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Route-choice and Signal Control: the Stability and Instability of Traffic Signal Controlled Networks

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ABSTRACT

Traffic signal controls, along with road pricing, are considered effective tools to manage traffic congestions in cities. There have been recent interests in the potential for traffic signals to influence traffic pattern and route choice, particularly when there is high congestion: using signals to discourage congestion-causing route-flows and encourage congestion-reducing route switches.

This paper studies the *stability of the interaction between traffic control and route choice* under several different control policies and investigate their performance benefits.

1 BACKGROUND

The performance of road transport infrastructure has significant environmental impact resulting from air and noise pollution, fuel consumption and for potential economic growth. Traffic signal control is an important tool to manage congestion in cities and efficient control has the potential to maximise existing road capacity and adverse effect from congestion.

There is a powerful tendency to seek automatic traffic systems that cause networks to react effectively to up-to-date information. Farhan and Martin (2011) have recently demonstrated in a large network model with 60 traffic signals that allowing for vehicle actuated signal timings reduces modelled distance travelled and total travel time by roughly 6%; thus not allowing for responsive signal controls gives skewed assessment results. In a small evacuation study Marciano et al (2012) suggest that a reduction of total travel time of 24% is feasible by readjusting traffic signals. Thus signals may have considerable effects.

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Studies of the *stability of the interaction between traffic control and route choice* have been much rarer; and this is the focus of this paper. The paper reports the *stability* of certain automatic control responses in traffic networks, and shows that stability and lack of stability may of course have profound performance implications. The work reported here is related to work by Dickson (1981), Smith (1979, 1987), van Vuren and Van Vliet (1992), Smith and Mounce (2011) and van den Berg et al (2009).

2 INTERACTION BETWEEN SIGNAL CONTROL AND ROUTE CHOICE

We consider simple models of routeing changes as drivers seek better routes and signal control changes as an adaptive control system responds to traffic flows, an evolution shown in Fig. 1.

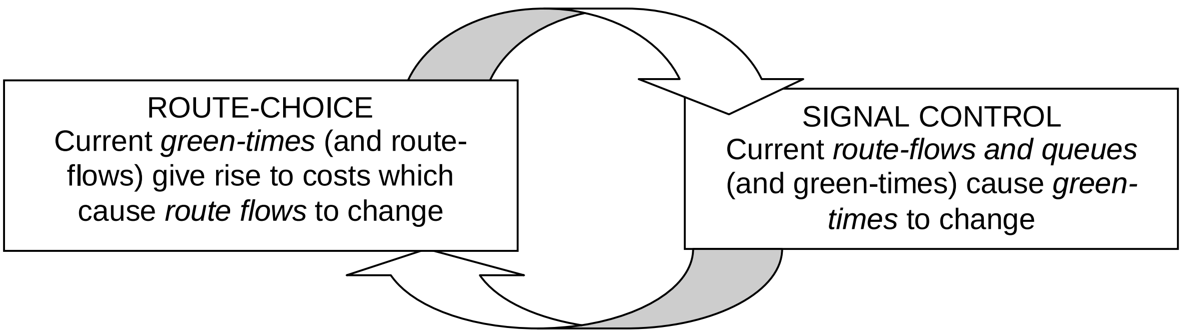

Figure 1. The dynamical system arising when a responsive control system is utilised. Current route-flows change current green-times (according to some responsive control policy) and current green-times change current delays and hence current flows (according to drivers' route-choices).

We consider a simple network of Fig. 2, with one O-D pair and two parallel routes meet at a signal-controlled junction (node 1). Assume symmetrical free-flow travel times on the two routes and equal saturation flows of the two approaches to the signal.

Smith and Mounce (2011) proved in theory that, under the standard equi-saturation policy, for each origin-destination (OD) load the symmetrical equilibrium is unstable and complicated dynamics arise, including the pitchfork bifurcation as shown in Figure 3.

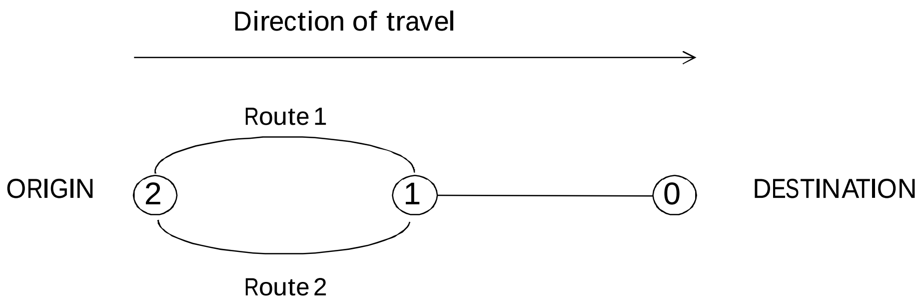

Figure 2. The simple 3-link symmetrical network with junction 1 signal-controlled.

