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## **Splitting rates equilibrium in Dynamic Traffic Assignment using the General Link Transmission Model to simulate network performances**

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### **Abstract**

This article presents a new model of Dynamic User Equilibrium that is particularly suited for ITS applications, where vehicle flows and travel times must be simulated on large road networks. The key feature of the proposed model is the different representation of time and space, less detailed on the demand side of the equilibrium problem, namely the route choice, and more detailed on the supply side, namely the dynamic network loading, that is here implemented through the General Link Transmission Model. This separation allows for a balanced investment of computing resources on the two sides of the equilibrium problem, by assuming that the pivot between them, also used as fixed-point variables, are the splitting rates, i.e. the turn probabilities at each node of the network evaluated without taking into account the destinations of vehicles. The proposed modelling framework is conceived for a seamless extension to real-time contexts.

**Keywords:** Dynamic User Equilibrium, Dynamic Network Loading, turn probabilities vs arc flows, fixed point problem, route choice trajectories.

### **Introduction**

Despite 30 years of intensive efforts produced by the transportation research community, Dynamic Traffic Assignment (DTA) is still one of the most challenging issues in network modelling. A satisfactory mathematical framework allowing for the existence and uniqueness of a dynamic user equilibrium, whose solution can be computed by a rapidly convergent algorithm, like that available for static assignment, is still to be achieved; unless, one is ready to pay a high price in terms of model realism on the supply side (e.g. simple point-queue models with no flow interaction at nodes and no congestion spillback) and/or on the demand side (e.g. instantaneous shortest paths). Indeed, fixed-point problems and MSA (Method of Successive Averages), with their poor mathematical and convergence properties, still seem to dominate the panorama of DTA models today. Our proposal makes no exception in this respect.

On the other hand, the need of reliable DTA models, that can be efficiently solved also for large applications, is constantly increasing, as they are now seen by most experts as fundamental tools to analyse congested networks in off-line transport planning, to provide simulations of design scenario (once recognized the limited realism of static models), as well as in real-time ITS (Intelligent Transport Systems), to provide short term traffic predictions.

However, the above two applications of assignment models have a fundamental difference. In ITS applications the main aim of simulations is to reproduce vehicle flows and travel times on the road network for given demand and supply modifications, possibly in real-time. In planning applications, instead, the main aim of simulations is to compare different design scenarios. This implies that while in the latter case it is very important to compute precise equilibrium solutions (Gentile, 2012), even at the price of adopting a rough representation of traffic phenomena (Gentile, 2010a) such as

that provided by static assignment models, the contrary is true in the former case.

Actually in the framework of ITS the paradigm of equilibrium path choices is not as stringent (is there a dynamic equilibrium in real-time?), while we need supply models capable to reproduce the essence of network congestion, that is the spillback of queues and their evolution in time.

In other words, although we still consider the user equilibrium paradigm a convenient modelling framework, we give more importance to the realism of dynamic network loading (consistency of flows, travel times and capacity constraints) including vehicle interactions at nodes, than to the possibility of reaching a highly precise consistency between path choices and arc performances (times and costs). At most, in traffic management applications the essential property of the DTA model is its sound sensitivity (reaction) to local changes of the main supply parameters, such as capacity reductions due to traffic events (Gentile and Meschini, 2011).

The main aim of this paper is to present a new DTA model where the dynamic network loading is performed by any traffic simulation tool (macro-, meso- or micro-scopic) where route choice is based on given *splitting rates*, i.e. the turn probabilities at each node evaluated without taking into account the destinations of vehicles. Although the proposed framework is quite general, our implementation is based on the General Link Transmission Model (Gentile, 2010b).

The key feature is the different representation of time and space, less detailed on the demand side of the equilibrium problem, namely the route choice, and more detailed on the supply side, namely the dynamic network loading. We will show that this separation allows for a balanced investment of computing resources on the two sides of the equilibrium problem, by assuming that the pivot between them are the splitting rates, that are also used as fixed-point variables.

### The model

In the following we present the main scheme of the fixed point problem that formalizes the proposed DTA model.

