Optimal routing and departure time choice for just-in-time deliveries in time-dependent networks with random delays

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1 Introduction

Supply of goods and services in many modern supply chains occurs as and when required to avoid unnecessary warehousing cost associated with keeping stock reserves. This approach, known as the just-in-time (JIT) strategy, requires a high level of certainty in the arrival times of deliveries.

Fowkes, Firmin et al. (2004) discuss a number of factors on both the demand and supply sides (e.g. restrictions on loading in city centres, vessel schedules at ports, technological requirements of processes) that shift the emphasis from the operational cost to the cost of delivery after the preferred arrival time (PAT). Their study of the valuation of journey time reliability and delay by logistics operators indicates that in JIT deliveries the cost of delay is about three times the operational cost, which is understandable, given that operators face penalties, or even loss of contract, if delays start to occur.

However, reliability of arrival times is difficult to achieve. Transport networks are not only congestible (i.e. travel times depend of traffic levels) and time-dependent (i.e. a small difference in the time of departure may result in large difference in experienced travel times), but also inherently unpredictable (i.e. travel times are different on different days, so the exact travel times become known only after completion of a journey) and have little resilience against service disruptions, so even small disturbances in available capacity, such as a lane closure or accident, may result in large delays.

Typically, if predictability of travel times is low, operators accommodate this by scheduling deliveries to start earlier.(McKinnon, Edwards et al. 2009) argue that the effectiveness of this approach is limited, because adding a significant buffer time can lead to underutilisation of a vehicle fleet if travel times turn out better than expected. At the same time, planning fleet utilisation based on optimistic estimates of expected travel times carries risk of delays.

A similar issue has been studied in the context of commuter journeys, where it was found that adjusting departure time is both a response to travel time uncertainty and a means of spreading the peak. 'Scheduling models' ((Noland and Small 1995), (Bates, Polak et al. 2001), (Noland and Polak 2002), (Bonsal 2004), (De Jong, Kroes et al. 2004), (Ettema and Timmermans 2006), (Batley 2007)) allow for calculation of the optimal buffer time and associated probability of arriving late. 'Bottleneck models' (e.g.(Vickrey 1969), (Ran, Hall et al. 1996), (Arnott, de Palma et al. 1990)) study the trade-off between arriving on time and experiencing congestion: those who want to avoid congestion must arrive too early or too late; those arriving at their preferred arrival time experience maximum congestion but no schedule delay.

Besides adjusting the departure time, travellers also try to choose routes with reliable travel times. The problem of finding best paths in networks with random delays has been subject to extensive research, resulting in many reliability criteria and related methods. For example: path with least possible travel time (Miller-Hooks and Mahmassani 1998); path maximising the probability of realising travel time not exceeding a given threshold ((Frank 1969), (Mirchandani 1976), (Fan, Kalaba et al. 2005); path maximising the probability of being shorter than all other paths (Sigal, Alan et al. 1980); path maximising probability of on-time arrival ((Fan and Nie 2006), (Nie and Wu 2009)); path minimising the risk of delay (Dangelmaier, Klopper et al. 2006); path minimising the maximum travel time ((Yu and Yang 1998), (Montemanni, Gambardella et al. 2004)). Usually more than one path can be identified with a positive probability of being the most reliable.

Consequently, an effective JIT delivery strategy in time-dependent networks with random delays requires combined routing and scheduling, and adoption of an appropriate risk-attitude towards delay, so that informed trade-offs are made regarding the choice of route (e.g. take a longer but reliable route, or a shorter but unreliable one) and buffer time (e.g. leave well in advance and arrive too early, or leave later and risk arriving too late). In addition, it is necessary to acknowledge that route choice made based on pre-trip information may be suboptimal in the light of actual traffic conditions. This has two major implications:

(1) travellers using adaptive path selection may achieve lower expected travel times than the corresponding a priori least expected time paths, (Hall 1986); and

(2) for multiple shipments, additional benefits can be achieved by allocating deliveries to several paths rather than one best path, as demonstrated by (Bell 2006) in relation to repeated hazmat transport.

Both assertions can be combined in the notion of a *hyperpath*, originally proposed by (Nguyen and Pallottino 1988) and (Spiess and Florian 1989) in the context of public transport services with uncertain arrival times, which in the context of traffic network we define as *a set of elemental routes with associated route-usage frequencies that collectively offer better expected travel time than any single path on its own*. A hyperpath with associated departure time is referred to as a *scheduled hyperpath*.

