

# Optimization of road pricing using heuristics

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## 1 Background

The purpose of this paper is to test different methods for improving the toll structure in a road network to achieve better socioeconomically results. The optimal for society would be the state in which the total amount of travel time for all users is minimized, the system optimum (SO, [1]). SO is not necessarily a stable condition and therefore not an equilibrium. In reality all drivers will try to minimize their personal travel time and this results in a user equilibrium, UE, which often is more costly than SO.

The introduction of a toll redistributes traffic flows. Road segments, that acts as bottlenecks in the network can have a toll added, making them less attractive for the travellers. Theoretically it would be possible to add tolls to a transport network that would make SO and UE the same situation by using marginal pricing on each road segment [2]. The problem is that every road is dependent on the other roads and finding the correct prices is a complex task.

Another approach is to divide the network into toll zones. This means that SO is not achievable, since the pricing is set at an average of the pricing needed on all the different roads within each zone. It is therefore only a second best pricing scheme. It should however be possible to narrow the gap between SO and UE.

## 2 Problem

The problem consists of two parts: There is finding the equilibrium and optimizing the toll structure. This is a bi-level optimization problem, where the transport model is a result of the toll structure chosen and the optimization of the toll levels are dependent on the amounts of traffic.

As transport models often are very time-consuming it is important to find efficient solutions to the problem. In practice, policy makers and planners are often trying to define the toll-levels by a trial-and-error method where they define a toll-structure, run a transport (equilibrium) model, evaluate the solution, and then retry with new toll-levels. This is both cumbersome in terms of the work effort; and it is not likely that the optimal solution is found (given the second best restrictions in the given configuration). The aim of the paper is to investigate different methods' abilities to find good solutions.

## 3 Method

In this project the different tolls will be the variables of the optimization. By changing these the total societal cost of transportation changes. Calculations are made for both link based and zone based tolls. And the calculations are tested both with and without a simple demand model.

Different approaches have been used. Initially some basic metaheuristics was applied. Metaheuristics are often used with many iterations [3] and using them with transport models has some problems. The long running time of the traffic assignments require the use of very efficient metaheuristics. Three metaheuristics are selected for this project. The metaheuristics used throughout the simulations are variations on Hill Climber [4, chapter 1], Simulated Annealing [4, chapter 7] and Taboo search [4, chapter 6].

For each different toll structure a transport model is used to find the stochastic user equilibrium, SUE; the situation in which no users can improve their perceived travel time by changing routes. This task is non-linear as congestion changes utility for users. Each user worsens the route for everybody else on that route. This means that the travel time of each user is a function of the number of travellers on that user's path. The number of traveller is then again a function of the travel time. To find SUE the method of successive averages is used.

If there is only one toll the problem of finding the optimal price is linear with one optimal solution for the society. When several more tolls are introduced the problem becomes increasingly complex. With only one toll the price could either be decreased or increased,

however with several zones some could increase and some could decrease and there are a high number of combinations of what could be changed.

An approach is to change the toll in a small number of the worst zones in the network and then find the new equilibrium. The main concern with this approach is to specify what the neighbourhood solutions is, meaning how the tolls should vary from iteration to iteration. Different sizes of toll changes are tested in this project.

Finally it is attempted to set a marginal price on each road in the network through a series of iterations.

## **4 Case**

The different methods were tried for two different networks. The first was small and consisted of only 16 road segments while the second was a version of the Copenhagen traffic model with around 30,000 road segments.

## **5 Results**

The small network had almost no calculation time and it was possible to use a high number of iterations. Two different tolls were added to the network and these were optimized. By using hill climber the calculations reached a local optimum fairly quickly and achieved poor results. Simulated annealing consistently produced better results. Taboo search performed similar to simulated annealing.

In the larger network the assignment time was half an hour, meaning that unlike the small network very few iterations can be run within a reasonable time period. This means that a very efficient heuristic is needed. Since the hill climber quickly was caught in local optima in the small network it was expected that it could happen again for the larger network. Therefore a fairly greedy simulated annealing was used for the larger network with a sort of taboo list implemented.

The methods gave some improvement for the larger network and some preliminary conclusions are made on which methods to proceed with. However more work should be done in finding more intelligent ways to determine each neighbourhood.

## References

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