A control strategy to prevent propagating delays in high-frequency railway systems

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1. Introduction

The railway system in Tokyo's metropolitan area is a dense network that operates within an overcrowded timetable (i.e., average headway is 2 to 3 minutes) enabling an enormous number of people to commute during morning peak periods. However, the overcrowded timetable does not permit sufficient slack time to enable train delays to be prevented; this destabilizes the railway system through delays that easy occur and propagate. As a result, train bunching arises even if train delays are only brief.

Propagation of delays in high-frequency railway systems is mainly caused through two types of mechanisms. The first is a delay propagation mechanism at *stations*, which is well known from past experiences and the theory of public transportation systems, e.g., Newell and Potts [1]. This shows that passenger boarding times for trains change according to the headway to the forward train. If the actual headway becomes longer than that scheduled in the timetable, the train faces more passenger and boarding times increase. As consequence, any following train tends to catch up. The second is a delay propagation mechanism on *railway tracks*, which was recently reported by Kariyazaki et al.[2]. In a field experiment held within Tokyo, they showed that congestion (i.e., stop-and-go motion) occurs on tracks with high train densities, very similar to road traffic congestions. Thus, there is a need to establish a control strategy that stabilizes high-frequency railway systems where both delay propagation mechanisms; such strategies have not yet been developed [1].

We propose a railway operation control strategy that prevents these propagating delays in the running of a high-frequency railway system. We first formulate a novel dynamical model of train motion within the system that includes the two types of delay propagation: one relates to passenger boarding at each station, and the other describes train dynamics on each railway track. We then introduce a feedback-control strategy that consistently combines both the holding strategy at each station and the control strategy along the railway tracks. We prove that the dynamical system under the proposed strategy is asymptotically stable at an equilibrium, i.e., timetable schedules can be maintained [2].

2. A dynamical model of the train motion

We consider a moving-block railway system, operating as a shuttle service, on a single line track with N trains and S stations. Let $t_n(s)$ be the arrival time of train n at station s. Then, a dynamical system that represents each train motion is given by

$$t_n(s+1) = t_n(s) + b_n(s) + c_n(s) \qquad \forall n \in N,\ \forall s \in S \tag{1}$$

where $b_n(s)$ is the passenger boarding time of train n at station s, and $c_n(s)$ is the trip time of train n between stations s and $s+1$, which are determined by the following sub-models.

The passenger-boarding model used is the same as Daganzo [3], i.e., the boarding time $b_n(s)$ is in proportion to the headway:

$$b_n(s) = \beta_s(t_n(s) - t_{n-1}(s)) \qquad \forall n \in N,\ \forall s \in S \tag{2}$$

[1]Daganzo et al. [3, 4, 5] have already proposed some dynamic holding strategies to stabilize bus system operations. However, these studies considered only delay propagations at bus stops. Therefore, these strategies may not be pertinent to high-frequency railway system operations when accounting for the two types of delay propagation.

[2]In recent years, a number of studies have been directed towards the problem of constructing robust timetables[6]. Although robust timetabling approaches are successfully aplicable in some practical settings, these models do not explicitly treat the two dynamic mechanisms of delay propagation stated above. Thus, we cannot verify that an “robust” timetable guarantees the stability of the railway system.

where β_s is the demand rate at station s, assumed to be time-independent. This sub-model accounts for the delay propagation each station.

Train motion on the railway track is modeled based on the linear car-following model proposed by Newell [7]. This model accounts for delay propagations along the railway track. Specifically, a trip time of each train between stations is determined according to the trip pattern that is related to the spacing between the train and the forward train: the trip times of all trains are given as the following simultaneous equation system:

$$c_n(s) = C_{n,s}\left(t_n(s), t_{n-1}(s), t_{n-1}(s+1), t_{n-2}(s+1)\right) \qquad \forall n \in N,\ \forall s \in S. \tag{3}$$

Finally, we obtain a complete dynamical system of all train motions by substituting the sub-models (2), (3) into the state equation (1). Furthermore, we can find a simple representation of the model by using state vectors that are defined so as to satisfy $k = n + s$:

$$\mathbf{t}^k \equiv [\ \cdots t_{n-1}(s+1) \quad t_n(s) \quad t_{n+1}(s-1) \cdots\ ]^T. \tag{4}$$