2 Aim

The paper examines two delivery strategies that are based on the concept of a scheduled hyperpath.

The first strategy, *optimal non-adaptive scheduled hyperpath,* defines a set of attractive routes together with assigned optimal route-usage frequencies. The dispatcher allocates elemental routes to drivers accordingly, and each driver follows the assigned route, irrespective of the encountered network conditions. Collectively the split between elemental paths conforms to the optimal path usage frequencies associated with the hyperpath.

The second strategy, *optimal adaptive scheduled hyperpath,* the dispatcher identifies and recommends to each driver a set of attractive routes. It is the driver who decides which of elemental routes included in the set to take, as the journey unfolds, based on the delays he encounters on the network.

We present the strategies in both game-theoretic and linear programming formulations to make the theoretical assumptions behind both models explicit.

The key contribution of this paper is a comparison between these two related strategies in specific contexts: multiple simultaneous deliveries, and repetitive shipments, which has not been specifically examined in the original formulations. Using a simple static network, some important properties of the strategies are identified, which then inform the extension to the time-dependent domain discussed later.

3 Method

3.1 Notation

$G=(I,A)$	directed time-dependent network
$I=\{i\}$	set of nodes with n nodes
$A=\{(i, j)\}$	set of links with m links
A_i^+	set of links (i,j) emanating from node i
A_i^{+*}	subset of attractive links $A_i^{+*} \subseteq A_i^+$
$AP=[0,\mathrm{T}]$	analysis period divided into equal time intervals of duration τ
$G_T=(I_T,A_T)$	time-expanded network
$I_T=\{(i, t)\}$	set of $n(T+1)$ nodes
$A_T=\{(i,t), (j,a_{ij}(t))\}$	set of $O(mT)$ links
$a_{ij}(t) = t + c_{ij}$	time of arrival at node j if one departs from node i at time t
$c_{ij} = c_{ij}(t)$	usual travel time on link (i,j)
$d_{ij} = d_{ij}(t)$	maximum expected delay on link (i,j)
$q_{ij} = q_{ij}(t)$	probability that delay d_{ij} occurs on link (i,j)
$p_{ij} = p_{ij}(t)$	probability that link (i,j) is used / frequency of using link (i,j)
$d_i = d_i(t)$	combined expected delay associated with node i
$u_i = u_i(t)$	expected travel time to reach destination s from node i

3.2 Inputs

In both strategies the potentially attractive routes belonging to the hyperpath are identified using historic travel times on the network. Taking the probability density distributions of link travel times throughout the day, for each time interval one can derive the usual link travel time c_{ij} as the 50$^{\text{th}}$ percentile.

Ordinarily, any increase in travel times above the 50$^{\text{th}}$ percentile is considered 'unpredictable / random delay'. Assuming that a risk-averse dispatcher would plan routes taking into account the maximum value of random delay, we can define d_{ij} as the difference between the 95$^{\text{th}}$ and 50$^{\text{th}}$ percentiles.

These values are used to identify a pessimistic estimate of the expected travel time, so that required departure time from the origin can be calculated for any PAT.

3.3 Non-adaptive hyperpath

Non-adaptive hyperpath has been originally formulated in a game-theoretic framework by (Bell 2000), with a zero-sum game between a dispatcher who wants to move shipments economically and reliably, and a second hypothetical player representing any factors that might prevent achievement of the dispatcher's goals.

In such a framework, the routing strategy is described by usage frequencies of elemental paths. The optimal strategy, which avoids worse-case delays, is determined by replacing the unknown delay probabilities with worst-case probabilities, as a risk-averse dispatcher envisages that disruption will specifically occur on routes used for deliveries (rather than elsewhere in the network). The path usage frequencies and worst-case delay probabilities are related to each other, while the expected cost is independent of the actual location of the incident.

