Linking the Properties of Mean-Variance Relations in Travel Times with the Hysteresis in Macroscopic Fundamental Diagram

Extended Abstract for hEART, Leeds, U.K.

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Recently, there has been a growing interest in understanding travel time variability to make decisions such as route choice and departure time. Recent studies show that people value both travel time and its predictability to choose their paths. Regarding this aspect, travel time reliability, a performance indicator of roadways, has an important effect on route and departure time choice, especially for time constrained trips (e.g. commute to work, trip to airport). Understanding travel time variability is quite important to develop the concept of reliability. Although travel time estimation models have been widely studied in transportation field, the variation in the mean and standard deviation of travel times has been much less investigated. Day-to-day travel time variability addressing the travel time variations of vehicles crossing the same route at the same period of time on different days plays a key role in travel decisions of people and has to be carefully investigated to further improve current travel time reliability concepts.

A temporary decrease in capacity (e.g. congestion caused by an active bottleneck) leads to a quite significant difference in the standard deviation of travel time for congestion onset and dissipation periods. This phenomenon results in hysteresis loops where the periods in congestion offset exhibit a higher travel time variance than the ones in congestion onset with the same mean travel time. Figure 1 presents mean-variance relationship for the afternoon and morning peak periods along with the departure time periods given next to the corresponding points. In this study, we analyze freeway data from California Performance Measurement System (PeMS) and Greece (Attiki Odos). We compare reliability indices for different levels of congestion in these study areas.

The aim of this paper is to identify empirical reasons that cause the hysteresis phenomenon in freeway travel times. These reasons are twofold in nature; due to network properties (i.e. traffic dynamics) and due to demand patterns. Gayah et al. (*1*) suggests that hysteresis behavior in travel time data is linked to the hysteresis loops observed in macroscopic fundamental diagrams (MFD) in urban networks, and it is, therefore, a network property. On the other hand, Fosgerau (*2*) provides a theoretical proof for counter-clockwise loop using Vickrey bottleneck model with random service rate and assuming Nash equilibrium in arrival times. The equilibrium state in this model dictates a concave cumulative arrival function or an arrival rate which is decreasing. This implies a particular

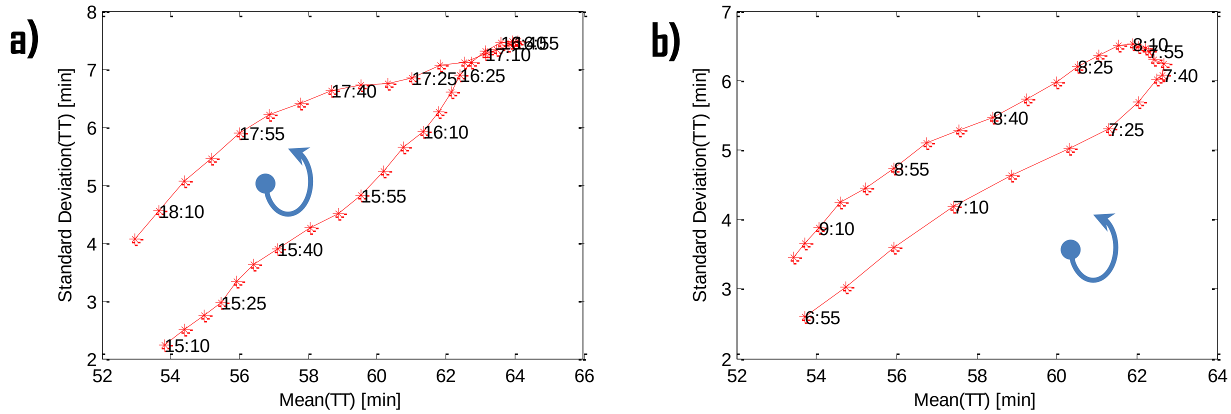

FIGURE 1: Mean-Standard Deviation Relation a. Afternoon Peak, b. Morning Peak

demand pattern in traffic networks and enables the theoretical proof of hysteresis loops in travel time data.

In this study, we investigate the above mentioned explanations using real freeway data and resulting traffic dynamics that result from these data. First of all, we analyze the effect of network traffic properties on the shape of mean-variance curve. We consider the hysteresis phenomenon which is very significant in freeway MFD, and investigate the impact of the existence or non-existence of the hysteresis phenomenon on mean-variance curves. Second of all, we analyze the demand patterns during the network loading and unloading phases. The significant difference in the standard deviation of travel time corresponding to the same mean travel time in the loading and unloading periods can be hypothetically explained by different demand patterns in these periods.

