A Comparative Evaluation of Gradient-based and Stochastic Approximation Algorithms for Estimation of Dynamic Origin-Destination Matrices

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Dynamic traffic assignment (DTA) models use various demand and supply related inputs and parameters to assign origin-destination (O-D) flows onto the network. These parameters need to be calibrated. Calibration of demand related inputs mainly refer to OD flows. Supply components are typically captured through representation of the network capacities and speed-density relationships. Calibration can be then considered as a problem of estimation of the aforementioned components from observed data. The demand and supply models were calibrated independently and sequentially. First the supply model calibration was performed using local data and then, given the supply parameters, the demand calibration followed ([1], [2]). However, recent literature has focused on simultaneous estimation of both the demand and supply inputs and parameters ([3]-[6]). The research focuses on the simultaneous estimation of demand and supply taking advantage of data available, not only from traditional sources (e.g. loop detectors but also mobile sensors such as FCD).

Estimation of time-dependent OD flows is very important. They are essential inputs to DTA models and provide spatial and temporal information of the distribution of the travel demand for a specific transportation network. The demand is difficult to be directly observed, therefore it is usually estimated using indirect measurements e.g. traffic counts from sensors.

Various solution approaches have been proposed in the literature addressing the OD estimation problem in two different application contexts: offline and real-time. Moreover, the proposed approaches are formulated in different ways. The most common formulation uses observed flows and expresses them as a linear function of the assignment matrix (proportions of OD flows crossing sensor locations over different time intervals). The assignment matrix maps the OD flows to traffic counts. However, other efficient approaches exist that do not rely on the assignment matrices [5], but use the output of any simulation model to capture the complex relationships between the OD flows and any available data.

The paper evaluates and compares the performance of two different OD estimation methods, which differ in their general solution scheme, as well as in the nature of the optimization technics utilized for solving this particular problem. The gradient based optimization algorithm [7], recently modified by [8] is the first method. It relies on the assignment matrix and its structure can easily incorporate on traffic counts. The second approach uses a quite general formulation and the simultaneous perturbation stochastic approximation (SPSA) algorithm ([9]-[10]) for its solution. This method does not rely on assignment matrix information [7]. A case study is used to evaluate the efficiency and robustness of the two algorithms in the context of the OD estimation problem. The two solution approaches are implemented using the mesoscopic event-based simulation model Mezzo [11] as a test bed.

The general problem is to find an OD matrix that best matches a set of observations. The problem can be formulated as an optimization problem expressed mathematically as follows:

$$\text{Minimize:} \qquad w_1 F_1(X, X^H) + w_2 F_2(Y, \tilde{Y}) \qquad (1)$$

$$\text{s.t. } X \geq 0 \qquad (2)$$

where F_1, F_2 are functions that measure the ''distance'' between the estimated quantities from their observed values. X is the estimated time-dependent OD matrix, X^H is the historic (''seed'') OD matrix, $\tilde{Y}$ is a set of observations and Y is a set of observations predicted by the model corresponding to the estimated X after assigned in the network. Factors w_1 and w_2 capture the uncertainty in the observations. Hence, their values reflect the quality of the available observations and the historical OD matrix.

The problem formulation is common for both algorithms. However, they differ in terms of their solution approach and nature of optimization technics. The key features of each algorithm, which are critical for their performance when implemented in the OD estimation problem, are specified below.

As mentioned earlier, the gradient-based algorithm makes use of the assignment matrix for the mapping between the OD flows and traffic counts at available sensor locations. In [8], the assignment matrix is approximated by a linear function in every iteration of the algorithm. The proposed solution approach has several advantages, as well as some limitations, which are mainly summarized below:

- The information provided by the gradient can significantly improve the speed of convergence compared to gradient-free algorithms.
- Gradient-based methods are more robust in finding a local optimum.
- Current gradient-based algorithms (e.g. [8]) use assignment matrix, which precludes an easy incorporation of other data sources besides counts. For example, relationships between speeds and OD flows can be rather complex, so approximations similar to that of the assignment matrix are not straightforward.

The SPSA algorithm has been used to solve the more general formulation that does not use the assignment matrix. The SPSA algorithm uses an approximation of the gradient, requiring only two function evaluations at each iteration. The main advantages and disadvantages of SPSA for the OD estimation problem are:

- Its general and flexible structure allows for the direct incorporation of any kind of data sources in the objective function.
- Its convergence can be slow but the computational cost per iteration is relatively small compared to more traditional algorithms that use numerical approximation of the derivative.
- Due to the stochastic nature of the algorithm, the solution is not very robust as in the gradient-based algorithm.
- Finally, SPSA has several algorithmic parameters, whose values are very critical for the performance of the algorithm.

The two algorithms were tested on a study network from Stockholm, Sweden (Fig. 1). The network consists of 1101 urban and freeway links. Synthetic data were used as the 'true' OD demand and traffic measurements for selected urban and freeway links in Stockholm. The mesoscopic traffic simulation model Mezzo was used to obtain the unknown traffic counts and speeds by assigning the 'true' OD demand to the network.

The experimental design was constructed in order to evaluate and compare the performance of the proposed algorithms. Consists of two historic (''seed'') OD demand matrices, one further off and a second one closer to the 'true' OD demand. Those ''seed'' matrices were generated by disturbing the 'true' OD matrix with the addition of random noise. Another factor in the experimental design is the level of sensor coverage in the network for the collection of traffic measurements. In reality, the percentage of links in a network covered with sensors is relatively low compared to the total number of links in the examined network. In order to capture how this impacts the OD estimation performance by each algorithm, two different levels of spatial sensor coverage were investigated; a sparse, and a denser coverage.

The last factor considered is related to the type of sensor data available. For the SPSA formulation, two separate OD estimations were conducted. An estimation with only OD flows and counts in the objective function, and an estimation with speed and count measurements. The main hypothesis of the described experiment is that in the case of dense sensor coverage (i.e., high amount sensor counts), the gradient-based algorithm is expected to have better performance. On the other hand, if the sensor locations are sparse, the availability of traffic count measurements decreases. Hence, the value of other observations increases and the general problem formulation can be used to improve the performance of the estimation.

Three statistical measures were selected in order to assess the performance of the estimation. The mean error (ME), the root-mean-square error (RMSE) and the Theil's inequality coefficient (U) for the OD flows, counts and speeds were calculated. The RMSE provides information about aggregated differences between the estimated results and the 'true' observations. The ME is another useful indication of the existence of systematic bias in the estimated measurements. The Theil's inequality coefficient is another useful measure to provide information on the relative error.

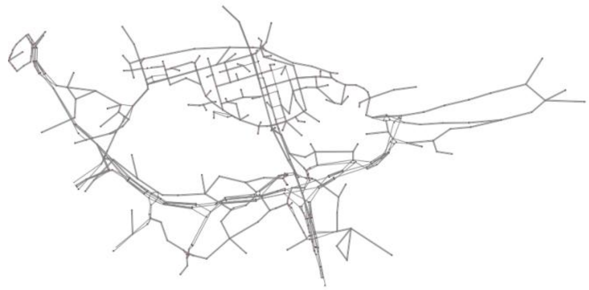

Figure 1. Södermalm, Stockholm, Sweden, network.

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