The non-adaptive hyperpath strategy arises specifically from a 'global game' with path selection only at the origin, formulated as the following maximin problem (Bell 2000):

$$\underset{q}{Max}\,\underset{p}{Min} \sum_{(i,j)\in A} c_{ij} p_{ij} + q_{ij} d_{ij} p_{ij} \tag{1}$$

subject to

$$\sum_{(i,j)\in A_i^+} p_{ij} - \sum_{(i,j)\in A_i^-} p_{ij} = g_i \quad \forall i \in I \tag{2}$$

$$\sum_{(i,j)\in A} q_{ij} = 1 \tag{3}$$

$$p_{ij} \geq 0 \quad \forall (i,j) \in A \tag{4}$$

$$q_{ij} \geq 0 \quad \forall (i,j) \in A \tag{5}$$

As shown by constraint (3) the delay may occur on any one (an only one) link in the network. The above problem can also be formulated as a linear program, as shown by (Schmöcker, Bell et al. 2009):

$$\underset{p,d}{Min} \sum_{(i,j)\in A} c_{ij} p_{ij} + d \tag{6}$$

subject to

$$\sum_{(i,j)\in A_i^+} p_{ij} - \sum_{(i,j)\in A_i^-} p_{ij} = g_i \quad \forall i \in I \tag{2}$$

$$p_{ij} d_{ij} \leq d \quad \forall (i,j) \in A \tag{7}$$

$$p_{ij} \geq 0 \quad \forall (i,j) \in A \tag{4}$$

Parameter d in (6) and (7) denotes maximum exposure to delay on any link in the network $d=\max\{p_{ij}.\,d_{ij}\}$.

The solution algorithm proposed by (Bell 2000) is based on a series of shortest-path searches combined with the Method of Successive Averages. The shortest path is always included in the hyperpath, though it may have low frequency of usage.

This approach is suitable for identification of critical links, and has been conceived and applied in the context of repeated hazmat transport and VIP routing by (Bell 2006), (Nagae and Akamatsu 2007) and (Bell, Kanturska et al. 2008).

3.4 Adaptive hyperpath

Adaptive hyperpath has been originally proposed in the context of public transport assignment, as a strategy minimising the total expected travel time, including waiting time at stops, by first determining attractive services and then boarding whichever arrives first. Adapted to the JIT context, the strategy to reach the destination is defined by a partial network that contains only those links that will be used as a consequence of this strategy. A dispatcher wishes to minimise the total expected travel time and determines pre-trip a set of attractive links, but the choice between them takes place on-route because of the random and dynamic character of delays. At each node, the driver selects among the attractive links that on which spare capacity becomes available first.

While generic expressions for the probability p_{ij} of capacity becoming available first on link (i,j) and for compound delay d_i are available in Kanturska, Trozzi et al. (pending), we focus on a special case where the waiting time on each link is exponentially distributed with mean d_{ij}. In this case, these variables can be calculated as follows:

$$p_{ij} = \frac{1/d_{ij}}{\sum_{(i,j)' \in A_i^+} 1/d_{ij'}} \tag{8}$$

$$d_i = \frac{1}{\sum_{(i,j) \in A_i^+} 1/d_{ij'}} \tag{9}$$

Under these assumptions, the adaptive hyperpath can be formulated as a linear program (Spiess and Florian 1989):

$$\underset{p,d}{Min} \sum_{(i,j)\in A} c_{ij} p_{ij} + \sum_{i \in I} d_i \tag{10}$$

subject to

$$\sum_{(i,j)\in A_i^+} p_{ij} - \sum_{(i,j)\in A_i^-} p_{ij} = g_i \quad \forall i \in I \tag{2}$$

$$p_{ij} d_{ij} \le d_i \quad \forall (i,j) \in A_i^+, \forall i \in I \tag{11}$$

$$p_{ij} \ge 0 \quad \forall (i,j) \in A \tag{4}$$

As shown by (Schmöcker, Bell et al. 2009), the adaptive hyperpath also arises from a sequence of 'local games' where an incident can occur at any of the links outgoing from each node of pre-selected routes, formulated as the following maximin problem:

$$\underset{q}{Max}\,\underset{p}{Min} \sum_{(i,j)\in A} c_{ij} p_{ij} + q_{ij} d_{ij} p_{ij} \tag{1}$$

subject to

$$\sum_{(i,j)\in A_i^+} p_{ij} - \sum_{(i,j)\in A_i^-} p_{ij} = g_i \quad \forall i \in I \tag{2}$$

$$\sum_{(i,j)\in A_i^+} q_{ij} = 1 \tag{12}$$

$$p_{ij} \ge 0 \quad \forall (i,j) \in A \tag{4}$$

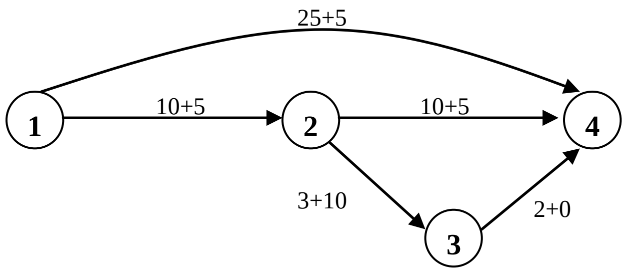
$$q_{ij} \ge 0 \quad \forall (i,j) \in A \tag{5}$$