Geroliminis and Sun (*3*) investigates the causes of the hysteresis in macroscopic fundamental diagram (MFD), where higher network flows are observed for the same network density in the onset and lower in the offset of the congestion. The explanation of the hysteresis phenomenon in the network level has two parts; the first reason is that distributions of individual occupancy measurements (congestion distribution) are different for the same level of network density. Another reason is the synchronized occurrence of hysteresis phenomenon at the individual detectors. This study investigates the potential link between the hysteresis observed at the network level (across space) on MFD and in the mean-variance relation of travel times (across time). Figure 2 compares mean-variance relations in freeway travel times resulting from different models. First, instantaneous travel times on a 20 km section of Attiki Odos freeway are calculated using the speed measurements at loop detectors. The resulting travel times are used to calculate mean and standard deviation of travel times across days, which exploit counter-clockwise loops in both morning and afternoon peaks (see Figure 2). On the other hand, the average speed in the network can be calculated using the relation between average density and average flow observed in MFD; $v = q/k$. Average unit travel time in the network can be calculated by simply taking the reverse of the average speed, and the unit travel time can be converted to the section travel time by multiplying it with the section length. MFD relationship between average flow and average density is defined in two different ways; with the empirical data collected from the detectors and with a parametric equation fitted on the empirical data. The former approach does not assign a functional relation between flow and density, and it includes the hysteresis effect described above. The latter fits a polynomial function on the empirical data, and avoids the clockwise loop observed in MFD. In the first approach, travel time is calculated using the actual average flow and density values in the freeway, while the second approach employs actual density measurements and connects them with the average flow values on the function. Of course, the second approach is not exactly consistent with traffic flow dynamics, but the purpose of this analysis is only to investigate the hysteresis phenomena observed in MFD and travel time data. Figure 2 presents the curves that result from these three models. Although they are not identical, all three travel time models indicate a counter-clockwise hysteresis loop, which implies that hysteresis on MFD is not the only reason of hysteresis in travel times. This result encourages us to further investigate the issue by employing dynamic MFD equations.

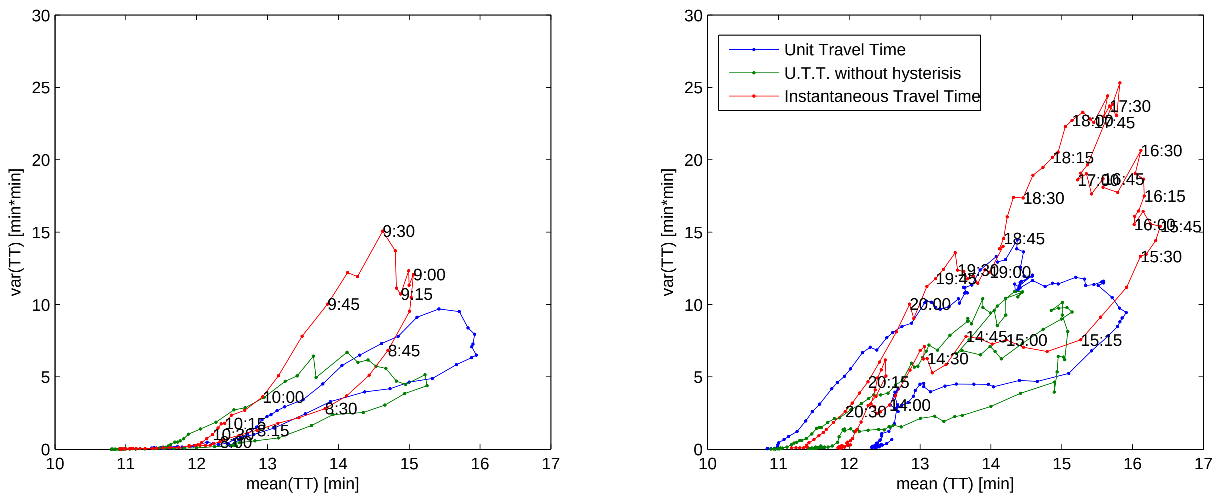

FIGURE 2: Mean - Variance Relations with Different Models

Dynamic equation of the state variable or the accumulation, n, for a single reservoir system:

