Demand Based Timetabling of Passenger Railway Service

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Railway Planning

- STRATEGIC - several years
- TACTICAL - >= 1 year
- OPERATIONAL - < 1 year

- Demand → Line Planning → Lines → Train Timetabling → Actual Timetables → Train Platforming → Platform Assignments
- Train Timetabling → Actual Timetables → Rolling Stock Planning → Train Assignments
- Actual Timetables → Crew Planning → Crew Assignments

TOC
IM
Line Planning Problem

Railway Infrastructure

Passenger Demand

Potentional Lines

Model

Min Cost

Max Direct Pass.

Trade-Off
Train Timetabling Problem – Non-Cyclic
Train Timetabling Problem – Cyclic

Railway Infrastructure

Cycle

Actual Timetable(s)

Model

SAFETY FIRST

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Arising Issues

Figure: Outside peak hour

Figure: Train station in China

Figure: Inside peak hour
Railway Planning Improved

- **STRATEGIC** - several years
- **TACTICAL** - >= 1 year
- **OPERATIONAL** - < 1 year

**Flowchart**:
- Demand → Line Planning
- Lines → Ideal Train Timetabling
- Ideal Timetables → Train Timetabling
- Train Timetables → Actual Timetables
- Actual Timetables → Train Platforming
- Actual Timetables → Rolling Stock Planning
- Actual Timetables → Crew Planning
- Platform Assignments
- Train Assignments
- Crew Assignments

**Abbreviations**:
- TOC
- IM

**Logos**:
- TRANSP-OR
- EPFL
Agenda

1. Motivation
2. Ideal Train Timetabling Problem
3. Conclusions
4. Future Work
1 Motivation

2 Ideal Train Timetabling Problem
   - Assumptions
   - Inputs
   - Decision Variables
   - Objective
   - Constraints
   - Cyclicity
   - Connections

3 Conclusions

4 Future Work
Assumptions I

User Cost

Ideal Time

Time
Assumptions II

SOURCE $\sigma$

Geneva + 33'
Lausanne + 66'
Berne + 60'
Luzern

$x_t$

0

1440
Inputs

\( t \in T \) – set of time steps

\( l \in L \) – set of lines

\( f \) – fraction by which it is better to be early

\( d_t \) – demand captured along the line \( l \), when scheduling a train at time \( t \)

\( d_t^{ll'} \) – connection demand captured along the line \( l \) and \( l' \), when scheduling a train at time \( t \) on the line \( l \)

\( n^l \) – number of trains available for line \( l \)

\( h_i^l \) – relative headway to reach a connection point of lines \( l \) and \( l' \) from the first station on line \( l \) and \( l' \)

\( c^l \) – size of the cycle on line \( l \)

\( s \) – preferred start of the planning horizon

\( M \in \mathbb{M} \) – set of sufficiently large numbers
Primary Decision(s)

\[ x_t^l = \begin{cases} 
1 & \text{if a train on line } l \text{ is scheduled at time } t, \\
0 & \text{otherwise.} 
\end{cases} \]
Secondary Decisions I

- \( y_{t}^{lb} \in \mathbb{R}^{+} \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \), when taking a closest train at \( t \) or before

- \( y_{t}^{la} \in \mathbb{R}^{+} \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \), when taking a closest train after \( t \)

- \( y_{t}^{l} \in \mathbb{R}^{+} \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \)
Secondary Decisions II

\[ z_t^l = \begin{cases} 
1 & \text{if passengers wanting to travel at time } t \\
& \text{on the line } l \text{ take the closest train} \\
& \text{after the time } t, \\
0 & \text{otherwise.} 
\end{cases} \]
Objective

\[
\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l
\]
Constraints I

\[
y_{t}^{lb} \geq \frac{(t - t')}{f} \cdot \left( x_{t'}^{l} - \sum_{t''=t'+1}^{t} x_{t''}^{l} \right) \quad \forall l \in L, \forall t, \forall t' \in T : t \geq t',
\]

\[
y_{t}^{la} \geq (t' - t) \cdot \left( x_{t'}^{l} - \sum_{t''=t+1}^{t'-1} x_{t''}^{l} \right) \quad \forall l \in L, \forall t, \forall t' \in T : t < t',
\]
Constraints II

\[ y_{t}^{lb} \geq M_{1} \cdot \left( 1 - \sum_{t' = s}^{t} x_{t'}^{l} \right) \quad \forall l \in L, \forall t \in T, \]

\[ y_{t}^{la} \geq M_{1} \cdot \left( 1 - \sum_{t' = t}^{T} x_{t'}^{l} \right) \quad \forall l \in L, \forall t \in T, \]
Constraints III

