

# Congestion and Vehicle Emission Pricing with a Bilevel Bi-objective Optimisation Model

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The cost of congestion and the environmental cost of vehicle emissions are two major externalities in transport. To enhance sustainability in transport, congestion pricing is a policy instrument that has been applied in many cities around the world, e.g. Singapore, Stockholm and London. To maximise the effectiveness of this policy instrument, it is only natural to consider internalising both congestion and vehicle emission costs by charging road users an appropriate toll.

The classical formulation of toll optimisation in a road network is a bilevel optimisation problem [7], whereby the upper level represents the decisions of the planner or policy decision maker, and the lower level represents the decisions of the travellers. The upper-level decision maker decides on the tolls, and given the tolls, the travellers make their route choices based on their preferences. Naturally this should be a multicriteria decision problem at both levels. For instance, by optimising the tolls, the upper-level decision maker would want to minimise not only the total travel time, but also the vehicle emission levels such that the environmental and subsequent health impacts can be minimised. On the other hand, given the travel time and toll on different routes for an origin-destination (O-D) pair, users might have different preferences as per their willingness to pay, yet it is only natural to think that all users will want to minimise travel time as well as the monetary cost.

In this paper, we propose a bilevel bi-objective approach to optimise the tolls in a road network. We believe that considering multiple objectives at both levels is important for several reasons. To support sustainability analysis, it is important to consider multiple objectives at the upper level. For example, since the tolls that minimise total travel time do not necessarily minimise the emission levels [4, 8], it is important to determine the *efficient* tolls such that neither the total travel time nor the total emissions can be reduced without worsening the other. For the lower level, it is important to model the variability of preferences among individuals in terms of their willingness to pay. The bilevel formulations in the literature e.g. [7, 8], are based on a user equilibrium (UE) formulation at the lower level, which might not represent route choice behaviour realistically.

Multiple objectives have been considered at either the upper or lower level in the literature in various ways but not both. For the lower level, Dial [1] was the first to consider bicriterion traffic assignment, minimising time and cost as the two objectives in a route choice model. In Dial's model [1], however, a simplification was made by adding time and toll cost in a linear choice function, which is essentially the same as the generalised cost function, but with the value of time being a probabilistic component. As discussed in [6], Dial's model only represents a special case of bi-objective user equilibrium (BUE) condition, whereby traffic arranges itself in such a way that no individual trip maker can improve either their toll or travel time or both without worsening the other component by unilaterally switching routes. Dial then further developed algorithms to optimise the tolls such that the system travel time can be minimised [2, 3], i.e. a single objective is considered at the upper level.

Yin and Lawphongpanich [8] were the first to apply bi-objectives at the upper level: (1) to minimise system travel time; and (2) to minimise total CO emissions, while the lower level is a classical user equilibrium (UE) traffic assignment model. The non-dominated frontier of the two objectives is identified, which consists of all combinations of total travel time and total emissions such that neither of the two can be reduced without worsening the other. Yin and Lawphongpanich [8] showed that the first-best pricing scheme does not necessarily lead to fewer emissions, which is consistent with the observations made by Nagurney *et al.* [4].

In our proposed model, we consider two objectives at both levels. At the upper level, we consider the two objectives as in [8], i.e. to minimise system travel time and total vehicle

emissions. For the lower level, we adopt a BUE model as proposed in [6]. We assume that all users have two objectives: (1) to minimise travel time; and (2) to minimise toll cost. Hence the general model is a bilevel bi-objective optimisation model with bi-objective equilibrium constraints, a very hard problem to solve.

We further specify the general BUE model as a time surplus maximisation bi-objective user equilibrium (TSMaXBUE) model as proposed in [5]. TSMaXBUE assumes that users have different preferences in the sense that for any specific toll, there is a limit on the time that an individual would be willing to spend for a trip from their origin to their destination. Each individual can have their own preferences represented by this indifference curve between toll and time. Time surplus is defined as the maximum time willing to spend minus the actual travel time. Given a set of paths, the one with the highest (or least negative) time surplus will be the preferred path for the individual. Hence, each individual can have a different preferred path, even though all individuals are considering the same choice set (the set of efficient paths). Users are all rational in the sense that they will only choose one of the efficient paths (those that do not allow an improvement of travel time or toll without deterioration of the other). This modified lower level problem can be formulated as a complementarity problem and solved with global optimisation methods. Based on the distribution of individual indifference curves, we can deduce an equilibrium solution satisfying the TSMaXBUE condition. That is, all individuals are travelling on the path with the highest time surplus value among all the efficient paths between this O-D pair. Based on our bilevel bi-objective optimisation model, we will be able to derive the set of efficient toll charges such that neither total travel time nor total vehicle emission level can be improved without worsening the other.

## References

- [1] Dial, R. (1979). A model and algorithm for multicriteria route-mode choice. *Transportation Research Part B*, **13**, 311–316.
- [2] Dial, R. (1999a). Network-optimised road pricing: Part I: A parable and a model. *Operations Research*, **47**(1), 54–64.
- [3] Dial, R. (1999b). Network-optimised road pricing: Part II: Algorithms and examples. *Operations Research*, **47**(2), 327–336.

- [4] Nagurney, A., Qiang, Q., and Nagurney, L. S. (2010). Environmental impact assessment of transportation networks with degradable links in an era of climate change. *International Journal of Sustainable Transportation*, **4**, 154–171.
- [5] Wang, J. Y. T. and Ehrgott, M. (2011). Modelling stochastic route choice with bi-objective traffic assignment. In *International Choice Modelling Conference 2011, Leeds, U.K., 4-6 July 2011*.
- [6] Wang, J. Y. T., Raith, A., and Ehrgott, M. (2010). Tolling analysis with bi-objective traffic assignment. In M. Ehrgott, B. Naujoks, T. Stewart, and J. Wallenius, editors, *Multiple Criteria Decision Making for Sustainable Energy and Transportation Systems*, pages 117–129. Springer Verlag, Berlin.
- [7] Yang, H. and Lam, W. H. K. (1996). Optimal road tolls under conditions of queuing and congestion. *Transportation Research Part A*, **30**(5), 319–332.
- [8] Yin, Y. and Lawphongpanich, S. (2006). Internalizing emission externality on road networks. *Transportation Research Part D*, **11**, 292–301.