

## **The Multiple Discrete-Continuous Extreme Value Model (MDCEV) with fixed costs**

Being carless is an option for many households in economies having a good system of public transportation. Thus, a good model should be able to map this option. In particular, it should also be able to map how the fixed costs of holding a car affects car ownership. So far, no model can be found in the literature that adequately maps this option. In this paper we present the theoretical model that fills this gap.

The drawbacks of the existing modelling techniques can be summarized as follows: The OLS fails to map carless households. The Tobit model is unable to map the impact of fixed costs. The sample selection model fails due to the lack of an instrumental variable: there is no variable that influences only the choice of whether or not to own a car whilst not influencing the demand for driving at the same time. An interesting candidate for solving this problem is the Discrete-Continuous Choice model introduced by Dubin and McFadden (1984).<sup>1</sup> This model can be used to explore the ownership of certain car types and their use. Unfortunately, the model only allows the choice of being carless to be captured if the annual mileage travelled using public transport is given in the dataset. Since this information is not available in most micro-census datasets, this model cannot be applied.

The Multiple Discrete-Continuous Extreme Value Model (MDCEV) with fixed costs overcomes the drawbacks of these models. As mentioned above, the proposed model can measure the impact of changes in the fixed costs of cars on driving demand and on the probability of households being carless. This ability to map the impact of income, fuel price and the fixed costs of car ownership on both car ownership and car use could not be found in the literature.<sup>2</sup> The MDCEV model makes it possible to compute the effects of policies such as taxes on fuel or car ownership on both the share of carless households and the average driving distance.

The MDCEV model was introduced by Bhat (2005).<sup>3</sup> This model consists of a direct utility function and a

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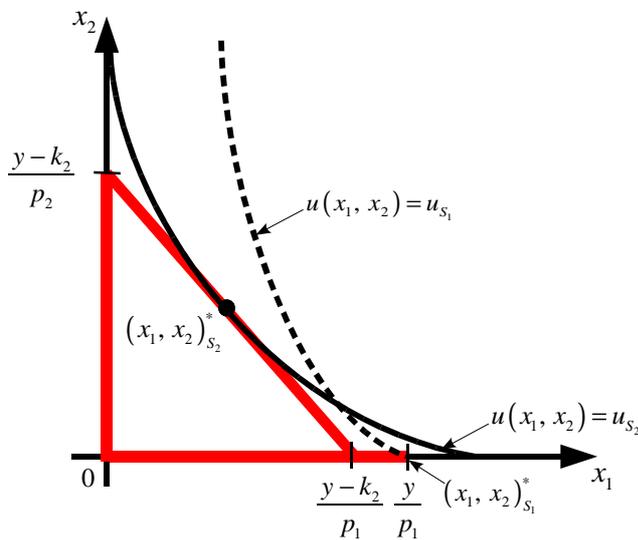
<sup>1</sup> Dubin, Jeffrey A. and Daniel L. McFadden, 1984, "An Econometric Analysis of Residential Electric Appliance Holdings and Consumption", *Econometrica*, Vol. 52, No. 2 (Mar., 1984), 345-362.

<sup>2</sup> One exception is the model of De Jong (1990), used later by Ramjerdi and Rand (1992) and Bjorner (1999). In contrast to our model, it is based on an indirect utility function instead of a direct function. Unfortunately, De Jong's (1990) model has an assumption that violates its compatibility with a microeconomic utility maximisation framework. In addition, it yields rather unrealistic results, particularly with respect to the impact of changes in fixed costs on car ownership. We believe that the MDCEV model with fixed costs maps reality much more effectively and lead to realistic results.

<sup>3</sup> The first application of Bhat's model was to explain the time tourists spend for different activities. The model reflects that each activity can be chosen or not and how many hours are spent for the activities, subject to the time restriction of 24 hours a day, Bhat (2005). Later, Bhat applied this modeling framework to the case where households can choose to own none, one or several cars of different car types and decide of the driving distances the different cars are used for, Bhat (2006). In this model, Bhat ignores the fact that holding cars causes fixed costs and thus according to the model it would not be irrational to hold a number of cars even when the preference for car driving is low. Thus, we want to overcome this drawback by introducing fixed cost in our MDCEV model.

budget restriction. It is assumed that it maps the utility maximisation process of a household and is based on the assumption that a household chooses certain amounts of goods from a set of goods including the possibility of a household choosing not to consume any good at all. This means that a household may choose not to consume any goods at all. In order to adapt the model for examining car ownership and car use, we modified this model in two ways: first, we restricted it to the case with only two goods. This means that households may only choose whether or not to own and use a car and spend the remaining income for a consumption basket containing any other good. Secondly, we extended this model to the case where driving a car requires car ownership, incurring fixed costs, which is our contribution to the theory.

We assume that all decisions are taken at the household level and each household compares the utility yielded from the following two options: first, it establishes the utility level it would gain if it owned a car. In this case, the household income would be reduced by the fixed costs of car ownership. Given that the household would then decide what annual distance  $x_2$  it would drive in order to yield maximal utility given the marginal driving costs  $p_2$ . The remaining income it spends entirely on good one  $x_1$ , which we consider to be a consumer basket containing all goods apart from car driving, e.g. housing, food, medical care, holidays, and so on. We assume that utility is driven exclusively by the kilometres driven and not by the car ownership. Second, we assume that the household establishes the utility in the case that it decides not to own a car and would spend all its income on good one  $x_1$ .



**Figure 1:** Optimum decisions of two households with different preferences

This figure illustrates the optimal consumption plan of two households with identical income but different car driving preferences. The solid lined iso-utility curve  $u(x_1, x_2) = u_{S_2}$  represents a household with a high preference for car driving that decides to own a car and the dashed lined iso-utility curve  $u(x_1, x_2) = u_{S_1}$  a household with a low preference for car driving that decides not to own car.

We use the utility function proposed by Bhat (2005):

$$U = (X_1 + a_1)^d + \exp(m + \beta \cdot \zeta) \cdot (X_2 + a_2)^d, \quad (1)$$

with  $m = \gamma \cdot s$ , representing the deterministic component of the relative preference and  $\zeta$  is the stochastic component of the relative preference which is logistically distributed.<sup>4</sup>

By use of Swiss household data we estimated the parameters. Given these parameters we could simulate some interesting results, e.g. that a tax on car ownership has a much lower impact on aggregate driving demand – per unit of tax revenues – than a tax on fuel or that the effect of a fuel tax is dominated by the households that keep their car but drive less and not from the households that sell their car.

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<sup>4</sup> This utility function is based on the utility function proposed by Bhat (2005:686):  $U = \sum \exp(m_i + \beta \cdot \xi_i) \cdot (X_i + a_i)^{d_i}$ , where the random terms are assumed to be iid Gumbel distributed:  $\xi_j \sim iid\ gu(0,1)$ ,  $f_\xi(x) = e^{-x} \cdot \exp(-e^{-x})$ . Transforming the utility function by multiplying by  $\exp(m_i + \beta \cdot \xi_i)^{-1}$  yields Equation (1). Note that the stochastic component  $\zeta$  in (1) corresponds to  $\zeta = \xi_2 - \xi_1$  and is therefore logistically distributed (for a proof see Appendix A1). Note that we use capital letters for  $X_1$  and  $X_2$ , because these variables are also stochastic since their solution in optimality will depend on the stochastic parameter  $\zeta$ .