Unscented Kalman Filter for freeway traffic state estimation using multiple data sources

D. Ngoduy *

Institute for Transport Studies,

The University of Leeds, Leeds, UK

***** Email: mailto:d.ngoduy@leeds.ac.uk

1 Introduction

Traffic state is defined as the classification of traffic flow situations in time and space: free-flow, congested, or some other complex phenomena such as stop-and-go waves, synchronized flow, oscillatory flow, etc. A model based traffic state estimator is defined as an optimization problem of combining model predictions from a traffic model and traffic measurements from sensors. Such model based estimators typically use a Kalman Filter (KF) algorithm and its family such as the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF). In addition to the KF type, other Monte Carlo type such as Particle Filter (PF) is also tried in the model based traffic estimation problems. Those filtering techniques are embedded into a macroscopic dynamic traffic model to form a model based real-time traffic estimator. It all depends on the type of the filter algorithm and the model used different real-time traffic estimators are obtained. For example, Wang and Papageorgiou [1] developed the estimator from the EKF algorithm combined with a second order macroscopic model. Mihaylova et al. [2] and Ngoduy [3] adopted the PF algorithm in other second order macroscopic traffic models. Ngoduy [4] investigated the applications of the UKF combined with a multiclass first order traffic model for the estimation problems of multiclass freeway networks.

In recent years rapid advances in information technology have led to various data collection systems which enrich the sources of empirical data for the traffic state estimation problems. In practice, traffic data are collected from loop detectors, floating cars, cell-phones, video cameras, remote sensing, etc. [5]. It has been argued that fusing data from multiple sources could result in better accuracy, increased robustness and confidence. As indicated in van Lint and Hoogendoorn [5], most approaches to traffic state estimation consider a single data source due to some constraints when they are used for heterogeneous data: highly increased computational demand for any realistic traffic network. Accordingly, van Lint and

Hoogendoorn [5] and subsequently Treiber and Kesting [6] have proposed an approach to reconstruct the traffic state from heterogeneous data. These methods belong to a class of non-model based (or data driven) assimilation technique. Nevertheless, the non-model based technique does not utilize the prediction power of a dynamic traffic flow model, therefore the prediction results will suffer significantly in case the data are missing and/or corrupted.

This paper is devoted to the development of a solution for the model based approach to deal with the problems of fusing traffic states from multiple data sources. More specifically, the proposed solution will employ a single measurement model for data from a group of sources and then derive recursive equations for integrating data from multiple sources into a common state estimate at the same time. The proposed solution belongs to the information filter (IF) method, which is essentially based on the family of the KF algorithm expressed in terms of the inverse of the covariance matrix. In the IF, or inverse covariance filter, the estimated covariance and estimated state are replaced by the information matrix and information vector respectively, which are defined as Fisher information [7]. The IF method has exhibited some advantages over the original KF in the estimation problems from multiple sources: computationally simpler and easier initialization without knowing a priori information of the state of the systems [8]. In this paper, the IF is embedded into the Unscented Transform (UT) method [9] to form a new real-time traffic estimator using data from multiple sources. The performance of the developed technique will be numerically studied using data from loop detectors and floating cars.

2 Methodology

Let us consider a discrete nonlinear dynamic system which is written in the following form:

$$\mathbb{Y}_{k+1} = f\left(\mathbb{Y}_k, \mathbb{B}_k, \Gamma_k\right), \quad (1)$$

where $\mathbb{Y}_k, \mathbb{B}_k, \Gamma_k$ denote, respectively, the vector of states, boundary conditions and model noises at time instant *k*. *f(.)* denotes the nonlinear function, which relates the future predictions of the states $\mathbb{Y}_{k+1}$ given the previous states, boundary conditions and model noises $\mathbb{Y}_k, \mathbb{B}_k, \Gamma_k$. In the model based assimilation method, equation (1) is combined with the measurement equation in order to optimally estimate the future states. The measurement equation for each data source is written as:

$$z_k^m = h^m\left(\mathbb{Y}_k, \Omega_k\right), \quad (2)$$

where z_k^m denotes the vector of observable outputs from source *m* ($m \in M$) and Ω_k is the vector of measurement noises. $h^m(.)$ denotes the nonlinear function, which relates the system states $\mathbb{Y}_k$ to the observable outputs z_k^m from source *m*.

