratio $R(m,p)$ in the weight calculation (3) is replaced by its consistent estimate the resulting 'adaptive' MP, which we call MP-adap, will inherit the stabilizing property of MP. We study the same network as before. The demand volumes are the same but the routing probabilities change at time 0, 1500s, and 3,000s. Every 900s, MP-adap calculates $\rho(l,m) = \frac{D(l,m)}{\sum_k D(l,k)}$ – the proportion of vehicles that travel on link l and turn into link m. The previous estimate $\hat{R}(l,m)$ is then updated as ($0 < \lambda < 1$ is the forgetting factor)

$$\lambda \hat{R}(l,m) + (1-\lambda)\rho(l,m) \leftarrow \hat{R}(l,m).$$

Fig. 3 plots the sum of queue lengths for MP-adap on the left and an FT control on the right. The performance of the former is much better. One could also construct examples in which there is no FT control that can stabilize demands with the same volume but different turn ratios.

Result 3 In a linear arterial network, queues can be minimized by coordinating signals. For FT control, successive signals are offset by the link travel time so as to create a 'green wave'. As described above MP-pract control does not include such an offset. It is therefore surprising that by following the stage-switching rule (5), MP-pract automatically creates a green wave. This property is illustrated in the 15-node one-way arterial of Fig. 4.

The only demand is from vehicles arriving from the left into node 1 at the rate 0.7 v/s, all other arrival rates are 0, all saturation flow rates equal 1.5v/s. Consider first the FT control with 60s cycle time, and green duration of 30s for the left-right movement. The offset between successive nodes is 30s. In Fig. 5 the middle (right) plot is the sum of queue sizes divided by 30, the number of queues, when the travel time is 60s (45s).

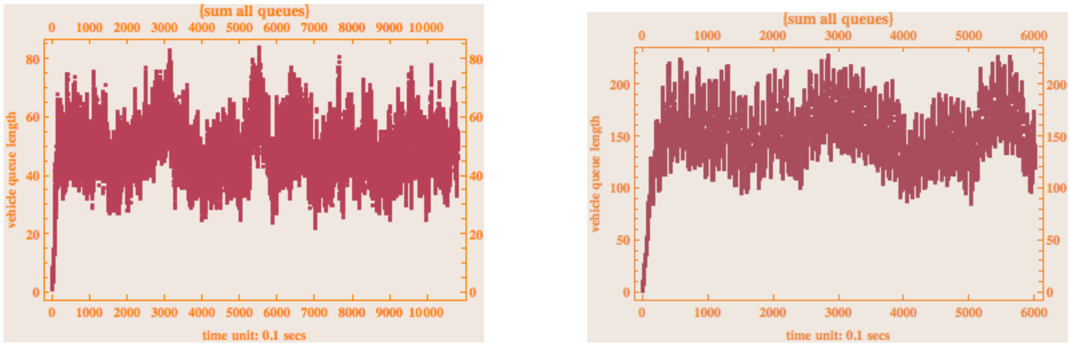

Figure 3: Sum of queue lengths for MP-adap (left) and for FT (right)

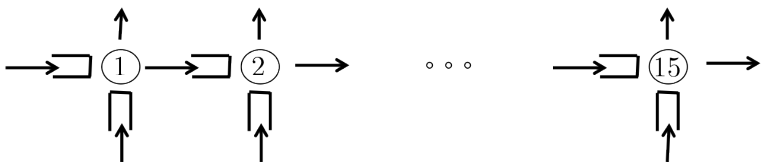

Figure 4: A long arterial.

In neither case is the offset ideal. The leftmost plot is for the ideal offset of 60s which is the travel time. Since the offset in the middle plot is further away from the ideal, it shows a larger travel time.

The upper plots of Fig. 6 display the average queue size for MP-pract with 10 decisions/cycle. As is evident and expected from Fig. 2, the queue size for MP-pract is about half of the queue size for FT. Much more intriguing are the lower plots which display the time intervals for which MP-pract at nodes 8-15 actuates the left movement (solid color). The pattern of intervals reveals a 'self-organizing' feature: the controllers determine an effective offset of 58s when the travel time is 60s and 47s when the travel time is 45s. Thus MP-pract adapts to correctly coordinate signals even though the control policy (3)-(4) involves *no* information at all about travel time. We can explain the pattern of actuation intervals in terms of the control law.

