

Determining Optimal Control Stop to Improve Bus Services Reliability

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1 Bus Bunching and Holding Strategy

Bus bunching is a common problem of public transport in cities and a vital factor of determining service reliability. As first studied in [1], the service schedule cannot remain stable because of both the uncertainty of the number of boarding and alighting of passengers, and the variability in traffic conditions and hence travel time in between stops. Furthermore, the driving behavior of the bus captain is also considered as one of the factors leading to bus bunching [2].

One approach to address bus bunching and to maintain the service schedule is to add slack time into the schedule. However, too much slack time may cause the slowness of the buses and the reduction in service frequency [3]. In [4], a model to determine optimal slack time inserted into the schedule is proposed by minimizing the expecting waiting times of the passengers.

In many cities, the majority of typical urban bus services run at least in peak times with a headway shorter than 15 minutes. In such a situation, the schedule based strategy may cause additional on-board waiting time. To avoid this, the service operators tend to adopt some other strategies, of which the most studied is the holding strategy [5–11].

Different from the previous studies, this paper will not focus on how but where to control services and propose a methodology to determine the optimal position of the control stop along a service route. The application to a real scenario is demonstrated based on a simulation which is sourced by detailed data on effective traffic patterns for a bus line in Singapore. It builds on an study by [9] which concluded that control stops should have the following properties:

1. A high level of boarding demand and
2. close to the middle of the service route.

The main contribution to this presented in this paper is a formalization and simulation-based implementations to find optimal control stops. To this means, two different models to determine the position of the control stop including demand patterns are presented.

2 Models to Determine the Holding Stop

For a certain route service with M stops and N buses running on, the departure time of bus k at stop i is defined as $t_{i,k}$. For any stop i , the headway between bus $k + 1$ and its preceding one (bus k) is determined by:

$$H_{i,k} = t_{i,k+1} - t_{i,k} \quad (1)$$

where $i = 1, 2, \dots, M - 1$, $k = 1, 2, \dots, N - 1$.

The number of passengers who board on stop i and alight on stop j is defined as $B_{i,j}$, which is the value from the OD matrix for the service.

In this study, if stop m is chosen as the control stop where buses will be rescheduled, the headway for stop m are set to be the same as the initial terminal (stop 1). If the a bus reach stop m within the headway interval, it will wait until the intended headway is reached, otherwise, an empty bus will depart from stop m . Under this assumption, determining a control stop is similar to the strategy of cutting a bus service into segments as studied in [12].

2.1 Waiting Time Based Model

This model tries to find the position of the control stop by minimizing average waiting time of all passengers. which includes both waiting time at the bus stop and the additional on-board waiting time for the passengers travel through the control stop.

If more than one bus arrives the control stop within the same interval, the passengers on the following buses are forced to transfer on the first bus without costing extra time to depart earlier.

As studied in [13], the expectation of the waiting time a certain stop is given by:

$$E(w_i) = \frac{1}{2}E(H_i) \left[1 + \frac{Var(H_i)}{E^2(H_i)} \right] = \frac{1}{2}E(H) \left[1 + \frac{Var(H)}{E^2(H)} \right] \quad (2)$$

The second part of Equ2 refers to bus operation of a whole day. The expectation headway for any stop i is:

$$E(H_i) = E(H) \quad (3)$$

As studied in [12], in reality the variances of the headways increase almost linearly with stop sequence if no control strategy is imposed. The analytical mathematical model presented in [14] also shows that the main factors which influence headway variance are bus loading conditions and traffic conditions along the service route. In this study, the variability caused by changing in traffic conditions is not considered.

For one stop, the increase in variance from preceding stop is assumed to be in proportion to the number of boarding passengers. The headway variances at start of the route (stop 1) and terminal (stop M) are readily available for service operators, which are $Var(1)$ and $Var(M)$ respectively. Thus, headway variances at other stops can be calculated as:

$$Var(i) = Var(i-1) + \frac{Var(M) - Var(1)}{\sum_{i=1}^{M-1} \sum_{j=i}^M B_{i,j}} \cdot \sum_{j=i-1}^M B_{i-1,j} \quad (4)$$

If stop m is chosen as the control stop, then it is assumed that $Var(m) = Var(1)$. For the following stops, the variance can still calculated by Equ(4).