A general Bayesian technique is used to recursively estimate the (augmented) $\mathbb{Y}_k$ states taking different values, given data $\mathbb{Z}_{1:k}$ up to time *k*, where $\mathbb{Z}_{1:k} = \left\{ z_{1:k}^1, z_{1:k}^2, ..., z_{1:k}^M \right\}$. To this end, it is essential to construct the probability density function (pdf) $p\left(\mathbb{Y}_k \mid \mathbb{Z}_{1:k}\right)$. It is assumed that the initial pdf $p\left(\mathbb{Y}_1 \mid \mathbb{Z}_1\right) = p\left(\mathbb{Y}_1 \mid z_1^1, z_1^2, ..., z_1^M\right) = p\left(\mathbb{Y}_1\right)$, which is also known as the *prior*, is available. Now suppose that the pdf $p\left(\mathbb{Y}_k \mid \mathbb{Z}_{k-1}\right)$ is available. Using Bayes' rule, the *posterior* pdf is:

$$p\left(\mathbb{Y}_k \mid \mathbb{Z}_k\right) = \frac{p\left(\mathbb{Y}_k \mid \mathbb{Z}_{k-1}\right) \prod_{m=1}^{M} p\left(z_k^m \mid \mathbb{Y}_k\right)}{\prod_{m=1}^{M} p\left(z_k^m \mid \mathbb{Z}_{k-1}\right)} \tag{3}$$

The denominator of equation (3) is independent of the state and can be set to some normalizing constant *A*. Equation (3) is known as the independent likelihood pool and the estimate $\hat{\mathbb{Y}}_k$ of $\mathbb{Y}_k$ will be determined by the *maximum log- likelihood method* as in the rest of this section.

Let $L\left(\mathbb{Y}_k\right) = \log\left[p\left(\mathbb{Y}_k \mid \mathbb{Z}_k\right)\right] \propto \sum_{m=1}^{M} \log\left[p\left(z_k^m \mid \mathbb{Y}_k\right)\right] + \log\left[p\left(\mathbb{Y}_k \mid \mathbb{Z}_{k-1}\right)\right]$. Let us assume that the model and measurement noises follow the Gaussian distribution (this is the basic assumption in all KF-type algorithms). Accordingly, the *pdf* of the numerator of equation (3) can be written as:

$$p\left(z_k^m \mid \mathbb{Y}_k\right) = \frac{1}{\left(\sigma_{z,k}^m\right)^2 \sqrt{2\pi}} \exp\left[-\frac{\left(z_k^m - \mathbb{Y}_k\right)^2}{2\left(\sigma_{z,k}^m\right)^2}\right], \; p\left(\mathbb{Y}_k \mid \mathbb{Z}_{k-1}\right) = \frac{1}{\left(\sigma_{y,k-1}\right)^2 \sqrt{2\pi}} \exp\left[-\frac{\left(\mathbb{Y}_k - \hat{\mathbb{Y}}_{k-1}\right)^2}{2\left(\sigma_{y,k-1}\right)^2}\right]$$

where $\hat{\mathbb{Y}}_{k-1}$ is the estimated value of $\mathbb{Y}_{k-1}$ (computed at the previous time step). $\sigma_{y,k-1}$ and $\sigma_{z,k}^m$ are, respectively, the errors associated with the model and measurement noises. To substitute this *pdf* into the log *posterior pdf* we obtain:

$$L\left(\mathbb{Y}_k\right) = \log\left[p\left(\mathbb{Y}_k \mid \mathbb{Z}_k\right)\right] \propto \sum_{m=1}^{M} \left(\frac{z_k^m - \mathbb{Y}_k}{\sigma_{z,k}^m}\right)^2 + \left(\frac{\mathbb{Y}_k - \hat{\mathbb{Y}}_{k-1}}{\sigma_{y,k-1}}\right)^2 \tag{4}$$