Constraint (12) indicates that the delay may occur on any one link emanating from a currently traversed node, so the driver makes on-the-spot decisions at every node as the trip unfolds. The solution, found by a label-setting algorithm (Spiess and Florian 1989), determines the attractive links, along with link selection probabilities at each node. Conceived for the individual's route choice in the assignment context, the approach translates link selection probabilities into route usage frequencies.

3.5 Numerical Example

Inputs

Both routing strategies are presented using a simple network, shown in **Figure 1**. Input values of usual travel times and worse-case delays have been selected so that the three alternative routes have the same worst-case travel time (c_{ij} +d_{ij}). We present the case with static travel times to emphasise certain properties of the solution and their implications for the design of the algorithm for time-dependent networks.



	c	*D*	*c+d*	
Route 1-4	25	5	30	slow but reliable
Route 1-2-4	20	10	30	
Route 1-2-3-4	15	15	30	fast but unreliable

Figure 1 Example network

Solution

For the above inputs, the solution algorithms lead to the following non-adaptive and adaptive strategies, with expected costs and path use probabilities shown in **Figures 2a** and **2b** respectively.

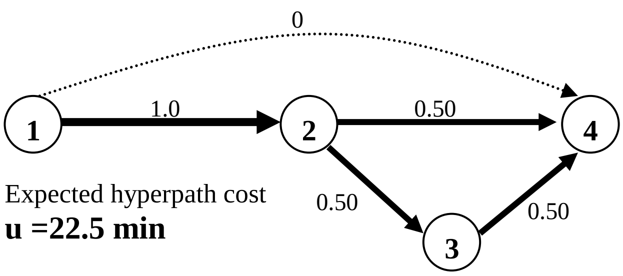


Figure 2a Non-adaptive strategy: cost and link usage

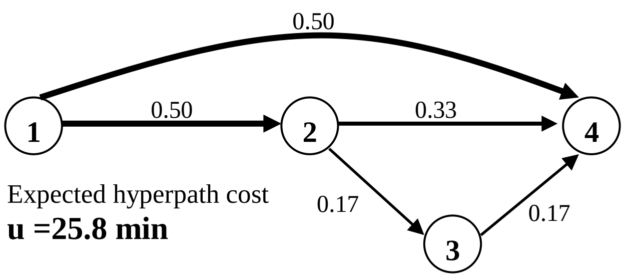


Figure 2b Adaptive strategy: cost and link usage

Taking 9:00am as the preferred arrival time at the destination, we can also calculate the corresponding required departure times from all nodes, shown in **Figures 3a** and **3b** respectively.

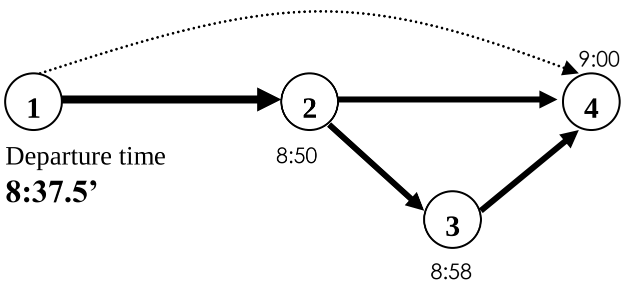


Figure 3a Non-adaptive strategy: departure times

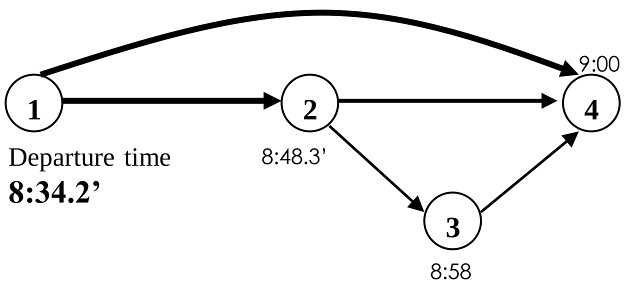


Figure 3b Adaptive strategy: departure times

The expected hyperpath cost in the non-adaptive strategy is 3.3 minutes lower than in the adaptive one, hence allowing later departure. This might suggest that the non-adaptive strategy is superior. However, the lower cost is a consequence of the assumptions underlying the methods: there is only one delay expected in the global game, as distinct from one delay at each node in the local game.

