On the use of entropy for the estimation of vehicle OD matrices within urban commodity-based models

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The distribution of goods to retailers and final consumers represents the last mile segment of a supply chain and is a key component of urban freight flows. The descriptive models developed to date for urban goods distribution can be classified according to the following categories (Comi et al. [1]): commodity-based, truck-based and delivery-based. The commodity-based approach models the quantities of goods that are moved between every origin-destination (OD) pair in the different chains. This approach has the advantage of capturing the fundamental mechanisms underlying freight transport demand because behavioural models can be used to represent the restocking and final consumption activities (as in the modelling framework proposed by Russo and Comi [2]).

The conversion of the OD quantity flows into OD vehicle flows is a critical step in the approach as modelling of the tours made by the vehicles is required. The simplest way to tackle this step is to assume that only direct deliveries occur, i.e. one delivery only per tour. Empirical evidence suggests that this is not the case in a number of chains. Several authors have dealt with the conversion step using different approaches (a review is in Comi et al. [1]). The present paper proposes an entropy-based approach.

The concept of entropy provides a useful paradigm for system modelling (a review is in Wilson [3]). There are two main ways of viewing entropy:

(i) Entropy as measure of probability of the macro-states of the system: the maximum entropy, i.e. the most probable, macro-state is the one that is associated with the highest number of micro-states; this view, representing Wilson's proposal [4], is derived from statistical mechanics.

(ii) Entropy as measure of uncertainty of a probability distribution: the maximum entropy probability distribution is the one about which the modeller is most uncertain subject to the partial information on the system with which she is endowed. This view, representing Jaynes' proposal [5], is derived from information theory.

In the area of transport applications, entropy has been used to provide a theoretical foundation to gravity models (Wilson [4]), to estimate the most likely trip matrices from traffic counts (Van Zuylen and Willumsen [6]; Fisk [7]), and to estimate the matrix of OD vehicle flows within a truck-based approach to urban goods distribution modelling (Wang and Holguín-Veras [8]). In all these applications the first of the two views of the entropy concept seen above has been used.

The paper formulates an entropy-based model for the estimation of the matrices of OD vehicle flows within a commodity-based approach. The model uses the second of the two views of the entropy concept. The model is formulated as a maximisation problem of an entropy-like objective function subject to linear equality constraints and non-negativity constraints on the decision variables. This class of optimisation problems has been studied extensively both from the theoretical properties' and from the computational point of view (a comprehensive review is in Fang et al. [9]). The optimal solution is unique if the feasible region is non-empty. Efficient algorithms are available.

First, we introduce a few assumptions and definitions. Deliveries occur in traffic area zones, each associated with a centroid (O or D). The network is represented by a graph composed by nodes, which include centroids and other nodes, and directed links. A feasible tour of order *r* is any ordered sequence of *r* centroids. The set of feasible tours identify the tour-zone incidence matrix and the tour-OD pair incidence matrix. Each feasible tour is associated with a probability of being made. The number of vehicle journeys following a specific tour is referred to as tour flow and is proportional to the tour probability. Given the tour flows, the associated matrix of OD vehicle flows is obtained by a set of structural equations where the coefficients are the elements of the tour-OD pair incidence matrix. Each link of the graph is associated with a cost. The tour follows the minimum cost route on the graph between each OD pair. The tour cost is additive with respect to the constituent links.

The decision variables are the tour probabilities. The objective function is the entropy measuring the uncertainty associated with a given tour probability distribution. Jaynes [5] has shown that there exists a unique family of functions that satisfy a set of desirable properties for an uncertainty measure. The functions of this family are unique up to a multiplicative constant which is irrelevant to the optimisation.

There are two sets of constraints. The first set represents structural properties: the tour probabilities are non-negative and their sum equals unity. The second set represents the information available to the modeller. This includes the matrices of OD quantity flows, provided by the upstream commodity-based models. This information translates into a set of demand constraints on the tour flows where the coefficients are the elements of the tour-zone incidence matrix.

The number of feasible tours needs to be reduced to a practically tractable size. To this aim, a sample of tours needs to be observed for each chain providing: the frequency distribution of the tour order, the frequency distribution of the number of distinct zones included in the tour, the maximum number of times a zone is re-entered along a tour. This information is used to reduce the set of feasible tours to the smaller set of candidate tours having a non-zero probability of being made. In addition, we need a cost constraint for the tours that include two or more zones. This constraint is derived from the sample information on the average tour cost.

The model has two notable special cases where the solution is in closed form. When all tours have the same order and one zone only is included in each tour the solution is obtained directly from the demand constraints and equals the minimum-cost solution for the given order. When all tours have the same order, if all feasible tours are considered and the cost constraint is not included the solution is a multi-proportional model.

Test networks are used to illustrate the model's application. A critical analysis of the approach makes evident a limitation in the use in forecasting. Similarly to gravity models, it is necessary to assume unchanged the quantities obtained from the observed sample that are used in the model's constraints. The approach might benefit from developments in research which is investigating how to obtain tour-related information from simulation.

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