Note that at each decision time, the pressure of stage 1 (left-right movement) is compared with the pressure of stage 2 (bottom-top movement). The latter pressure is always 0 since those queues are empty. Thus at each node, MP will actuate stage 1 whenever its pressure is positive, which happens when and only when the upstream queue exceeds the downstream queue. Since node 15 has no downstream queue stage 1 will always be actuated. As we traverse nodes 10-14 it takes more and more time for their downstream queues to become sufficiently large so as to force a stage switch. The regular pattern of switches in nodes 8-14 is a consequence of the fact that MP tries to balance upstream and downstream queues. However, one needs further investigation before one can assert the existence of this 'self-organizing' feature.

Conclusion Modifications are proposed that make max pressure practical. Three results are described to show that the modifications are indeed practical with no performance loss.

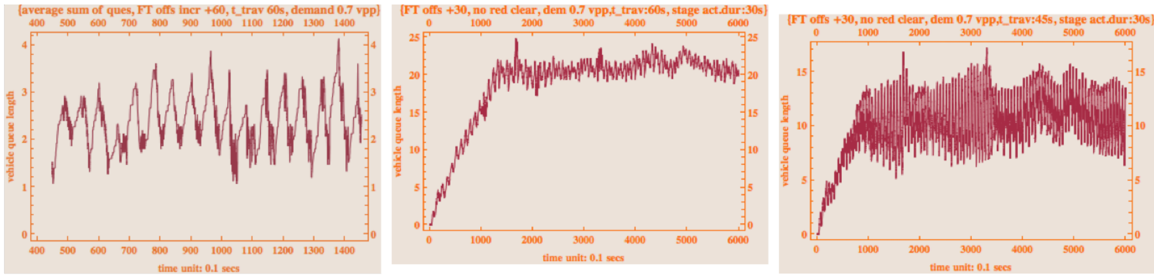

Figure 5: Average number of vehicles per queue with travel time of 60s (left and middle) and 45s (right) for FT.

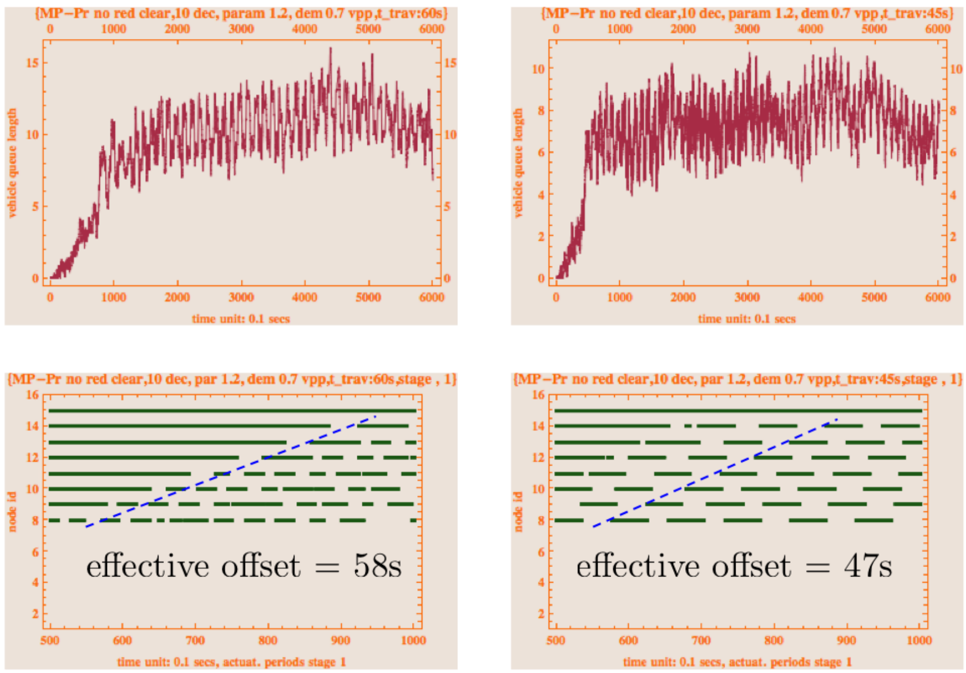

Figure 6: Stage actuations and average number of vehicles per queue with travel time of 60s (left) and 45s (right) for MP-pract.

References

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