Non-linear time and cost effect in a large scale travel demand model – experience from the Danish National Transport Model

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In December 2009 the Danish Ministry of Transport initiated the development of a new Danish National Transport Model (NTM). The principal objective was to develop a unified transport model framework for Denmark. The entire model framework, which has been released in a first version in the spring 2013 covers the travel of all Danish citizens, in Denmark and outside of Denmark, as well as foreigners travelling in Denmark. The following paper presents the first results from this model. We will focus on the specification of non-linearities in the cost-time domain and implications for elasticities. These are important aspect especially in a large-scale demand that covers a relative large span of travel distances. An early discussion about the need for non-linear specification is presented by Gaudry et al. (1989) and a recent discussion about empirical findings related to cost damping is given in Daly (2010).

The overall NTM passenger model consists of four transport demand models (to the right in Figure 1 - the weekend model has not yet been developed) and a strategic model (to the left). The models are linked with a route choice model in order to properly represent congestion effects and the fact that increased demand is counteracted by lowered accessibility due to congestion. The route choice and demand model is iterated using a weighted MSA scheme\(^1\) until system convergence is reached. The models are linked in a traditional random utility framework\(^2\) in which the strategic model at the upper level dictates the choice process at lower levels and where lower levels feed accessibility measures (e.g. logsum variables) to the strategic model. An illustration of the model framework is presented in Figure 1.

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1. The convergence was improved very considerably by using a weighted MSA compared to a simple repeated approximation (MRA).
2. A set of discrete choice models where the perception of individuals in each model assumes a division into a deterministic indirect utility function and a random component.
A choice in the design has been to consider all weekday trips, within Denmark and below 24 hours, in one set of models divided by trip purpose. As a result, we do not introduce a separate long-distance model. There are several reasons for this. Firstly, Denmark is a relative small country connected with a set of tolled bridges between islands. Hence, defining a distance threshold would not be easy if the purpose is to capture trip preferences. Rather one should define models between regions and inside regions separately, however, this would leave only few observations for the long-distance segment. Secondly introducing a separate model for long-distances would make it difficult to deal with substitution effects across this imposed distance band. To focus the analysis, the current paper will look specifically on the national weekday model and in particularly on the model estimation for mode and destination choice.

There are two sets of models, a set of models for primary tours and a set of models for secondary trips. Both primary and secondary trips are segmented into 6 trip purposes \{commute, education, escort, shop, leisure/other, business\} and represents a tour based design with 6 modes \{walk, bike, car, car passenger, public and airplane\}. The demand model is estimated in a standard nested logit model framework.

Let in the following $s$ represent segments, $d$ destination (907 i total) and $i$ income intervals (currently 11 income bands for gross personal income). Monetary costs for all modes are deflated by an income dependent exogenous value-of-time estimate $VoT(i, s)$. Furthermore, for car and car passenger we introduce an occupancy regression, which determine the occupancy rate $OR(d)$ (number of people in the car) depending on trip and socio-economic characteristics and conditional on the whether the choice is car as driver or car as passenger. This leads to the following cost (we only present three of these) where $1_i$ represents an indicator function of being in income interval $i$. 

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**Figure 1: Overall structure of the passenger demand model.**

Long-term choices

- Strategic model
- Below 24h
- Above 24h
- All

Trip duration

- Below 24h
- Above 24h
- All

Geography

- National
- International
- National and international

Model

- Week-day
- Weekend
- Day model
- Over-night model
- Transit model
The concept of introducing a (lower-level) occupancy rate model implies that the occupancy rate is an implied attribute of the choice of mode and destination in a similar way as time and cost are. An example would be that three persons playing golf together, if choosing a golf club close to where they live, will go with their own cars. However, if choosing a club far away, the likelihood of going together increases as the cost savings out weight cost of arranging car pooling. The same mechanism works in other segments as well. We will include more details on the occupancy rate model in the paper however it is worth noting that the introduction of $L F_{car}(d|s)$ in itself leads to a cost-damping effect as the cost is deflated more for longer distances where the occupancy rate is higher.

