On the scale parameter in random regret minimization models

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Motivation

Random regret minimization (RRM) models have since their recent introduction (Chorus, 2010) been used to explain and predict a wide variety of transport-related choices. Examples of choice types that have been analysed using the RRM model include departure time, route, mode and destination choices, activity choices, and vehicle type purchases. Although much focus has been put on the empirical comparisons between RRM and linear-additive Random Utility Maximization (RUM) models, there has also been considerable progress in exploring the RRM-model’s properties (e.g. Hess et al., in press). However, until now no attention has been paid to the scale parameter in RRM models; in fact, in all empirical studies on RRM models that have been published to date the scale parameter has been normalised to one. The aim of this paper is to clarify the relationship between the scale parameter and the degree of semi-compensatory behaviour modelled in random regret minimization models. We show theoretically and illustrate empirically that the scale parameter is of fundamental importance in regret minimization models.

Methodological contributions

The methodological contributions of this paper are threefold. Firstly, we show how – in RRM models – the scale parameter, taste parameters, and the decision rule are related to one another. To see this, note first that in RRM models a large (small) scale parameter implies small (large) taste parameters – just like is the case for linear-additive RUM models. At first sight, this correspondence suggests that the scale parameter in RRM models can be treated the same way as in linear-additive RUM models. However, this is not the case. In contrast to a linear-additive utility function, the regret function is not a scale invariant function. More specifically, in RRM models a large scale parameter not only leads to small taste parameters, but also results in a model which generates only small asymmetries in the regret function. Vice versa: an RRM-model having a small scale parameter (i.e., close to zero) yields not only large taste parameters, but as a consequence also results in a model which generates very strong asymmetries in the regret function. In sum, in the context of RRM models, a different scale parameter implies a different choice model.

Secondly, and motivated by this observation, we propose a generalization of the RRM model: the $\mu$RRM model. This model has the scale parameter $\mu$ as an additional degree of freedom. As such, it accommodates for different degrees of regret minimization behaviour. We show that the $\mu$RRM model has three special cases: 1) if $\mu$ is very large, then the resulting model exhibits linear-in-parameters RUM behaviour (i.e., the function that maps attribute differences onto regret becomes linear); 2) if $\mu$ is insignificantly different from one, then the conventional RRM
model is obtained; and 3) if $\mu$ is arbitrarily close to zero, then the resulting model exhibits pure regret minimization behaviour. The pure regret minimization model postulates an very strong differential between performing better with regard to a specific attribute and performing worse on that attribute.

Thirdly, we present an attribute-specific ex post measure of the profundity of regret: $\alpha_k$. This measure (between zero and one) essentially measures the extent to which regret minimization has been imposed for a given attribute. Finding $\alpha_k$ equals one implies that pure regret minimization behaviour has been imposed with regard to that attribute, whereas finding $\alpha_k$ equals zero implies that no regret minimization behaviour has been imposed at all (i.e. RUM behaviour).

**Empirical contributions**

We re-analyse ten data sets that have been used in the literature to compare RRM and linear-additive RUM models. Firstly, we find very substantial improvements in model fit on four out of ten data sets when we use the proposed $\mu$RRM model. A related and important empirical finding is that the relatively minor differences in model fit between RRM and RUM that have previously been reported in the literature (e.g., Chorus et al., 2014) can be attributed to the fact that by assuming a scale of one, the profundity of regret imposed by the RRM model is typically limited. This is probably best illustrated by an example. Figure 1 shows the results of a re-analysis of the parking choice data used to compare RUM and RRM in Chorus (2010). On the left, a histogram is shown of attribute differences observed in the data (for just one attribute: number of parking spaces), on the right we show the associated regret function within that range (i.e. given the estimated taste parameter). As can be seen in the upper row, for the RRM model reported in Chorus (2010), the regret function is almost linear. As such, the profundity of regret that is imposed by the RRM model for this attribute is actually very limited. This is also indicated by the very low measure of profundity of regret: $\alpha = 0.04$. In the lower row, the same results are shown, but now for the $\mu$RRM model. We see that the regret function is now highly non-linear. This is in line with expectations given the very small scale parameter: $\mu = 0.03$. Furthermore, on these data, the $\mu$RRM model improves the model fit by over 24 log-likelihood points as compared to the RRM model (and by over 25 log-likelihood points as compared to the corresponding linear-additive RUM model).
Figure 1: Re-analysis of parking choice data, using the $\mu$RRM model

References