Assuming drivers swap toward the cheaper route Figure 3 shows the swap directions for all points within the triangular feasible set. As the arrows are too small to be seen a few large arrows, agreeing in direction with the tiny arrows, have been added.

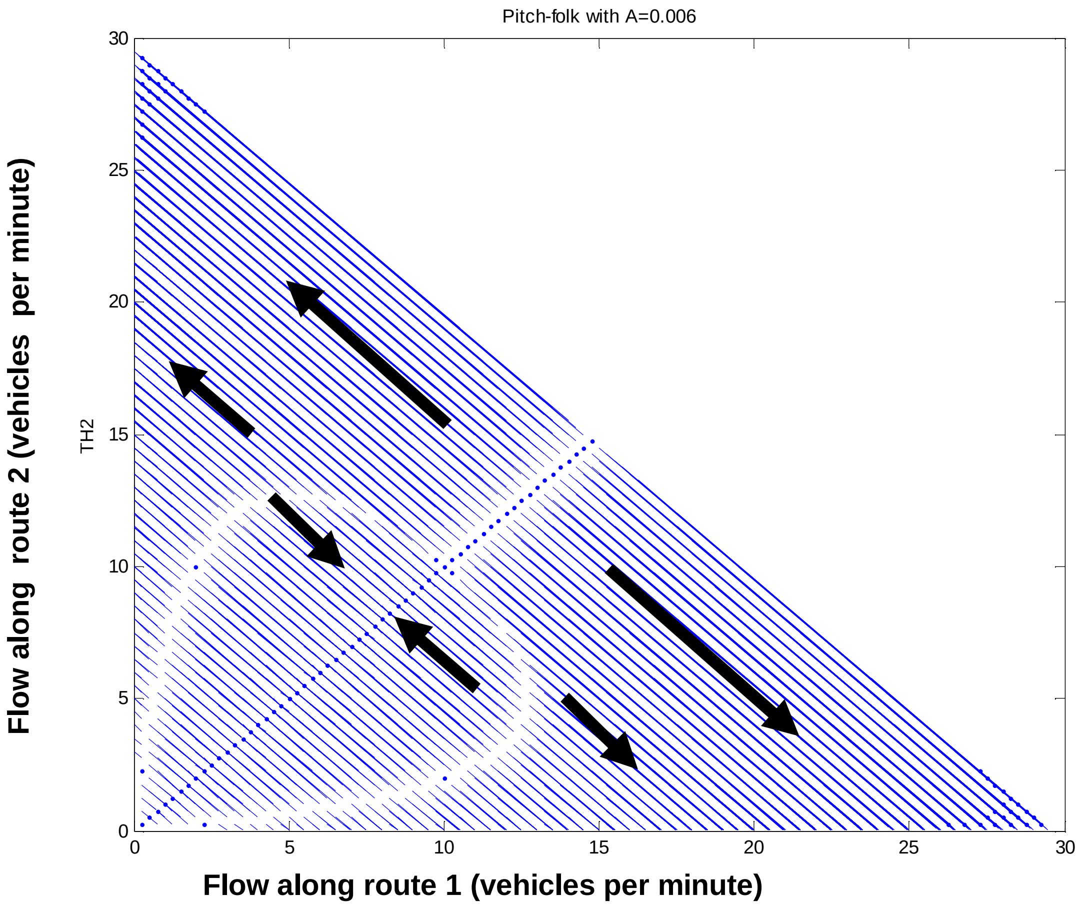

Figure 3. Arrows show the direction of motion of flow vectors under the equi-saturation policy: white areas show points of approximate equilibrium consistent with the equi-saturation policy.

3 STABILITY AND PERFORMANCE BENEFITS OF ALTERNATIVE CONTROL POLICIES

In this paper, we consider the stability of the Figure 2 network under various different assumptions. These are: (a) using a *new* traffic control policy ("P_1") very similar to the P_0 policy (Smith 1979, 1987) and the Webster's delay formula; (b) using P0 and with explicit queueing delays, but with no blocking back; and (c) using P0 and with explicit queues which take up space on the link. *In this extended abstract we give a flavour only of the results mentioned above.*

We show that in each case the P0 policy is shown to give substantial stability and performance benefits compared to standard policies.

