Inter-city Bus Schedule Planning: An integrated model for bus scheduling and routing with no passenger transfers

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1 Introduction

In developing countries, with large cities that concentrate the bulk of economic activity, inter-city bus passenger transport tends to play a very relevant role, given its relatively high flexibility and low fares. Chile, where the economic activity is concentrated in a few cities, which attract labor from various regions of the country, is not an exception. In Chile, most intercity bus trips are served by two major operators, even though the industry also includes several other small carriers playing a less important role.

Planning services (timetabling, bus routing/scheduling) for bus intercity services is quite different from the urban case. While quite often urban bus services aim at minimizing costs while providing a required level of service, interurban bus service planning is oriented towards maximizing profits. Except for some recent works ([4], [5], [1]), research on intercity passenger transportation services is relatively scarce. Furthermore, some assumptions regarding the supply of services differ from the Chilean case, so their methods can not be directly applied to the focus of this study.

In this paper we propose a model aimed at optimizing the timetable for an intercity bus firm providing a nation-wide service, through a set of multi-commodity models. The demand is disaggregated by i) origin destination pair, ii) desired time for their journey, and iii) class of seat (e.g. economic or business). The purpose is to maximize its utility while providing a certain level of service.

2 Proposed Model

The proposed model jointly defines the bus services to be offered and assigns passengers to each service. The model is built on a time-space network framework and is based on the following assumptions:

(i). While passengers can travel on a bus making multiple stops along its way, passengers will only be assigned to trips that allow them to reach their destination without transferring.

(ii). The demand is assumed fixed, independent of the set of services to be offered.

(iii). Passengers plan their trips in advance, so that waiting costs can be considered independent of the services offered and therefore can be neglected.

(iv). The demand follows a weekly sequential pattern so operations can be planned over a planning horizon of a single week.

(v). Every single service (defined by an itinerary, at a certain time, provided by a certain type of vehicle) can be operated by one specific bus or not, so the services will be associated by binary variables.

The time-space network of buses is a multi-commodity network, similar to those used for allocating air transport flights (see for example [2] and [3]), where each commodity is a type of bus, with its own setting for seats of each class. Each service is represented by an arc starting at the first node on the bus route and ending at its last one, so that the various stops in between these two nodes are not represented in this network.

Passenger trips are also modeled through this time-space multi-commodity network, similar to the one used in [4], where each commodity represents passengers from a given origin-destination pair and seat class. The proposed network is different from the one used in [4] in terms that it prevents passengers from transferring between services, by means of not considering the intermediate stops in each trip.

By joining both networks, one can formulate the following network flow model with mixed-integer variables:

$$\text{Max} \sum_{\omega j s} \bar{P}_{\omega j s} f_{\omega j s} - \sum_{s,b} C_{sb} x_{sb} \tag{1a}$$

$$\sum_{\omega \in U^-_{qj\{e_1 e_2\}}} u_{\omega j} + \sum_{\omega \in V^-_{qj\{e_1 e_2\}}} v_{\omega j} + w_{q^-} = \sum_{\omega \in U^+_{qj\{e_1 e_2\}}} u_{\omega j} + \sum_{\omega \in V^+_{qj\{e_1 e_2\}}} v_{\omega j} + w_q$$
$$\forall q \in M_{j,\{e_1,e_2\}}, j \in J, \{e_1, e_2\} \in E^2 \tag{1b}$$

$$u_{\omega j} = \sum_{s \in S} f_{\omega j s} \quad \forall \omega \in \Omega, j \in J \tag{1c}$$

$$v_{\omega j} \leq D_{\omega j} \quad \forall \omega, j \tag{1d}$$

$$\sum_{s \in X^-_{pb}} x_{sb} + y_p = \sum_{s \in X^+_{pb}} x_{sb} + y_{p^-} \quad \forall p \in N_b, b \in B \tag{1e}$$

$$\sum_{s \in T_A(b)} x_{sb} + \sum_{p \in T_N(b)} y_p \leq A_b \quad \forall b \in B \tag{1f}$$

$$\sum_{\omega \in \Omega_s} \delta^l_{\omega s} f_{\omega j s} \leq \sum_{b \in B} K_{jb} x_{sb} \quad \forall l \in L_s, s \in S, j \in J \tag{1g}$$

$$x \in \{0,1\}^{S \times B}; y, f, u, v, w \geq 0 \tag{1h}$$

The set of variables $u_{\omega j}$ and w_q are related to the flow of passengers from the origin to the destination of each trip, and with the waiting times suffered at each station, respectively (even though, under assumption (iii), these waiting times will be neglected in the objective function). The set of variables $v_{\omega j}$ represent the demand for traveling in each origin-destination pair and seat class, at a certain time. These variables can also be interpreted as the return arcs in a maximum flow network problem in which we maximize revenue sales $\sum_{\omega j s} \bar{P}_{\omega j s} f_{\omega j s}$ in (1a), which are bounded not to exceed the maximum demands $D_{\omega j}$, according to (1d). The constraints (1b) represent the flow balance that must be satisfied in every node of the passengers' network. The passengers flow $u_{\omega j}$ are decomposed into flows $f_{\omega js}$ using the set of services s, according to (1c).

On the other hand, the sets of variables x_{sb} and y_p are related to the bus flow between stations and waiting times at stations respectively. The constraints (1e) represent the flow balance of buses in every node. The left hand side of constraints (1f) represents the number of buses crossing a *count time*, so that the number of buses of each type used does not exceed the number of buses available A_b.The set of binary variables x_{sb} represent whether service s, will be operated by a bus type b or not. The total cost of providing the set of services is given by $\sum_{s,b} C_{sb}x_{sb}$ in (1a).

Finally, the number of seats of class j offered by the different types of bus b, associated with a service s is given by $\sum_{b\in B} K_{jb}x_{sb}$. The sections of a service s that each of the flows $f_{\omega js}$ request are given by the $\delta^l_{\omega s}$; thus, constraints (1g) link the trips $f_{\omega js}$ made in a service s with the capacity of the buses operating that service, for each one of the service's sections.

The model (1) is linear, based on mixed-integer variables, and usually quite large. In Chile, the model is particularly large given its geography since in such a large country most services have several intermediate stops, increasing the number of sections per service.

3 Case study

The proposed model was implemented for a test scenario with a single type of bus, and a single class of seat for trips joining 14 terminals from an isolated area of an intercity bus firm. The demand was obtained from historic travel data from low season week. The model was solved on a workstation using a commercial optimization software relatively fast. Even though the scenario does not consider the whole region served by the company, the results suggest that the firm could make a profit in an area where it has typically faced losses. Our team is now in the verge of running the model for the whole network of the firm, which considers 9 bus types, 4 seat classes and operates in 180 cities.

On the other hand, the structure of the model allow us to consider a specialized decomposition algorithm, such as Lagrangian relaxation or Branch & Price in order to obtain solutions for large instances, in relatively low time.

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