In reality, whether a strategy is superior depends on how close its assumptions conform to realisations of travel times in the actual network. We illustrate this below, using three examples of possible realisations of travel times in the network. In each example we discuss the performance of the two strategies for two cases:

- 10@1 – ten concurrent shipments departing together; and
- 1x10 – an individual shipment repeated over ten days

In the non-adaptive strategy, drivers are oblivious to the actual realisation of travel times on the network, so path split probabilities are respected in all examples. In the adaptive strategy, path split will arise from the encountered delays, which in case of concurrent shipments (10@1) are the same for all drivers using the same hyperpath. Consequently, all drivers would make the same route-diversion decisions and arrive together. In the case of the repetitive shipments (1x10), we expect that the non-adaptive strategy would result in a collective performance as indicated in **Figure 2a**. However, in the case of the adaptive strategy, different elemental paths are likely to be chosen on particular days as different manifestations of traffic conditions arise; these choices over the long term should result in path use frequencies as indicated in **Figure 3b**.

Example 1

The first example examines how both strategies perform if no delays occur in the network.

Travel time realisations				***Strategy adopted***							
				Non-Adaptive				***Adaptive***			
				10@1		***1x10***		***10@1***		***1x10***	
LINKS	***C***	***d***		***P***	***Cost***	***p***	***Cost***	***p***	***Cost***	***p***	***Cost***
Link (1,4)	25	0		-	-	-	-	0.5	12.5	- / - /1	- / - /25
Link (1,2)	10	0		1.0	10	1	10	0.5	5	1 / 1 / -	10/10/ -
Link (2,4)	10	0		0.5	5	1 / -	10/ -	0.25	2.5	- / 1 / -	- /10/ -
Link (2,3)	3	0		0.5	1.5	- / 1	- / 3	0.25	0.75	1 / - / -	3 / - / -
Link (3,4)	2	0		0.5	1	- / 1	- / 2	0.25	0.5	1 / - / -	2 / - / -
ROUTES	***c + d***			***p***		***p***		***p***		***P***	
Route 1-4	25			-		-		0.5		- / - / 1	
Route 1-2-4	20			0.5		1 / -		0.25		- / 1 / -	
Route 1-2-3-4	15			0.5		- / 1		0.25		1 / - / -	
			Total cost	**17.5**		**20 / 15**		**21.25**		**15 / 20 /25**	

<u>10@1</u>

If the *non-adaptive strategy* had been adopted, shipments would have been split between two routes. Five drivers would be sent along Route 1-2-4 and experience travel time of 20 min (arriving 2.5min before PAT). The other half of the drivers would be sent along Route 1-2-3-4 and experience travel time of 15 min (arriving 7.5min before PAT). Collectively, the experienced travel time would average 17.5 min, with arrival on average 5 min before PAT.

If the *adaptive strategy* had been adopted, drivers would split in equal proportions at each node, because delays on the outgoing links are equal to each other. Consequently, some drivers will experience travel time of 15 min, some of 20 min and some of 25 min. The collective average travel time would be 21.25 min, with arrival on average 4.55 min before PAT. Notably, despite the higher total cost than in the non-adaptive strategy, the adaptive approach achieves similar arrival time reliability.

1x10

If the *non-adaptive strategy* had been adopted, path split probabilities require that shipments are assigned between two routes interchangeably on consecutive days. On any given day only one shipment occurs, and only one route is used, so the experienced travel time may be either 20 min or 15 min, depending on the route assigned to this particular day.

If the *adaptive strategy* had been adopted and delays are equal on all links emanating from a node, driver is likely to pick any route, so the experienced travel time may be 15 min, 20 min or 25 min, depending on which route a driver happens to choose.

Note that for both concurrent and repetitive shipments, where the adaptive strategy is adopted and the encountered delays on links emanating from a node are equal (and not necessarily zero), drivers are always better off choosing the elemental path that has lower non-delayed travel time.