$$\frac{dn}{dt} = I'(t) - O(n(t)) \tag{1}$$

where $I'(t)$ is the exogenous demand or arrival flow and $O(n(t))$ is the outflow which is calculated with the MFD function and corresponds to the accumulation $n(t)$. In order to produce consistent traffic parameters, dynamic equation, along with the arrival flow taken from the detector measurements, is applied throughout the study period. Note that the arrival flow, in this case, is the sum of entering flow from all the on-ramps and the upstream point of the freeway section in hand. Similarly, the outflow is the sum of exiting flow from the off-ramps and the downstream point of the freeway section. This analysis allows us to produce outflow values consistent with the traffic dynamics defined by MFD relation (without hysteresis), while the results from the parametric MFD presented in Figure 2 are based on the actual density measurements and fictitious flows that correspond to them on the MFD function. This analysis is repeated for different days with arrival flow values that correspond to them. Resulting day-to-day time dependent accumulation values are converted to the average unit travel time, and mean-variance values of the section travel times are calculated. Figure 3 presents the mean-variance curve and the counter-clockwise loop that result from this model. The existence of hysteresis simply proves that clockwise loop on MFD is not the only reason of the counter-clockwise loop in mean-variance relation on travel times. However, its existence might still affect the size and the shape of the hysteresis on travel times.

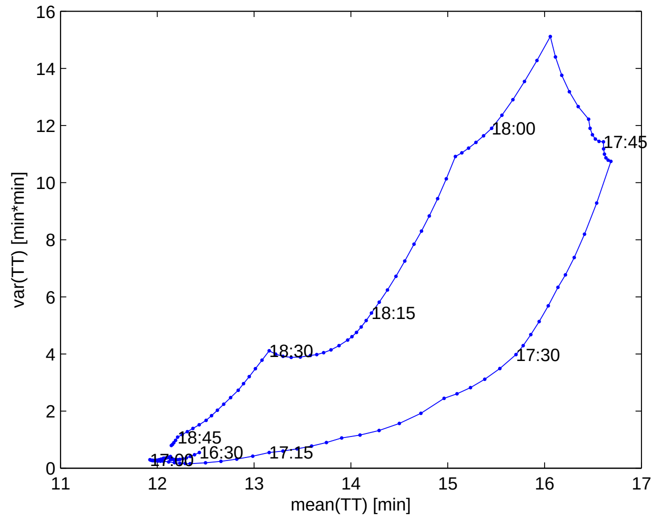

FIGURE 3: Mean - Variance Relations with Dynamic Equations

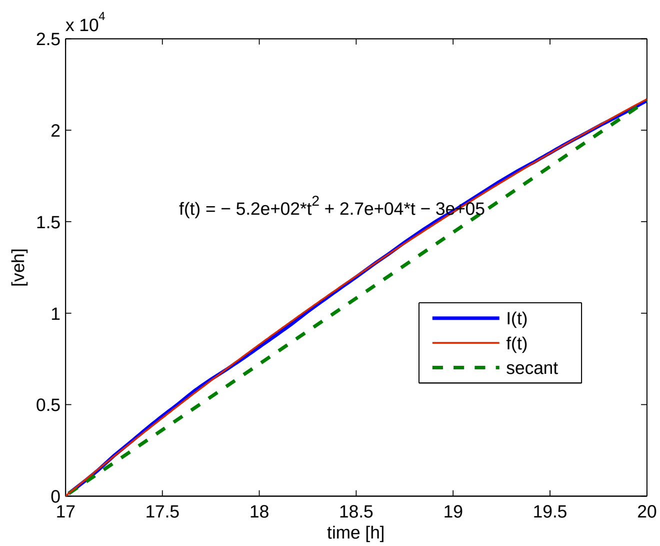

FIGURE 4: Cumulative arrival flow during a congested period

The second part of this study investigates the demand pattern assumptions and empirical implications of Vickrey bottleneck model and equilibrium state which allows Fosgerau (*2*) to provide the theoretical proof of counter clockwise loops. We investigate the assumption that the cumulative arrival flow during the congested period is a concave function. Figure 4 presents an empirical cumulative arrival flow curve $I(t)$, a second-degree polynomial function fitted on the curve

$f(t)$, and the secant line that connects the first and last data points. The function $f(t)$ constitutes an accurate statistical model considering the goodness of fit results, and it is a concave function, as $f''(t) = -5.2 < 0$. Note that this analysis is not complete and it does not prove the existence of such conditions in general. Ongoing work attempts to investigate new empirical aspects that result from Vickrey equilibrium assumptions. It is obvious that the demand pattern which is an implication of Vickrey bottleneck model can affect the change in accumulation, and this might influence the variance level in congestion onset and offset periods. These issues will be further discussed in the full paper.

REFERENCES

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