Under the new P1 control policy, the natural adjustment of routeing and green-times in Figure 1 becomes stable. We show that with P1, if there is at least one flow-control pair (not necessarily an equilibrium) with a finite total travel cost, then the (flow, control) pair in the loop in Figure 1 converges to a set of flow-control pairs which are at equilibrium consistent with the P_1 policy; and that total travel costs are finite at each point of this set. We present the proof of stability utilising the standard Pollaczek-Khintchine (P-K) formula for delays at the traffic signal (Pollaczek, 1930). *The proof is then generalised so that it applies to a general network.*

For condition (b), with the P0 control policy and vertical queueing, we show that, again, the "natural" or standard control policy may be unstable in a further very damaging way: the loop in Figure 1 causes the (flow, control, queue) triple converges to a distribution of flows and green-times where delays are infinite, i.e. *these policies fail to maximise the capacity of the simple network with point queues.*

For condition (c), we show that *when there is blocking back, the previously guaranteed capacity-maximisation result of the P_0 /P1 family of policies may fail.* However we show that other policies are often worse.

In the paper, we further discuss how the above (positive) P_1 stability results may be extended to cater for networks with blocking back and elastic demand.

Our theoretical analysis is confirmed by simulation results in the paper; an example of which is illustrated in Section 4.

4 EXAMPLE ILLUSTRATION

We show here how the instabilities of the equi-saturation policy illustrated in figure 3 deform with an unsymmetrical network.

For the same network in Figure 2, but now

- The free-flow times on Route 1 and 2 are K_1 and K_2 where $K_1 < K_2$; and
- The saturation flows to the two approaches to the signal are s_1 and s_2 where $s_1 < s_2$.

Figure 4 illustrates the direction of motion of flow vectors now if the equi-saturation policy is followed responsively. The loop in Figure 1 causes the flow to converge to the unfeasible boundary where delays are infinite; this is much worse than Figure 3.

