A stochastic theory of passenger traffic along a transit line, with operations policy and relativity effect

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Summary

Along a given transit line, passenger traffic is shaped by the line structural characteristics in terms of vehicle capacity, headways and spatial structure of both the infrastructure and the pattern of demand flows between stations. It is also submitted to a number of variability sources, related to the distribution (meaning heterogeneity) of, respectively, headways, order in schedule, vehicle types, demand levels, individual passengers. Furthermore, there is a relativity effect since the users experience travel conditions distributed specifically because of the vehicle load. The paper extends the stochastic model of Leurent et al (2012) to discrete passenger flows and a range of operations policy, with special emphasis on stopping policy.

1. Background

The operations of a transit line are submitted to not only the operator's policy but also to a set of variability sources that induce stochasticity. On the side of the operator, the infrastructure design determines the physical path, the right of way along it, the stations, whereas the operational design determines the fleet composition hence vehicle capacity, the stopping policy, the schedule of runs and also the distribution of headways. The specification of both the infrastructure and the quality of service (via the headways and the operating speed) determine the spatial structure of the demand flows with respect to the access-egress station pairs (the so-called legs). Indeed, these characteristics are structural properties of the transit line as a traffic system: they shape its passenger traffic in terms of leg flows and also quality of service.

On the other side, variability or stochasticity is inherent to some specific features in the system: passenger arrivals are random at each station and in fact by leg; their distribution is conditional on the station headways between the previous and the incoming vehicle run, which is itself more or less regular – thus complying to a specific distribution; the order in schedule determines the propagation of incidents; vehicle capacity may be heterogeneous, yielding variable passenger exposure to congestion; passengers have idiosyncrasies in their sensitivity to quality of service; lastly, the demand level varies within day as well as from day to day.

The interplay between structural properties and variability sources calls for stochastic models specific to transit lines. Queuing theory provides a standard model of passenger waiting at a station (the bulk model), which is useful for stationary analysis (i.e. to deal with a set of sufficiently homogeneous periods, e.g. peak hours for a given type of days) only if passengers are queuing with priority – as opposed to mingling, cf. Kurauchi *et al* (2003) and for services that are all attractive whatever the number of waiting passengers, which is not the general case (Leurent, 2011). Nonetheless, many of the basic assumptions and model components in

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stochastic modeling can be used to deal with a transit line. Leurent *et al* (2012) have modeled the vehicle journey trips and passenger load along the line, as Random Variables (RVs) derived from the local distribution of headways, the order in schedule, the vehicle capacity, the level of demand and passenger idiosyncrasies. They have elucidated a relativity effect that distinguishes the passenger exposure to station waiting and vehicle crowding, from the operator's evaluation of the line performance that is based on the statistical population of vehicle runs.

2. Objective

The paper's objective is to generalize the stochastic model of a line in the following two respects. First, the passenger flows are modeled as discrete RVs rather than continuous, thus enabling to distinguish strictly positive values from null. Second, some characteristics are introduced to distinguish between transit modes, notably the train vs. the bus: beyond vehicle capacity and infrastructure settings, the distinction pertains to vehicle operations at stations for passenger alighting and boarding and, in the bus case, the eventuality of stopping at a given station depending on passenger demand.

The paper provides a theoretical framework to model the state variables as RVs, with analytical formulae for the basic statement of each RV, its CDF, its expectation and standard deviation. Specific attention is devoted to stopping events and their consequences on journey times, to local passenger load and to passenger exposure to quality of service.

3. Summary of contents

The paper is organized in five parts, namely (1) Basic model with random flows, (2) User exposure, (3) The effects of headway variability and demand level, (4) Numerical illustration, (5) Conclusion.

3.1 Basic model with random flows

In the basic model, the vehicle journey time between station pairs and its local passenger loads and access-egress flows are formulated as functions of local times or of leg flows, respectively, in relation to the particular local headways and a particular level of demand. The local load function is an RV due to random leg flows, taken from leg-based Poisson processes with leg-specific intensity and independence between legs. A postulate of headway correlation between vehicle runs is made to derive the CDF, mean and standard deviation with respect to a given shape of local headway distributions. Consequences are drawn on the local times including those at stations, on the stop probability and the stop time if stopping is optional on the basis of passengers' demands.

3.2 User exposure

About user exposure, every passenger is submitted to a station wait time related to the headway of the incoming vehicle and to a vehicle load which is essentially proportional to the headway, the demand level and the pattern of origin-destination flow matrix by leg. The influence of headway and load re-shape the distribution of travel times and experienced load.

3.3 The effects of headway variability and demand level

The relativity effect between operator's performance and users' perceptions is mainly carried by the distribution of headways and by that of demand level. Analytical formulae are established for the general case and the independent case.

3.4 Numerical illustration

In order to investigate the respective magnitude of mean and standard deviation of the main variables, the case of a bus line under alternative scenarios of operating policy and passenger demand is addressed.

3.5 Conclusion

The conclusion discusses the applicability of the stochastic line model in the framework of traffic assignment to a transit network.

4. References

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