\[ y_t^l \geq y_{t}^{lb} - z_{t}^l \cdot M_2 \quad \forall l \in L, \forall t \in T, \]
\[ y_t^l \geq y_{t}^{la} - \left(1 - z_{t}^l\right) \cdot M_2 \quad \forall l \in L, \forall t \in T, \]
\[ M_2 > M_1 \]
Constraints IV

\[ \sum_{t \in T} x_t^l \leq n^l \quad \forall l \in L, \]
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Introducing Cyclicity

\[ x_{t+c}^l = x_t^l \]

\[ \forall l \in L, \forall t \in T : t + c^l \leq T : t \geq s, \]

\[ \min(t+c^l, T) \sum_{t'=t+1} x_{t'}^l \leq (1 - x_t^l) \cdot M_3 \]

\[ \forall l \in L, \forall t \in T : t \geq s, \]
Introducing Cyclicity

\[ x_{t+c}^l = x_t^l \quad \forall l \in L, \forall t \in T : t + c^l \leq T : t \geq s, \]

\[ \min(t+c^l, T) \sum_{t'=t+1} x_t^l \leq (1-x_t^l) \cdot M_3 \quad \forall l \in L, \forall t \in T : t \geq s, \]
Motivation

Ideal Train Timetabling Problem
- Assumptions
- Inputs
- Decision Variables
- Objective
- Constraints
- Cyclicity
- Connections

Conclusions

Future Work
Extra Decisions I

- $y_t^{ll^b} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time $t$ on the line $l$, when taking a closest train at $t$ or before and connecting to line $l'$
- $y_t^{ll^a} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time $t$ on the line $l$, when taking a closest train after $t$ and connecting to line $l'$
- $y_t^{ll'} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time $t$ on the line $l$ and connecting to line $l'$
**Extra Decisions II**

Let

\[ z_t'' = \begin{cases} 
1 & \text{if passengers wanting to travel at time } t \\
& \text{on the line } l \text{ take the closest train after the time } t \text{ and connecting to line } l', \\
0 & \text{otherwise.}
\end{cases} \]
Objective

$$\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l + \sum_{l \in L} \sum_{l' \in L} \sum_{t \in T} y_t^{ll'} \cdot d_t^{ll'}$$
Extra Constraints I

\[ y_{t''}^{bb} \geq (t - t') / f \cdot \left( x_{t'}^l - \sum_{t'''=t'+1}^{t} x_{t'''}^l \right) + \left( t'' - (t' + h_l'') \right) \cdot \]

\[
\left( x_{t''}^l - \sum_{t'''=t'+h_l''+1}^{t''-1} x_{t'''}^l \right) - M_4 \cdot \left( 1 - x_{t'}^l + \sum_{t'''=t'+1}^{t} x_{t'''}^l \right)
\]

\[ \forall l, \forall l' \in L : l \neq l', \]

\[ \forall t, \forall t', \forall t'' \in T : t \geq t' \text{ and } t' + h_l'' < t'', \]

\[ y_{t''}^{aa} \geq (t' - t) \cdot \left( x_{t'}^l - \sum_{t'''=t'+1}^{t'-1} x_{t'''}^l \right) + \left( t'' - (t' + h_l'') \right) \cdot \]

\[
\left( x_{t''}^l - \sum_{t'''=t'+h_l''+1}^{t''-1} x_{t'''}^l \right) - M_4 \cdot \left( 1 - x_{t'}^l + \sum_{t'''=t'+1}^{t'-1} x_{t'''}^l \right)
\]

\[ \forall l, \forall l' \in L : l \neq l', \]

\[ \forall t, \forall t', \forall t'' \in T : t < t' \text{ and } t' + h_l'' < t'', \]
Extra Constraints II

User Cost

Ideal Time $t$

Regular Time Step

Departure

Arrival to Line $l'$ at time $t'+h$

Time $t''$
Extra Constraints III

\[ y_{t}^{ll'} \geq y_{t}^{ll''} - z_{t}^{ll'} \cdot M_{2} \quad \forall l, \forall l' \in L : l \neq l', \forall t \in T, \]

\[ y_{t}^{ll'} \geq y_{t}^{ll''} - (1 - z_{t}^{ll''}) \cdot M_{2} \quad \forall l, \forall l' \in L : l \neq l', \forall t \in T, \]

Constraints to add

- Beginning and the end of horizon, when no connections are possible
1 Motivation

2 Ideal Train Timetabling Problem

3 Conclusions

4 Future Work
Conclusions

- New planning phase, based on the demand
- User cost rather than demand to capture (no need for discrete choice model)
- Can handle both non- and cyclic timetables
- Connections are demand imposed
1 Motivation

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4 Future Work
Future Work

- Methodology design (cyclic is tighter than the non-)
- Actually solving the problem
- Analysis of the general results
- Analysis of the connections
Thank you for your attention.