The total waiting time is the sum of three parts. Thus, the objective is to minimize the average waiting time γ :

$$\min \gamma = \gamma_1 + \gamma_2 + \theta \cdot \gamma_3 \quad (5)$$

$$\text{where } \gamma_1 = \frac{\sum_{i=1}^{m-1} \left(E(w_i) \cdot \sum_{j=i}^M B_{i,j} \right)}{\sum_{i=1}^{M-1} \sum_{j=1}^M B_{i,j}}, \gamma_2 = \frac{\sum_{i=m}^M \left(E(w_i) \cdot \sum_{j=i}^M B_{i,j} \right)}{\sum_{i=1}^{M-1} \sum_{j=1}^M B_{i,j}}, \text{ and } \gamma_3 = \frac{\sum_{i=1}^{m-1} \left(\frac{E(H)}{2} \cdot \sum_{j=m+1}^M B_{i,j} \right)}{\sum_{i=1}^{M-1} \sum_{j=1}^M B_{i,j}}.$$

θ is the weigh factor and γ_3 is the additional waiting time to the the passengers who pass through stop m caused by bus holding at control stop. As several studies ([15]) suggest that passengers are more sensitive to at-stop waiting time than the riding time on bus, it can be given that $\theta \in [0, 1]$.

Based on this objective function, the optimal control stops for both directions can be found. Same as [12], the arrival and departure time for every stop are estimated based on the fare card data records and interpolation of trajectories if there are no records.

2.2 Demand Based Model

In this part, a simplified model which takes demand information (OD matrix) into consideration is proposed and discussed.

A bus service with stop m as a designated control stop, can be interpreted as two distinct route services. Intuitively, the control point should have the ability to have large direct flows in those new routes and try to reduce number of transfers.

The model is proposed based on actual demand patterns as observed by records of smart card fare collection data. δ_1 , δ_2 and δ_3 represent the demand parameters of direct flow for previous stops, direct flow for after stops and flow passing the control point. Then, the objective function of this model is:

$$\max \delta = \delta_1 + \delta_2 - \delta_3 \quad (6)$$

$$\text{where } \delta_1 = \frac{\sum_{i=1}^{m-1} \sum_{j=i}^{m-1} B_{i,j}}{m}, \delta_2 = \frac{\sum_{i=m}^M \sum_{j=i}^M B_{i,j}}{M-m+1}, \text{ and } \delta_3 = \frac{\sum_{i=1}^{m-1} \sum_{j=m+1}^M B_{i,j}}{\sqrt{m(M-m+1)}}.$$

2.3 Case Study

The two proposed models are tested in a case study based on service route in Singapore which has 71 stops in one direction and 74 stops in the other direction. This route has already previously been selected for the case study on service reliability [12], which provided the information on headway variability.

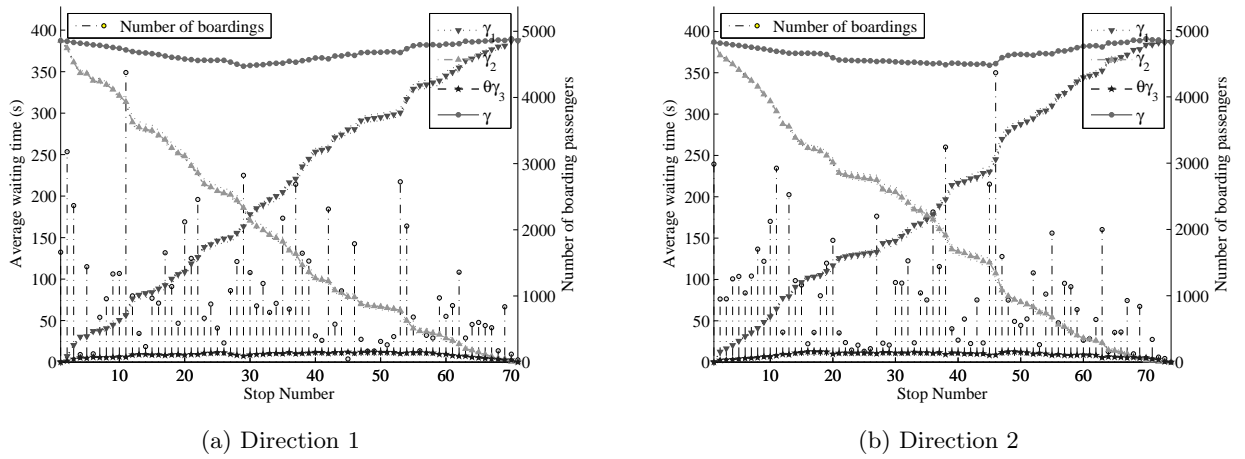


Figure 1: Waiting time model

2.3.1 Waiting Time Based Model

In this case, variances for start and final terminal are $1 \times 10^4 s^2$ and $22 \times 10^4 s^2$ respectively. Since $\theta = 0$ means passengers ignore the additional waiting at control stops and $\theta = 1$ means there's no difference between the on-boarding waiting and at-stop waiting, in this case, θ is defined as 0.2. Analysis of θ will be presented in future works.

