A proposition of algorithm for modeling air traffic and road networks

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Extended Abstract

This article suggests an algorithm, based on geographical principles, aimed at building different kind of undirected simple networks between given nodes located in an Euclidean space. The resulting networks turn out very similar to some classes of real transportation networks, and hence provide interesting models aimed at understanding and analysing the fundamental structure of those networks. The present algorithm bears strong similarities to the work of M.T. Gastner and M.E.J. Newman (*The spatial structure of networks*, 2006, The European Physical Journal B), who propose, however with a differing point of view, very similar network models, yet obtained by solving a combinatorial optimization problem, rather than with an iterative algorithm as in the present case.

The initial configuration needed for the present algorithm is a set of n fixed points in a $\mathbb{R}^p$ space with relative weights $w_i > 0$. In a geographical context, those points can be seen as cities of different sizes, located on a map ($p = 2$). Our algorithm will create or destroy edges between those points, with two conflicting objectives in scope: the first one will be to minimize a quantity, called here the *cost* (C) of the network, which is simply the sum of all edges Euclidean length d_{ij}, i.e. $C = \sum_{i,j=1}^{n} d_{ij} a_{ij}$ where A is the adjacency matrix of the graph. The second goal will be to maximize another quantity, called here the *accessibility* (H) of the network, which is the sum of the inverse of generalized geodesic distances δ_{ij} (defined below) between all nodes, weighted by w_i, i.e. $H = \sum_{i,j=1}^{n} \frac{w_i w_j}{\delta_{ij}}$. In geographical terms, the cost of a traffic network, whose primary purpose it to bring cities mutually closer, is proportional to its total length. To fix an equilibrium between the opposite objectives, we introduce a quantity $T > 0$, which arbiters between the latter, and devise an algorithm aimed at minimizing the functional $F = C - T \cdot H$. This formula is inspired from the thermodynamical equilibrium formula `Free Energy = Energy - Temperature · Entropy`. In the present case, the parameter T can be interpreted as a "development index" of the network, and will be fixed from the start.

The algorithm is defined as follows:

1) We create a permutation of all possible dyads in the networks, in order to introduce randomness in the process.
2) We scan all dyads one by one. If there is an existing edge, we remove it if F decreases. If there is no edge, we create one if F decreases.
3) If we have made any changes during point (2), we go back to point (1). If not, we stop.

We can easily show that this algorithm converges in a polynomial time and that the resulting network is a local minimum of F.

We define the generalized geodesic distance δ_{ij} between two vertices i and j as $\min_{\gamma \in \Gamma_{ij}} \sum_{e_{ab} \in \gamma} l(e_{ab})$, where Γ_{ij} denotes the set of all paths between i and j and e_{ab} denotes an edge part of a path. We define in turn $l(e_{ab})$, the length of the edge between a and b, as $l(e_{ab}) = d_{ab}^{\lambda}$, where $\lambda \geqslant 0$. A particularly interesting feature about the algorithm consists in observing the dramatic change in the resulting network by varying the value of λ. When $\lambda = 0$, the geodesic distance simply counts the number of edges in the shortest path between i and j, and with $\lambda = 1$, it expresses the "real" traffic distance along the edges. In the first case, the resulting networks exhibit scale free properties, creating a few high degree "hubs" as well as many 1-degree nodes. These networks look very similar to air traffic networks, as if the latter networks did not attempted so much to minimize the flight distances, but rather the number of different flights needed to reach a new city. In the second case, the resulting networks are planar, and look a lot more like a road network. In this case the emphasis is put on minimizing the actual distance driven along the roads, no matter if a city is crossed of bypassed.

This paper will study different aspect of this algorithm. We first explore the behavior of the algorithm, by examining the evolution of some network statistics, like degree distribution, betweenness centrality distribution, diameter and spectrum, under change of the free parameters of the algorithm, namely T and λ, or under change of the initial configuration parameters, namely n, p and the points' distribution in space. Then we will show that those networks share characteristics similar to real world network, with all proper reserves, as the real networks are of course also influenced by many other factors completely ignored there. In this part we will also propose estimation strategies for the free parameters of the algorithm, allowing us to simulate as closely as possible a given network. The estimated value of the parameters permits in addition to characterize the observed network; typically, a large value of T characterize an highly higher ”developed” network. Finally, limitations of the model and possible improvements will be discussed. One of the main difficulties with the algorithm consists of its extensive use of the geodesic distance, a quantity difficult to manipulate formally, thus making the production of proofs intricate. Some clues aimed at overtaking the problem and to improve the analytical formulation of the algorithm will be mentioned.