That is,

$$\mathbf{t}^{k+1} = \mathbf{F}(\mathbf{t}^k, \mathbf{t}^{k-1}) \tag{5}$$

$$\mathbf{F}(\mathbf{t}^k, \mathbf{t}^{k-1}) \equiv [\ \cdots F_{n-1}(s+1) \quad F_n(s) \quad F_{n+1}(s-1) \cdots\ ]^T \tag{6}$$

where $F_n(s)$ is the right hand side of the equation (1). This dynamical model is simple and captures the aforementioned delay propagation mechanisms, which enables us to establish a train control strategy both analytically and systematically.

3. Stability analysis of railway systems with and without control strategy

A railway system operating without control strategy (5), and thus operates according to the timetable, is unstable in the sense of local stability (see Figure 1 (a)). Specifically, we define an equilibrium for train motion as the actual arrival time for each train at each station $t_n(s)$ which equals to the time $T_n(s)$ in the timetable schedule. Then, the maximum absolute eigenvalue of the following Jacobian matrix $\mathbf{J}(\mathbf{T}^k, \mathbf{T}^{k-1})$ of the dynamics (5) at equilibrium is above 1.

$$\mathbf{J}(\mathbf{T}^k, \mathbf{T}^{k-1}) \equiv \left[\begin{array}{c|c} \nabla_{\mathbf{t}^k}\mathbf{F}(\mathbf{T}^k, \mathbf{T}^{k-1}) & \nabla_{\mathbf{t}^{k-1}}\mathbf{F}(\mathbf{T}^k, \mathbf{T}^{k-1}) \\ \hline \mathbf{I} & \mathbf{0} \end{array}\right] \tag{7}$$

To stabilize the railway system, we need to introduce a state-feedback control strategy. On implementing this control strategy, denoted $\mathbf{P}(\mathbf{t}^k, \mathbf{t}^{k-1})$, the train dynamics and its Jacobian matrix at equilibrium are given by

$$\mathbf{t}^{k+1} = \mathbf{F}(\mathbf{t}^k, \mathbf{t}^{k-1}) + \mathbf{P}(\mathbf{t}^k, \mathbf{t}^{k-1}) \tag{8}$$

$$\hat{\mathbf{J}}(\mathbf{T}^k, \mathbf{T}^{k-1}) \equiv \left[\begin{array}{c|c} \nabla_{\mathbf{t}^k}\mathbf{F}(\mathbf{T}^k, \mathbf{T}^{k-1}) + \nabla_{\mathbf{t}^k}\mathbf{P}(\mathbf{T}^k, \mathbf{T}^{k-1}) & \nabla_{\mathbf{t}^{k-1}}\mathbf{F}(\mathbf{T}^k, \mathbf{T}^{k-1}) + \nabla_{\mathbf{t}^{k-1}}\mathbf{P}(\mathbf{T}^k, \mathbf{T}^{k-1}) \\ \hline \mathbf{I} & \mathbf{0} \end{array}\right]. \tag{9}$$

Among the various control strategies possible in practice, we propose a quite simple one that consistently combines the holding strategy at the station and the control strategy on the track. Our control strategy is based on a weighted average $E_n(s)$ between delay and forward headway:

$$\begin{aligned} E_n(s) &\equiv (1-\alpha)\epsilon_n(s) + \alpha\beta_s(\epsilon_n(s) - \epsilon_{n-1}(s)) \\ &= (1 + \alpha\beta_s - \alpha)(t_n(s) - T_n(s)) - \alpha\beta_s(t_{n-1}(s) - T_{n-1}(s)) \end{aligned} \tag{10}$$

where α is a weight parameter and $\epsilon_n(s) \equiv t_n(s) - T_n(s)$ is a deviation from the timetable. More specifically, our control strategy operates to make $E_n(s)$ to be zero (i.e., delay is eliminated) by holding the train within the slack time at the station and by speed-up so as not to catch up to the forward train.

