Dynamic User Equilibrium Hyperpaths in bus networks with passenger queues at stops

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1 Introduction

Transit assignment models, which describe and predict the network usage in terms of passengers flows, differ on the assumptions made about traveler behavior, congestion effects, and level of service supplied by the transport system. For instance, if high-frequency and low-reliability services are provided, *frequency-based* (FB) models are usually preferred, because it is plausible to assume that passengers do not choose a particular run and do not time their arrival at the stop to coincide with vehicle arrivals. Furthermore, these models are capable of handling easily the *common lines* problem, and, since the seminal works of Nguyen and Pallottino [1] and Spiess and Florian [2], they have been successfully exploited to model passenger travel strategies (*hyperpaths*) in public transport networks.

Although FB models have increasingly received attention in recent years, the effect of overcrowding on the public transport network has often been neglected. However, transport systems in large cities are frequently affected by recurrent passenger congestion during peak periods and this leads to a significant increase of total travel time, for example, due to the queuing time that passengers experience at stops because of limited available capacity on board. Consequently, this paper proposes a Dynamic User Equilibrium model for FB networks, where passenger congestion results in FIFO (*First In First Out*) queues at bus stops and travelers choose their routes in terms of *optimal strategies* (or, equivalently, *hyperpaths*).

2 Background research

Most of the research carried out in the realm of FB transit assignment with hyperpaths, has dealt with overcrowding assuming passengers mingling at the stop ([3],[4], [5], [6] [7], [8]) that implies no waiting priority is respected and, in case of oversaturation, all waiting passengers have the same probability to board the next carrier to approach (provided it is attractive). Thus a possible simple solution [3] would consider the effective frequency, namely the line frequency perceived by waiting passengers that decreases as the probability of not boarding its first arriving carrier increases.

In general, when passenger congestion occurs, the queuing protocol followed by travellers is determined by the stop layout. For example, for stations and stops with large platforms, it is correct to assume passengers mingle, so the probability distribution function (pdf) and the expected value of the waiting time, as well as passengers' distribution among the attractive lines can be calculated according the methods presented in the afore-mentioned studies. However, in urban bus networks the stop layout is usually such that, when passenger congestion occurs, users arriving at the stop would join a FIFO (first-in first-out) queue and board the first line of their attractive set that becomes available. Early attempts to describe this queuing process in the realm of FB hyperpath assignment are made in [9] and [10] by using a bulk queue model, but its complexity prevents any practical application. Meschini *et al.* [11] present a FB assignment model for multimodal network where in-vehicle transit congestion is explicitly considered and the additional queuing time at stops, due to transit demand exceeding supplied capacity, is calculated by means of a bottle-neck queuing model.

The major drawback of the latter study is that travel strategies are not considered in the route choice model and this will be the main contribution of the present paper. In other words, we prove that, in the context of commuting passengers who know by previous experience the number of carrier passages they must let go before being able to board, the route choice can still be represented as the selection of optimal travel strategy and, thus, expected waiting times and passenger distribution among attractive lines may be calculated by extending the formulas presented in [1] and [2]. On the other hand, the number of carriers that travellers must let go before boarding is calculated by means of a *Bottleneck Queue Model* that exploits the results of [11].

3 Methodology

Figure 1 schematically presents the structure of the Dynamic User Equilibrium described in this work.