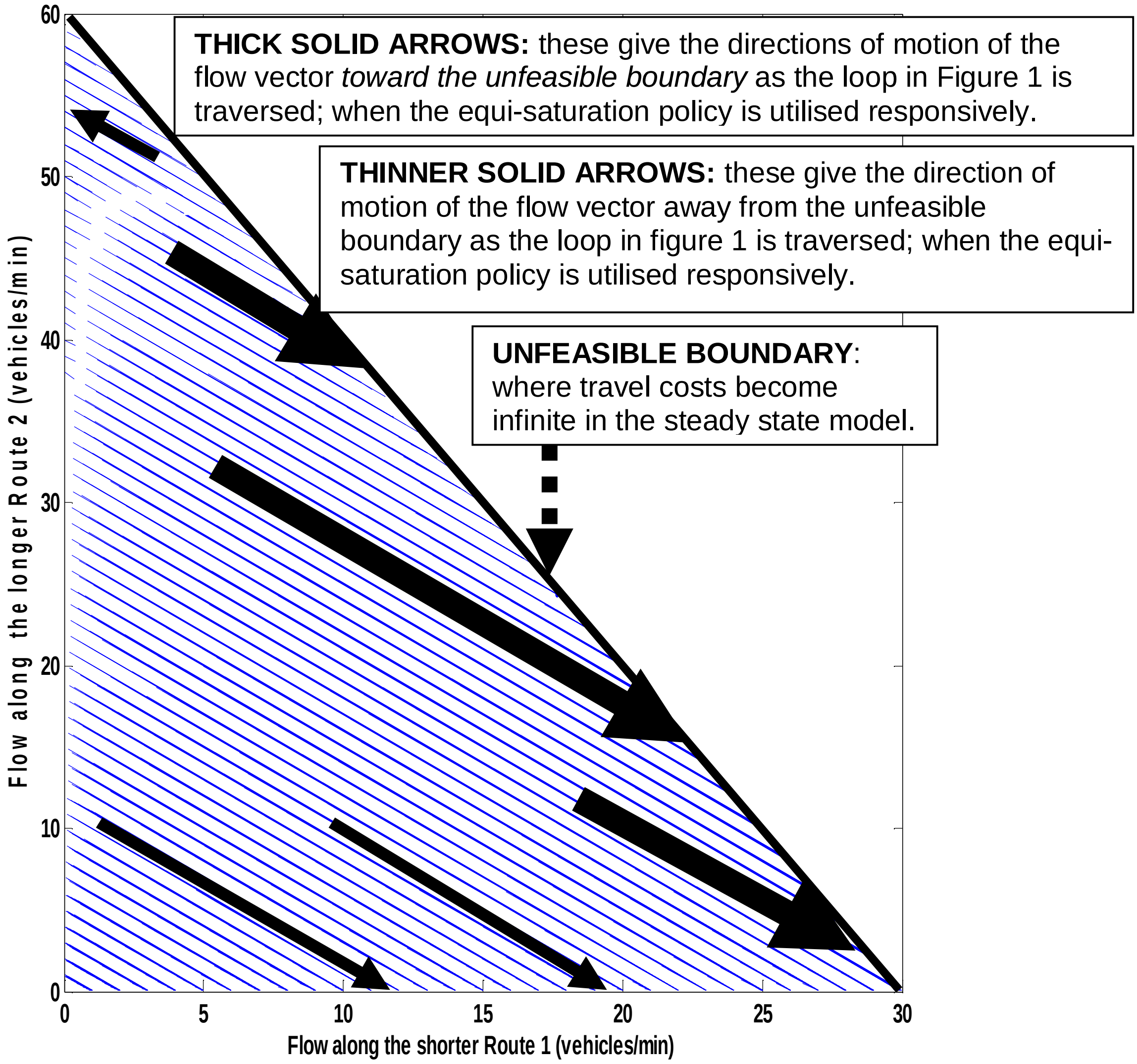

Figure 4. Trajectories of flow vectors arising from natural route-swapping with the responsive equi-saturation policy. In this figure very many start points in the feasible triangle were utilised and a vector plotted to show the direction of route swaps (toward the cheaper route) from each start point. The vectors are too small to see the direction; so the bold vectors have been added; these agree in direction with the small arrows. The white areas comprise near equilibria consistent with the equi-saturation policy.

REFERENCES

Dickson, T. J. (1981) A note on traffic assignment and signal timings in a signal-controlled road network. *Transportation Research* B, 267 – 271.

Farhan, M., Martin, P. (2011). Implementation of combined traffic assignment and control frwmework based regional model for the Wasatch front region. Asian Resarch Publishing Network Journal of Engineering and Applied Sciences. 6 (12) (December 2011).

Marciano, F. A., Musolino, G., Vitetta, A. (2012). Signal setting optimisation with route choice behaviour and acyclic profiles of vehicular flow in road networks. Proceedings of the EURO Working Group on Transportation international scientific conference (2012).

Pollaczek, F., (1930). Uber eine Aufgabe der Wahrscheinlichkeitstheorie. Mathematische Zeitschrift 32, 64 – 100.

Smith, M. J. (1979). A local traffic control policy which automatically maximises the overall travel capacity of an urban road network. Presented at the International Symposium on Traffic Control Systems, Berkeley, California (August, 1979); published in the Proceedings, Volume 2A, 11-32, and in Traffic Engineering and Control, 21, 1980, 298-302.

Smith, M. J. (1987). Traffic control and traffic assignment in a signal-controlled network with queueing. Proceedings of the Tenth International Symposium on Transportation and Traffic Theory (Editors: Gartner, N. H. and N. H. M. Wilson), Elsevier, 61 - 68.

Smith, M. J. and Mounce, R. (2011). A splitting rate model of traffic re-routeing and traffic control. Transportation Research Part B, 1389 - 1409. (Also presented at and published in the Proceedings of the 19th International Symposium on Transportation and Traffic Theory (Berkeley, Editors: Cassidy, M. and Daganzo, C. F.), Elsevier, 316 – 340.

Tettamanti, T., Varga, I., Peni, T. (2010). MPC in urban traffic management. Available from http://www.interchopen.com/books/model-predictive-control/mpc-in-urban-traffic-management.

van den Berg, M., De Schutter, B., Hegyi, M. and Hellendoorn, H. (2009). Day-to-day route choice control in traffic networks with time-varying demand profiles. Proceedings of the European Control Conference 2009 (ECC'09), Budapest, Hungary.

van Vuren, T., Van Vliet, D., 1992. Route Choice and Signal Control. Avebury.