We prove that the railway system under the proposed control strategy is asymptotically stable at equilibrium: the maximum absolute eigenvalue of the Jacobian matrix $\hat{\mathbf{J}}(\mathbf{T}^k, \mathbf{T}^{k-1})$ is below 1. We also demonstrate through systematic numerical experiments that the railway system under our control strategy not only prevents delay propagation but can also recover from delays quickly (see Figure 1 (b)). Furthermore, we show numerically that the train dynamics under the extended version of our control strategy globally converges to the equilibrium (i.e., timetable schedule) at moderate demand levels.

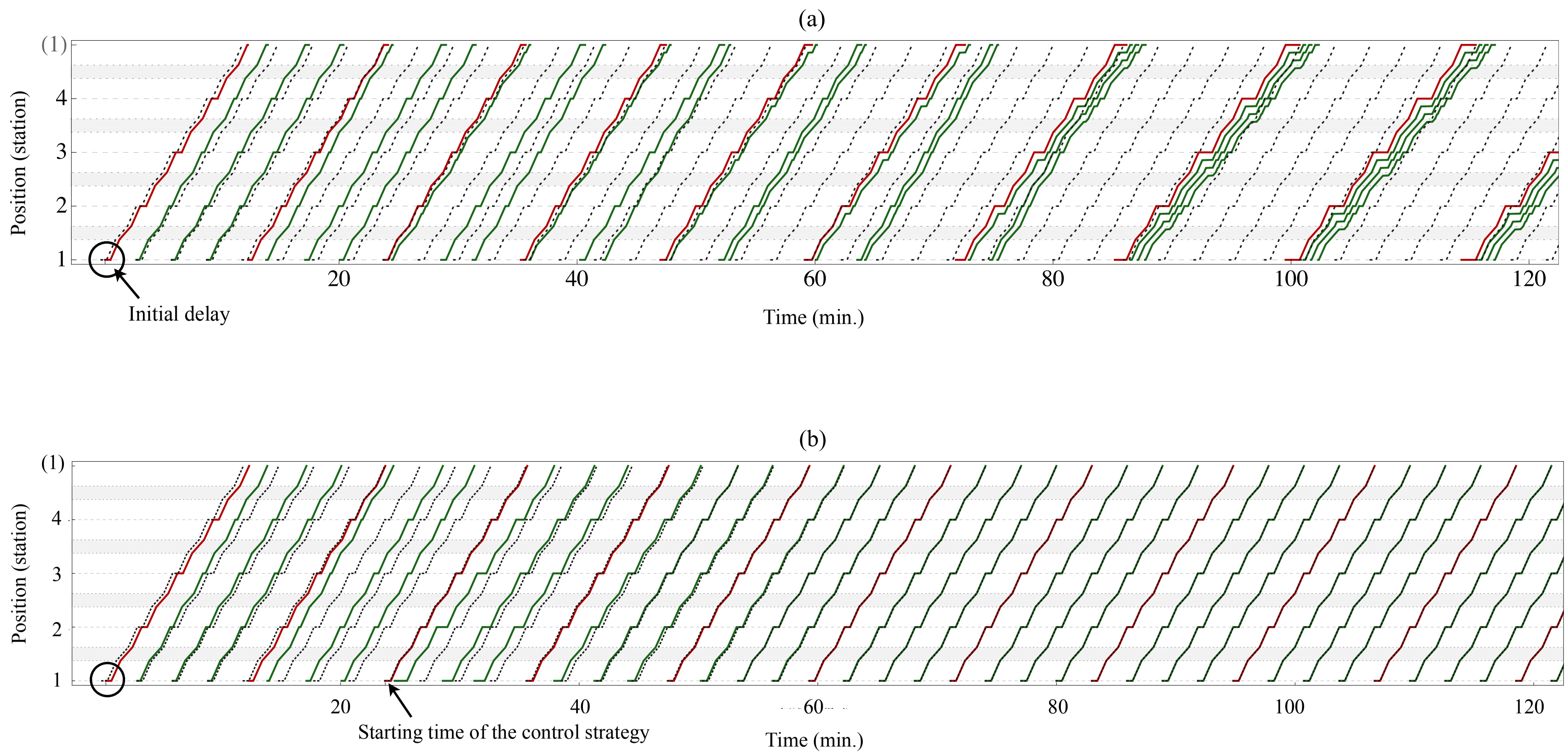

Figure 1: Trajectories of 4 trains on closed loop with 4 stations. (a) Without control strategy; (b) with proposed control strategy.

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