Figure 1 and Figure 2 show the result from the waiting time based model. As indicated in Figure 2, stop 29 and stop 45 are the best control points for the two directions respectively.

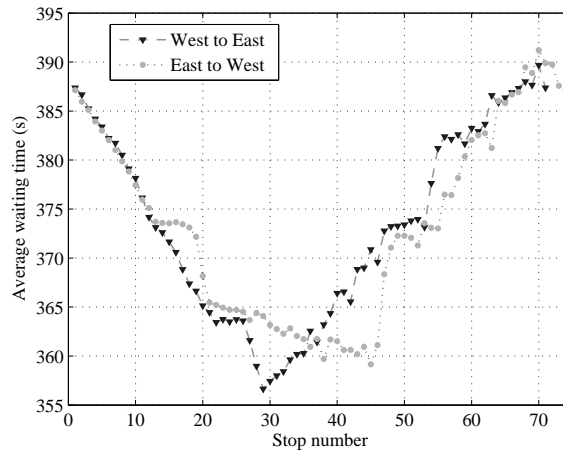


Figure 2: Waiting time against control stop for two directions

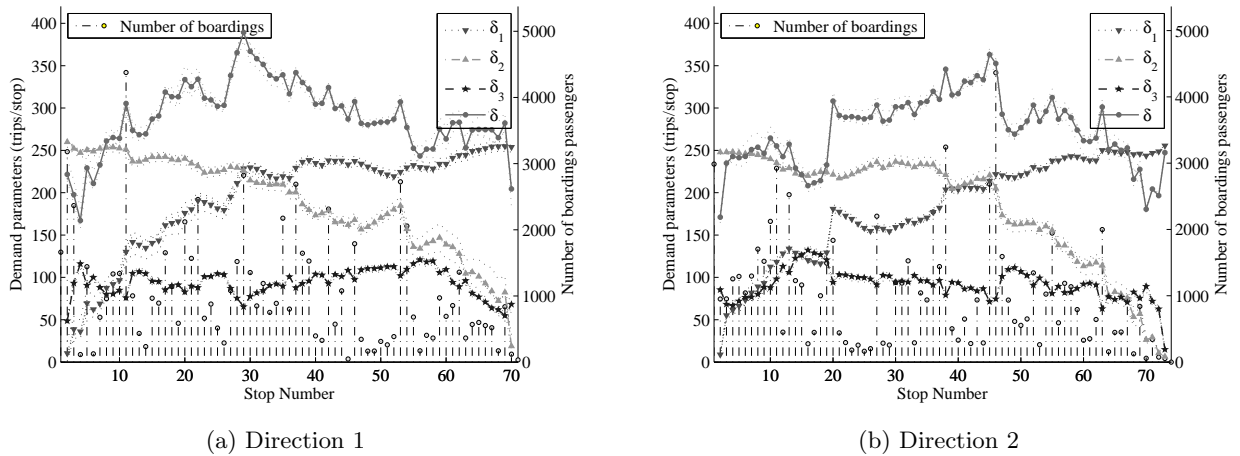


Figure 3: Demand model

2.3.2 Demand Based Model

Figure 3 shows the parameters calculated from the demand model. Based on the value of δ , it obtains the same optimal control stops as the waiting time model.

2.3.3 Implication

Simulations are also conducted to compare the scenarios with and without the control stops (i.e. stop 29 and stop 45 on two directions). Without inserting control stops, a fleet of 26 buses are needed. For the scenario with two control stops, the size of the fleet should be 27, i.e. one addition bus is needed compared with the original service schedule. Simulations also show that both of the control stops should have the capacity to store 3 empty buses.

Practically, for a service with two directions sharing the same route, it would be more easily to apply if the two control stops are close enough to each other. Regarding to the case service with a length of $27,6km$, stop 29 and stop 45 are located at $12.4km$ and $12.3km$ on direction 1 respectively. The distance between two stops is about $140m$ which is adequately short to combine the control stops into a terminal. For future research, multi-services sharing the route will be considered to study the availability of building the terminal.

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