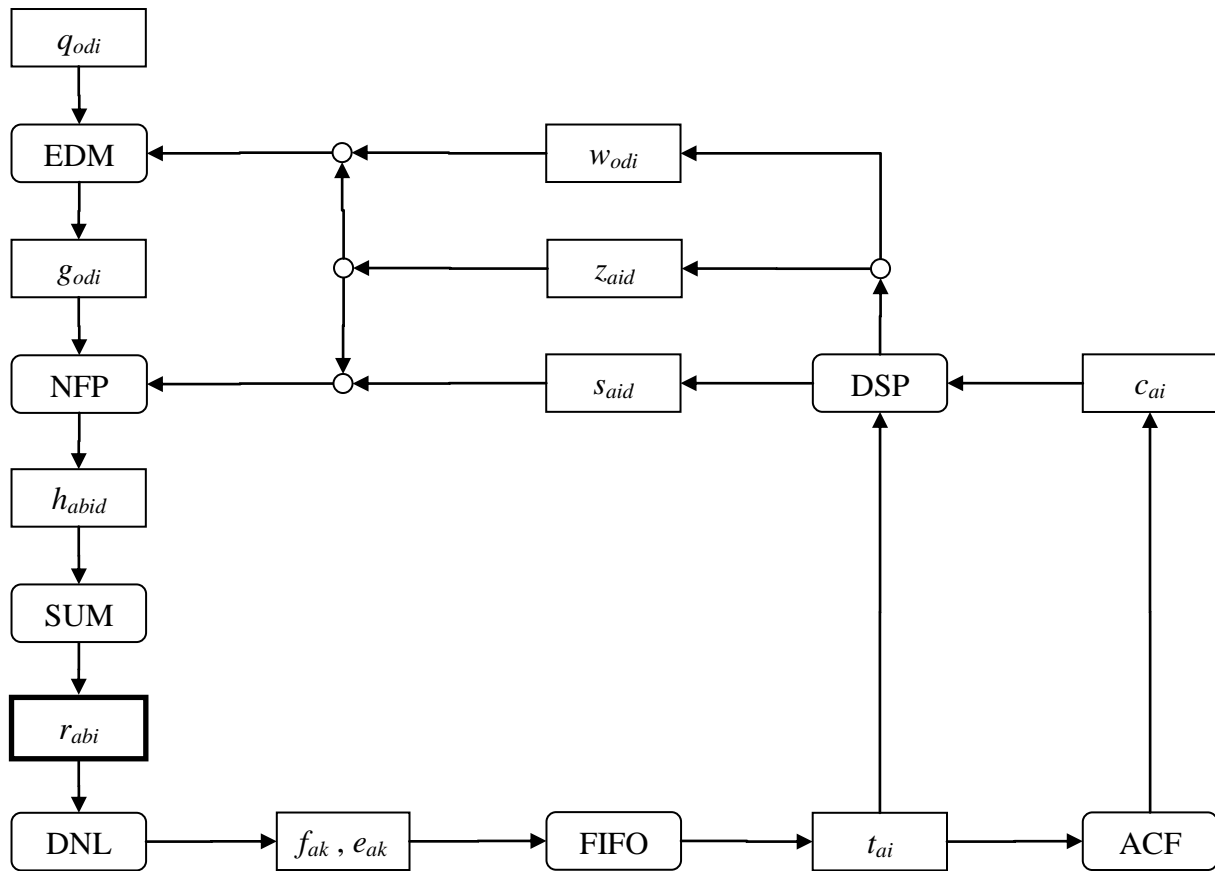
#### Variables

$q_{odi}$	demand of users from origin $o$ to destination $d$ during the $i$ -th DTA interval
$g_{odi}$	flow of users travelling from origin $o$ to destination $d$ during the $i$ -th DTA interval
$h_{abid}$	turn flow from arc $a$ to arc $b$ during the $i$ -th DTA interval of users travelling toward destination $d$
$r_{abi}$	splitting rate from arc $a$ to arc $b$ during the $i$ -th DTA interval
$f_{ak}$	entry flow of arc $a$ during the $k$ -th simulation interval
$e_{ak}$	exit flow of arc $a$ during the $k$ -th simulation interval
$t_{ai}$	entry time of link $a$ for users exiting it at the end of the $i$ -th DTA interval
$c_{ai}$	cost of arc $a$ for users exiting it at the end of the $i$ -th DTA interval
$s_{aid}$	successive arc of arc $a$ for users travelling to destination $d$ and arriving there at the end of the $i$ -th DTA interval
$w_{aid}$	minimum cost to travel from the end of arc $a$ to destination $d$ for users arriving there at the end of the $i$ -th DTA interval
$z_{aid}$	instant at the end of arc $a$ to travel along the minimum cost path to destination $d$ for users arriving there at the end of the $i$ -th DTA interval

#### Functionals

DNL	Dynamic Network Loading model, in our case the GLTM
FIFO	First In First Out rule, travel times are obtained as in Gentile et al. (2005)
ACF	Arc Cost Function
DSP	Dynamic Shortest Paths

EDM Elastic Demand Model  
 NFP Network Flow Propagation, based on trajectories as in Gentile and Meschini (2004)  
 SUM Aggregation and ratio



Interestingly, two kind of arc flows are considered in the model, the first one produced by the network flow propagation model on the basis of route choices to determine the splitting rates is consistent with drivers' behaviour but inconsistent with congestion phenomena, the second one is produced by the dynamic network loading model for given splitting rates, is consistent with congestion phenomena but inconsistent with drivers' behaviour. Only at equilibrium these two variables become mutually consistent, although they are referred to two different discretization used for time aggregation: the DTA interval (of some minutes), the simulation intervals (of few seconds). However, while in Gentile et al. (2007) the consistency of the dynamic network loading between flows and travel times (including spillback and capacity constraints) is achieved only jointly with the equilibrium, in the proposed framework a proper DNL based on splitting rates is obtained at each iteration. This conveys robustness to the model and allows for an easier interpretation of results, even when the level of convergence achieved through the MSA is not fully satisfactory.

**References**

The short list of references presented below is only mainly aimed at delineating the research stream followed by the authors. A proper literature review goes beyond the scope of the extended abstract.  
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