\[
\begin{align*}
\text{cost}_{\text{car}}(d|s) &= \frac{c_{\text{car}}(d|s)}{O R_{\text{car}}(d|s)} \left[ \sum_i \frac{1_i}{V o T(i,s)} \right] \\
\text{cost}_{\text{carp}}(d|s) &= \frac{c_{\text{carp}}(d|s)}{O R_{\text{carp}}(d|s)} \left[ \sum_i \frac{1_i}{V o T(i,s)} \right] \\
\text{cost}_{\text{pub}}(d|s) &= c_{\text{pub}}(d|s) \left[ \sum_i \frac{1_i}{V o T(i,s)} \right]
\end{align*}
\]

For public transport time consists of many components, which is estimated separately. However, the main component is the in vehicle-time $\text{Inv}_{\text{pub}}(d|s)$.

\[
\text{time}_{\text{pub}}(d|s) = \text{Inv}_{\text{car}}(d|s)
\]

In the estimation we have applied Box-Cox $BC(x, \mu)$ transformations for both time and cost components in order to capture “damping preferences”. A stripped-down representation of the utility functions is shown below

\[
\begin{align*}
V_{\text{walk}}(d|s) &= \beta_{t_{\text{walk}}} \text{time}_{\text{walk}} ... \\
V_{\text{bike}}(d|s) &= \beta_{t_{\text{bike}}} \text{time}_{\text{bike}} ... \\
V_{\text{car}}(d|s) &= \beta_c BC(cost_{\text{car}}, \gamma_c) + \beta_{t_{\text{car}}} BC(time_{\text{car}}, \gamma_t) ... \\
V_{\text{carp}}(d|s) &= \beta_c BC(cost_{\text{carp}}, \gamma_c) + \beta_{t_{\text{car}}} BC(time_{\text{car}}, \gamma_t) ... \\
V_{\text{pub}}(d|s) &= \beta_c BC(cost_{\text{pub}}, \gamma_c) + \beta_{t_{\text{pub}}} BC(time_{\text{pub}}, \gamma_t) ... \\
V_{\text{air}}(d|s) &= \beta_c BC(cost_{\text{air}}, \gamma_c) + \beta_{t_{\text{air}}} BC(time_{\text{air}}, \gamma_t) ... 
\end{align*}
\]

The following setup superimposes the relationship between VoT and income. It was tested if this relationship could be empirically estimated, however, for income groups over 75.000 Euro / Year we experienced identification problems due to the low number of observations. To have a robust specification...
we have therefore maintained the current formulation. However, by introducing a time and cost parameter in the model we estimate a scaling between cost and time in order to express that the mean VoT (in the data) may be different from the impose mean value of the VoT table. It turns out to be important to estimate time parameters by mode as a generic formulation will tend to over-estimate time-effects for public transport in particular.

Below we present a sub-set of parameter for the commuter segment.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Std.Dev.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_c$</td>
<td>-0.9974E-01</td>
<td>0.445E-02</td>
<td>-22.4</td>
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<td>$\gamma_c$</td>
<td>0.79</td>
<td>*</td>
<td>*</td>
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<td>$\beta_{\text{walk}}$</td>
<td>-0.9810E-01</td>
<td>0.441E-02</td>
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<td>$\beta_{\text{bike}}$</td>
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<td>-28.3</td>
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<tr>
<td>$\beta_{\text{pub}}$</td>
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<td>0.236E-02</td>
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<td>$\gamma_t$</td>
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<tr>
<td>$\mu \text{ (logsum)}^3$</td>
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<td>0.703E-02</td>
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<td></td>
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<td>Likelihood</td>
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<tr>
<td>$\rho^2$</td>
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</tbody>
</table>

Table 1: Selected parameters for the primary commuter model. Parameters for $\gamma_c$ and $\gamma_t$ has been approximated using a bi-section approach and standard errors are therefore not directly available.

References


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$^3$ The presented model is nested as $p(m|d)p(d)$, however, the nesting structure varies with trip purposes.