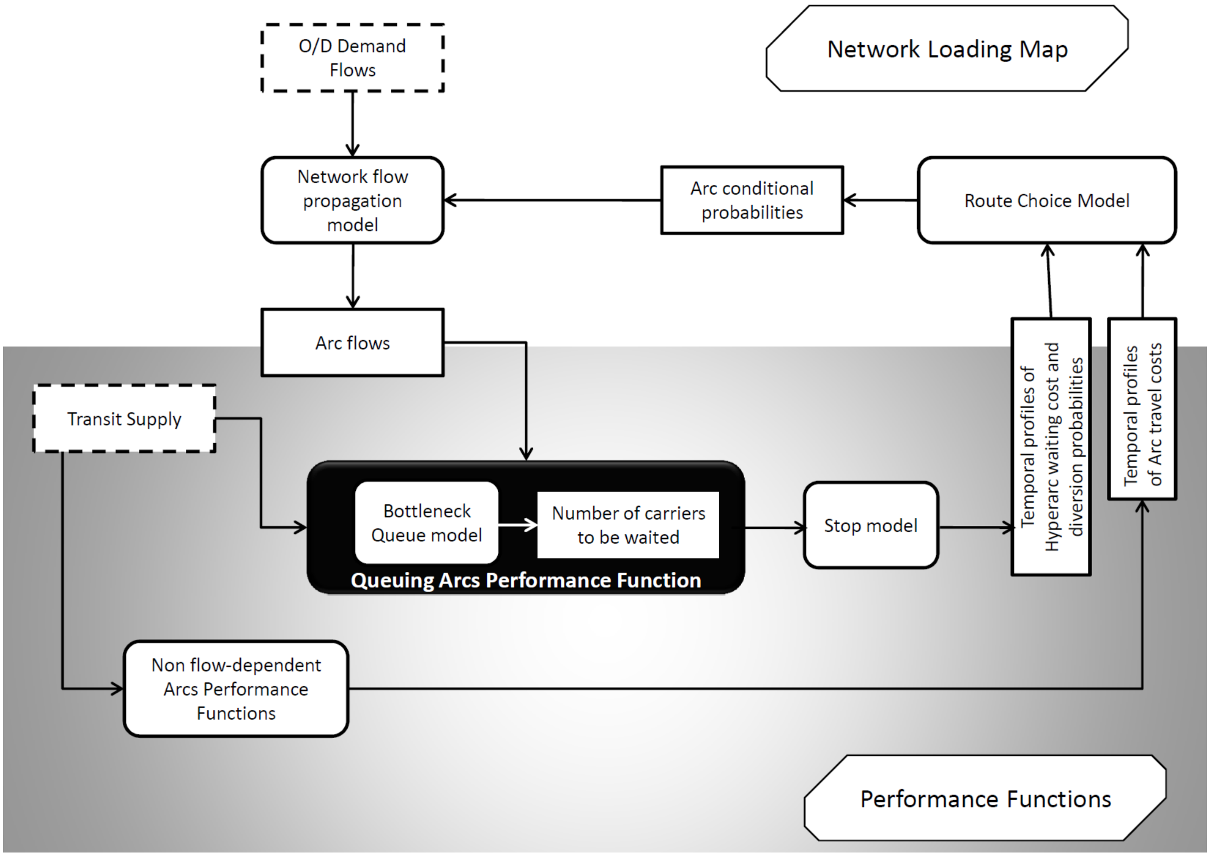

Figure 1: Deterministic Dynamic user Equilibrium: model structure

The main inputs of the model are the time-varying origin-destination (OD) demands and the frequency-based transit supply, which provides time-dependent values of service frequency, dwelling time and in-vehicle travel time. On the other hand, in order to develop a dynamic assignment for public transport networks where time-dependent recurring congestion occurs, the following components need to be specified:

- the *Performance Functions*, which yield the exit time from any arc for given entry time, transit supply characteristics and passenger flows over the network. In the present study it is assumed the *Performance Function* of queuing arcs is the only one affected by passengers flows, through a *Bottleneck Queue Model*, and influences the *Stop Models* by means of the calculation of congestion parameters (number of carriers to wait before boarding);

- the *Stop Model* and deterministic *Route Choice Model*, which reflect the behavior of a rational passenger travelling from an origin to a destination on the transit network for given *Performance Functions*;
- the *Network Flow Propagation Model*, which aims at finding time-varying link flows that are consistent with line capacities for given route choices.

As in [11], the Dynamic Assignment is regarded as a Dynamic User Equilibrium and is formalized as a system between continuous time-dependent *Performance Functions* and dynamic *Network Loading Map* [12]. The major innovation with respect to the original formulation is the inclusion of a *Stop model,* that describes the behavior of passengers at bus stops where several travel alternatives are available, and leads to modeling *Route Choice* as a dynamic hyperpath search [13] in transit networks with passenger congestion, where the traditional assumption that passengers *always* board the first line of their choice set approaching the stop [1] does not hold because of the formation and dispersion of FIFO passenger queues. The *Stop Model* is, in turn, affected by the results of the *Performance Function* of queuing arcs that yields the total queuing time for each line and, thus, the number of buses that passengers have to wait before being able to board.

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