Note also, that if no delays occur, for concurrent shipments both strategies result in travel times that are lower than those in **Figure 2**, while for the repetitive shipments this can only be guaranteed through the non-adaptive strategy, since it always includes the non-delayed shortest path.

Example 2

The second example examines how both strategies perform if the maximum delay occurs on links (1,2) and (2,3). The results are presented in the table below:

Travel time realisations				***Strategy adopted***							
				Non-Adaptive				***Adaptive***			
				10@1		***1x10***		***10@1***		***1x10***	
LINKS	***C***	***D***		***p***	***Cost***	***p***	***Cost***	***p***	***Cost***	***p***	***Cost***
Link (1,4)	25	0		-	-	-	-	1	25	1	25
Link (1,2)	10	5		1	15	1	15	-	-	-	-
Link (2,4)	10	0		0.5	5	1 / -	10/ -	-	-	-	-
Link (2,3)	3	10		0.5	6.5	- / 1	- /13	-	-	-	-
Link (3,4)	2	0		0.5	1	- / 1	- / 2	-	-	-	-
ROUTES	***c + d***			***P***		***p***		***p***		***p***	
Route 1-4	25			-		-		-		-	
Route 1-2-4	25			0.5		1 / -		1		1	
Route 1-2-3-4	30			0.5		- / 1		-		-	
			Total cost	**27.5**		**25 / 30**		**25**		**25**	

10@1

If the maximum delay had occurred on only one link in the network, the *non-adaptive* strategy would result in the expected cost shown in **Figure 2a**, as this is in line with its assumptions. However, if delay occurs on more than one link in the network, this strategy generally performs worse than its expected cost. Because link (1,4) is not included in this strategy, all ten drivers would be sent through the other two routes, with none taking advantage of route 1-4. The long travel time on route 1-2-3-4, designated for half of the drivers, would result in arrival on average 5 min after PAT.

In contrast, if the *adaptive strategy* had been used, the decision taken by each driver at node 1 would be to avoid the delay on link (1,2) and use route 1-4. All ten shipments would experience a travel time of 25 min, and arrive 0.8min before PAT.

1x10

As in Example 1, with the *non-adaptive strategy* on any given day only one shipment occurs, and only one route is used. The experienced travel time would be either 25 min or 30 min, depending on the route assigned to this particular day. If the *adaptive strategy* had been adopted, route 1-4 would have been selected, just as in the case of 10@1.

This example demonstrates that the adaptive strategy is likely to perform better in networks where multiple delays occur on links that normally have short non-delayed travel times. Both in concurrent and repetitive shipments, the actual travel times resulting from this strategy are better than those expected (**Figure 2b**).

Example 3

In this example we examine a scenario where the whole network is congested and delays on all links approach their maximum value of d_{ij}.

Travel time realisations				***Strategy adopted***							
				Non-Adaptive				***Adaptive***			
				10@1		***1x10***		***10@1***		***1x10***	
LINKS	***C***	***d***		***p***	***Cost***	***p***	***Cost***	***p***	***Cost***	***p***	***Cost***
Link (1,4)	25	4		-	-	-	-	-	-	-	-
Link (1,2)	10	3		1	13	1	13	1	13	1	13
Link (2,4)	10	3		0.5	6.5	1 / -	13/ -	1	13	1	13
Link (2,3)	3	6		0.5	4.5	- / 1	- / 9	-	-	-	-
Link (3,4)	2	0		0.5	1	- / 1	- / 2	-	-	-	-
ROUTES	***c + d***			***p***		***p***		***p***		***P***	
Route 1-4	29			-		-		-		-	
Route 1-2-4	26			0.5		1 / -		1		1	
Route 1-2-3-4	24			0.5		- / 1		-		-	
			Total cost	**25**		**26 / 24**		**26**		**26**	

10@1

If the *non-adaptive strategy* had been adopted, drivers would be split between two shorter routes, which would result in average travel time of 25 min (arriving on average 2.5 min after PAT). With the adaptive strategy, the decision at node 1 would be to take link (1,2) rather than link (1,4) where the delay is greater, so the longest route 1-4 would be avoided, which is advantageous. Yet, route 1-2-3-4 would also not have been used, despite the fact that it remains the shortest even when delayed. This is because the decision at node 2 would have favoured that adjacent link on which the delay was smaller, even if greater delays awaited further downstream. This myopic character of decision-making is a disadvantage of the adaptive strategy, and in this example results in all drivers using route 1-2-4, experiencing travel time of 26 min and arrival 0.2 min after PAT.

