Family of macroscopic node models

Erik-Sander Smits*[1,2], Michiel Bliemer[2], Adam Pel[1], and Bart van Arem[1]

[1]Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of Transport & Planning
[2]The University of Sydney, Institute of Transport and Logistics Studies

March 15, 2013

Abstract

The complete family of theoretically sound macroscopic node models is presented and analysed. Based on the behaviour of drivers approaching and passing through intersections, an analytical model family is presented. Properties like the invariance principle and the conservation of turning fractions are satisfied. The model family is analysed using max-plus algebra and the complete set of feasible solutions is determined. The node models proposed by Tampère et al. (2011), Gibb (2011), and Flötteröd & Rohde (2011) are members of the family. Furthermore, efficient methods to determine solutions of the node model are presented. Such methods are very useful in dynamic network loading which is in turn key for dynamic traffic assignment.

1 Introduction

Dynamic network loading (DNL) is an important procedure in for example dynamic traffic assignment and network assessment. DNL models commonly consist of a link and a node model. The link model traces traffic flow along homogeneous road stretches and the node model connects these traffic flows at places where such homogeneous road stretches are connected (e.g. bottlenecks and intersections). Micro-, meso- and macroscopic approaches exist for these models; this study will focus on the macroscopic approach where traffic is represented with flows. Based on behaviour at intersections, we present and analyse a family of node models in which all previously prosposed proper[1] node models are contained.

The full importance of node models within DNL was observed only a few years ago by [Tampère et al., 2011]. In the transition from static to dynamic network models the focus was on phenomena like shockwaves and spillback and these were addressed with link models. Furthermore, in the static approach traffic is assigned instantaneous and will never be blocked at nodes, so no node model existed. However, most congestion originates at nodes, and the node will determine its severity. [Lebacque and Khoshyaran, 2005] identified the invariance principle, the first big issue for node models, which is derived from properties of traffic dynamics [2]. Later [Tampère et al., 2011] derived a set of requirements for node models to make them compatible with traffic flow theory.

Only two models exist that satisfy all the requirements of Tampère et al. A capacity proportional node model is presented in [Tampère et al., 2011] and [Flötteröd and Rohde, 2011]; the capacity of outgoing links is divided competing flows proportional to the capacity of the corresponding incoming links. In [Gibb, 2011] a model based on capacity consumption equivalence is presented; this basically assumes that traffic towards saturated links consume more capacity of their incoming link. The main drawback of the first model is the lack of behavioural foundation, more specifically, introducing a small flow on an unused turn towards a saturated link may cause major delays on its origin link. The latter model is more realistic from a behavioural point of view, but lacks an efficient non-iterative solution method.

2 Contribution

The presented node model family has the following characteristics:

*Corresponding author: e.smits@tudelft.nl

[1]i.e., satisfying the requirements outlined in [Tampère et al., 2011]

[2]The invariance principle states that if a node model predicts spillback on a link, this result must be invariant to an increase of the demand of that link.

- Behaviour at intersections, specifically the waiting time per vehicle, determine the family.
- All solutions that satisfy the set of requirements from Tampère et al. are members of the family. In other words, the family consists of all possible theoretically sound models.
- Resulting flows have closed form formulations and are differentiable almost everywhere[3], which is a useful property when analysing the effect of changing demand for routes on networks.
- The set of feasible solutions is given by an inequality in max-plus algebra.
- Members of the family are easily identified, including the two existing node models.
- Existing methods for polynomials in max-plus algebra can be used to develop efficient solution methods
- A new node model that has the advantages of both existing node models without any of the disadvantages will be identified; it is behaviourally more plausible and efficient to solve without iterating till convergence.

3 Macroscopic node problem formulation

Let I be the set of all incoming links (*inlinks*) and J be the set of outgoing links (*outlinks*) of a node. Define a turn ij for each pair of inlink i with outlink j, and let $\mathcal{T} = \{ij | i \in I, j \in J\}$ be the set of all turns. Let Q_i be the capacity of link $i \in I \cup J$, let S_i be the traffic demand (or sending flow) from inlink $i \in I$, and let R_j be the available supply (or receiving flow) at outlink $j \in J$. Turnfraction α_{ij} is the fraction of flow coming from inlink $i \in I$ which is heading for outlink $j \in J$, so $\sum_{j\in J} \alpha_{ij} = 1 (\forall i \in I)$.

Following the formulation of [Tampère et al., 2011], the node problem is to find flows f_i for each inlink $i \in I$, such that

$$f_i \geq 0 \quad \forall i \in I, \qquad \text{(Non-negativity)}$$

$$f_i \leq S_i \quad \forall i \in I, \qquad \text{(Demand constraints)}$$

$$\sum_{i\in I} \alpha_{ij} f_i \leq R_j \quad \forall j \in J, \qquad \text{(Supply constraints)}$$

$$\text{for all } \varepsilon > 0 \text{ either } f_i + \varepsilon > S_i \text{ or}$$

$$\exists j \in J \text{ such that } \alpha_{ij}\varepsilon + \sum_{i' \in I} \alpha_{i'j} f_{i'} > R_j \quad \forall i \in I, \qquad \text{(Individual flow maximization)}$$

$$\text{if } f_i < S_i \text{ then } \frac{\partial f_i}{\partial S_i} = 0 \quad \forall i \in I. \qquad \text{(Invariance principle)}$$

In the full paper we will also include the internal node constraints. In this formulation with turnfractions the 'conservation of turning fractions' requirement is automatically satisfied. It can be shown that there are multiple feasible solutions to this set of constraints, the capacity proportional model and capacity consumption equivalence model being examples.