By differentiating *L* with respect to $\mathbb{Y}_k$ and set this differential to zero, we will obtain the best estimate $\hat{\mathbb{Y}}_k$ of $\mathbb{Y}_k$ which optimizes *L*:

$$\frac{dL\left(\mathbb{Y}_k\right)}{d\left(\mathbb{Y}_k\right)} \propto \sum_{m=1}^{M} -2\frac{z_k^m - \mathbb{Y}_k}{\left(\sigma_{z,k}^m\right)^2} + 2\frac{\mathbb{Y}_k - \hat{\mathbb{Y}}_{k-1}}{\left(\sigma_{y,k-1}\right)^2} \tag{5}$$

$$\text{Set } \left.\frac{dL\left(\mathbb{Y}_k\right)}{d\left(\mathbb{Y}_k\right)}\right|_{\hat{\mathbb{Y}}_k} = 0 \Rightarrow \hat{\mathbb{Y}}_k = \frac{\dfrac{\hat{\mathbb{Y}}_{k-1}}{\left(\sigma_{y,k-1}\right)^2} + \sum_{m=1}^{M} \dfrac{z_k^m}{\left(\sigma_{z,k}^m\right)^2}}{\dfrac{1}{\left(\sigma_{y,k-1}\right)^2} + \sum_{m=1}^{M} \dfrac{1}{\left(\sigma_{z,k}^m\right)^2}} \tag{6}$$

It can be seen from equation (6) that the estimate $\hat{\mathbb{Y}}_k$ takes the form of a weighted mean where data from each source with low variances (e.g. from high reliable sources) carry more weight than those with high variances (e.g. from low reliable sources).

Let $\frac{1}{\left(\sigma_{y,k}\right)^2} = \frac{1}{\left(\sigma_{y,k-1}\right)^2} + \sum_{m=1}^{M} \frac{1}{\left(\sigma_{z,k}^m\right)^2}$, $\hat{\mathbb{Y}}_k = \frac{\left(\sigma_{y,k}\right)^2}{\left(\sigma_{y,k-1}\right)^2} \hat{\mathbb{Y}}_{k-1} + \left(\sigma_{y,k}\right)^2 \sum_{m=1}^{M} \frac{z_k^m}{\left(\sigma_{z,k}^m\right)^2}$. The updated distribution $p\left(\mathbb{Y}_k \mid \mathbb{Z}_k\right)$ is defined by the mean $\hat{\mathbb{Y}}_k$ and variance $\left(\sigma_{y,k}\right)^2$, which is updated with the variances of data from all sources at one time step. To recursively estimate $\hat{\mathbb{Y}}_k$, the information filter technique will be embedded in the Unscented Transform method for the non-linear estimation problems of traffic states, described in the full paper.

References

[1] Wang, Y. and Papageorgiou, M. Real-time freeway traffic state estimation based on Extended Kalman Filter: a general approach. {Transportation Research Part B}, 39, 141–167, 2005

[2] Mihaylova, L., Boel, R. and Hegyi, A. Freeway traffic estimation within particle filtering framework. {Automatica}, *43*, 290-300, 2007.

[3] Ngoduy, D. Kernel smoothing method applicable to the dynamic calibration of traffic flow models. {Computer Aided Civil and Infrastructure Engineering}, 26, 420-432, 2011.

[4] Ngoduy, D. Applicable filtering framework for online multiclass freeway network estimation. {Physica A}, 387, 599-616, 2008.

[5] van Lint, J. & Hoogendoorn, S.P. A robust and efficient method for fusing hetergeneous data from traffic sensors on freeways, {Computer-Aided Civil and Infrastructure Engineering}, 24,1-17, 2009.

[6] Treiber, M. and Kesting, A. Reconstructing the traffic state by fusion of heterogeneous data. {Computer Aided Civil and Infrastructure Engineering}, 26, 408-419, 2011.

[7] Frieden, B. Roy. *Science from Fisher Information: A Unification*. Cambridge Univ. Press, 2004.

[8] Mutambara. A.G.O. and Durrant-Whyte, H.F. (2000). Estimation and control for a modular wheeled mobile robot. {IEEE Tran. Control Syst. Technol}, 8, 35-46, 2000.

[9] Julier, S.J. and Uhlmann, J.K. Unscented Filtering and Non-linear Estimation. Proceedings of IEEE Symposium, vol. 92, pp. 401–422, 2004.