1x10

If the *non-adaptive strategy* had been adopted, the experienced travel time would be either 25 min or 30 min, depending on the route assigned to this particular day. With the *adaptive strategy*, route 1-2-4 would have been selected, just as in the case of 10@1.

3.6 Extension to the time-dependent domain

The numerical example demonstrates that if the actual realisations of travel times differ significantly from those used to develop delivery strategies, the experienced travel times will differ from those expected. While in static networks this does not affect the strategy itself, in time-dependent networks where c_{ij} and d_{ij} are generally time-dependent, the optimal strategies for different time intervals will not necessarily be the same.

An extension of the *non-adaptive strategy* to the time-dependent context has been proposed by (Szeto and Sumalee 2009) as an application over a space-time expanded network. The series of static network replicas for each departure time allows for application of solution algorithms by (Bell 2000), but it is inefficient. Future work will search for a non-trivial extension of the global game to the dynamic domain.

The more challenging problem is the extension of the *adaptive strategy* where the difference between the expected and actual arrival time at intermediate nodes requires re-optimisation of the strategy en route. Such an extension has been proposed by Kanturska, Trozzi et al. (pending) who adapted the dynamic Bellman equations by (Trozzi, Gentile et al. 2012) into the JIT context as:

$$u_i = \begin{cases} 0 & \text{if } i = s \\ \min\limits_j \left[d_i\left(a_{ij}(t)\right) + \sum\limits_{(i,j)\in A_i^{+*}} p_{ij}\left(a_{ij}(t)\right) \cdot \left\{ c_{ij}(t) + u_j\left[a_{ij}(t) + d_{ij}\left(a_{ij}(t)\right)\right]\right\} \right] & \text{if } i \neq s \end{cases} \tag{19}$$

With this Bellman-based formulation, the total expected travel time of the time-dependent hyperpath is defined recursively. The first component equals d_i, and the second is the expected remaining cost of a hyperpath. The solution algorithm presented in Kanturska, Trozzi et al. (pending) returns hyperpaths from all nodes to destination for all possible arrival times at destination by exploiting the properties of a space-time expanded network.

However, instead of considering network expansion for a given departure time and trying to establish arrival at particular nodes, the proposed approach considers each time interval in isolation. Using appropriate time-dependent link lengths, it is established at which time intervals the end nodes of these links fall. As this is always in a time step greater than that currently considered, and the procedure is carried out in a decreasing order of time, the remaining distance to destination is known from the label established in earlier steps. The main loop of the algorithm processes time layers in decreasing order of time. Within each time layer, the internal loop of the algorithm updates node labels by processing the nodes one by one in any order (topological order is redundant given that only one time layer is considered at a time).

This approach is inspired by the Decreasing-Order-of-Time (DOT) procedure developed by (Chabini 1998), who analytically proved that DOT is the most efficient solution method for finding all-to-one shortest paths for all possible arrival times in time-dependent networks.

Given application in the JIT context, it might seem more efficient to calculate a hyperpath corresponding to PAT only, rather than for arrival times. However, a hyperpath established based on the expected travel time would prove optimal only if the actual travel times turn out equal to the expected ones. As discussed above, this generally will not be the case, and drivers will need to receive updated hyperpaths once the actual arrival times at intermediate nodes become known. This can be done efficiently only if hyperpaths are readily available for all arrival/departure times, which the proposed method facilitates.

4 Conclusions and future work

The paper has compared two hyperpath-based strategies for routing and departure time choice for just-in-time deliveries in time-dependent networks with random delays. Two delivery scenarios have been considered: (1) concurrent, with several deliveries departing together towards the same destination; and (2) repeated, with individual deliveries repeated over a number of days.

The numerical example demonstrates that, despite offering a better expected value of travel time, the non-adaptive strategy generally offers worse arrival time reliability. Future work will concentrate on formulating the mathematical conditions for the superiority of each strategy, and on testing the strategies on the UK road network and associated historic records of travel times. Also, the current assumptions imply that the actual realisations of travel times are not expected to fall below the 50th percentile. This needs to be explored in future work on the formulation of the delay and choice model.

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