4 Behavioural perspective of turning delays at intersections

We will discuss the behaviour of vehicles arriving at intersections to address the behavioural implications of node model solutions. Consider a turn $ij \in \mathcal{T}$ in an intersection and assume a vehicle arrives at the front of the inlink i. This vehicle will continue to outlink j only if there is space available. Given the supply rate R_j, we know that this space will be available if the previous vehicle departed at least $\frac{1}{R_j}$ time ago. In other words the inter departure time of vehicles going onto j should be at least $\frac{1}{R_j}$. Furthermore, if multiple vehicles are waiting to enter j, it can be that the vehicle on link i has wait for multiple other vehicles. The order in which the vehicles depart into j can be determined by priority rules and other intersection characteristics. Nevertheless, it is clear that for each turn the vehicles have to wait a certain amount of time at the front of their inlink. Define this delay as turndelay d_{ij}.

A second observation is that while vehicles are awaiting their turn to traverse through, vehicles behind it can also be delayed. Given the demand rate S_i, we know that the next vehicle can be at the front of the link $\frac{1}{S_i}$

[3] the set of input variables for which the solution is not differentiable has measure zero.

time later than the arrival of the current vehicle. However, it might be that a queue formed behind the current vehicle, in that case the next vehicle can be at the front of the link $\frac{1}{Q_i}$ time later than the departure of the current vehicle. We will use these simple observations on the micro-level behaviour to formulate behaviourally plausible macroscopic node models.

5 Macroscopic node model derived from behavioural principles

If the turndelays d_{ij} introduced in the previous section are known, then it is possible to determine the inter-arrival time at inlinks and the inter-departure time at outlinks. Since these are easily translated to flow, we get – given a set of turndelays – the following flow equations hold:

$$f_i = \frac{1}{\max\left\{\frac{1}{Q_i} + \sum_{j\in J} \alpha_{ij} d_{ij}, \frac{1}{S_i}\right\}} \quad \forall i \in I, \tag{1}$$

where the summation term is the average waiting time for each vehicle (i.e., the weighted average over all turn delays). When equation (1) is substituted into the requirements formulated in section 3, the problem of finding flows is transformed into a problem of finding – behaviourally plausible – turndelays.

6 Family of solutions

In the full paper we will show that each set of positive turndelays $\{d_{ij} > 0 | i \in I, j \in J\}$ that satisfies

$$\max_{j\in J}\left\{-R_j + \sum_{i\in I|\alpha_{ij}>0} -\max\left\{\alpha_{ij} S_i, \frac{\alpha_{ij} Q_i}{1+\sum_{j'\in J}\alpha_{ij'} d_{ij'}}\right\}\right\} \leq 0, \tag{2}$$

is a solution that meets all requirements except for individual flow maximization. However, we will also show how such a solution can be individual flow maximizing by lowering turndelays for specific inlinks.

In the full paper equation (2) will be fully analysed. The equivalent formulation in max-plus algebra is provided; actually, the relation was found using the the max-plus algebra formulations of the requirements. The resulting multivariate maxpolynomial is then solved by using methods like the one presented in [Cuninghame-Green, 1995]. The relation between turndelays and capacity based reduction factors (as presented in [Tampère et al., 2011]) is derived. Also, the precise behavioural meaning of the capacity consumption factors from [Gibb, 2011] is given. This analysis gives rise to new solution methods.

References

[Corthout et al., 2012] Corthout, R., Flötteröd, G., Viti, F., and Tampère, C. M. (2012). Non-unique flows in macroscopic first-order intersection models. *Transportation Research Part B: Methodological*, 46(3):343 – 359.

[Cuninghame-Green, 1995] Cuninghame-Green, R. (1995). Maxpolynomial equations. *Fuzzy Sets and Systems*, 75(2):179–187.

[Flötteröd and Rohde, 2011] Flötteröd, G. and Rohde, J. (2011). Operational macroscopic modeling of complex urban road intersections. *Transportation Research Part B: Methodological*, 45(6):903 – 922.

[Gibb, 2011] Gibb, J. (2011). Model of traffic flow capacity constraint through nodes for dynamic network loading with queue spillback. *Transportation Research Record: Journal of the Transportation Research Board*, 2263(-1):113–122.

[Lebacque and Khoshyaran, 2005] Lebacque, J. and Khoshyaran, M. (2005). First order macroscopic traffic flow models: intersections modeling, network modeling. In *Proceedings of the 16th ISTTT*.

[Tampère et al., 2011] Tampère, C. M., Corthout, R., Cattrysse, D., and Immers, L. H. (2011). A generic class of first order node models for dynamic macroscopic simulation of traffic flows. *Transportation Research Part B: Methodological*, 45(1):289 – 309.