The Time Between: Continuously-defined accessibility functions for schedule-based transportation systems

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Abstract

Accessibility is traditionally considered to be a property of a point or region in space, and to be invariant over time (or at least over some computationally convenient time interval). However, a location’s accessibility can vary over time on a wide range of scales. This temporal variation is especially significant for schedule-based transportation systems. Current integral measures of accessibility generally reflect the accessibility only at points in time corresponding to the departures of one or more trips; accessibility between these time points remains unconsidered and undefined. Consequently, these measures are insensitive to changes in route frequency and the distribution of trip departure times. Furthermore, these approaches ignore the disutility experienced by a system user who is limited to departing or arriving at scheduled times rather than at preferred times. As a result, they systematically overestimate the accessibility experienced by users of scheduled transportation systems. This paper establishes new methods for representing the accessibility provided by a schedule-based transportation system from a specific location as a continuously-defined accessibility function (CDAF) of desired departure time, defined for all time points. Using schedule and route information from metropolitan transit providers, applications of these methods are demonstrated to gain new insight into the accessibility provided by real-world transportation systems. Four examples are developed to represent common service types in metropolitan transit networks. The results confirm that accessibility is significantly overestimated by measuring single points and show that trip frequency is more valuable for sustained accessibility than high accessibility on individual trips.

1 Introduction

Accessibility, the ease of reaching destinations, is traditionally considered to be a property of a point or region in space, and to be invariant over time (or at least over some computationally convenient time interval). The concept has been well-described in the literature, and there are numerous definitions (Geurs and van Wee, 2004; Handy and Niemeier, 1997; Krizek and Levinson, 2007; 2011). These definitions highlight the importance of accessibility in transportation planning and policy making.

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2009; Kwan and Weber, 2003; Levine et al., 2012; Levinson and Krizek, 2008; Ottensmann and Lindsey, 2008; Scott and Horner, 2008). However, a location’s accessibility can vary over time on a wide range of scales.

There are many works on space-time accessibility in the literature, beginning with Hägerstrand (1970). Hägerstrand advanced the idea of space-time prisms to represent an individual’s accessibility. His theory incorporated the idea that activities occur at specific locations for limited time periods. Kwan (1998) drew a distinction between space-time measures, which follow individuals throughout the day, and integral (or place-based) measures and concluded that the two types are independent of each other. Miller (1999) looked at applications of Hägerstrands’s space-time prisms, and incorporated users’ preferred start and end times and overall time budget. This brought the theory closer to the user experience, as commuters in particular frequently have a fixed arrival time in the morning peak and a fixed departure time in the afternoon peak. In a later work, Miller (2005), Miller discussed the idea of fixed and flexible activities. Building on this idea, users going to and from fixed activities are more sensitive to the temporal constraints imposed by scheduled transportation services.

Weber and Kwan (2002) concluded that temporal effects from both the time-dependence of automobile travel times and from business hours have significant effects on accessibility. Temporal variation for schedule-based transportation systems is even more significant than for car travel because it becomes an issue of service availability in addition to speed. Current integral measures of accessibility generally reflect the accessibility only at points in time corresponding to the departures of one or more trips; accessibility between these time points remains unconsidered and undefined. Consequently, these measures are insensitive to changes in route frequency and the distribution of trip departure times. Furthermore, these approaches ignore the disutility experienced by a system user who is limited to departing or arriving at scheduled times rather than at preferred times. As a result, they systematically overestimate the accessibility experienced by users of scheduled transportation systems. One work that looks at this issue is Kim and Kwan (2003), which shows that space-time accessibility measures that trivialize the issues of schedule availability and network topology exhibit this problem of overestimation.

This paper establishes new methods for representing the accessibility provided by a schedule-based transportation system from a specific location as a continuously-defined accessibility function (CDAF) of desired departure time, defined for all time points. Using schedule and route information from metropolitan transit providers, the application of these methods is demonstrated to gain new insight into the accessibility provided by real-world transportation systems. Here, the aim is to explore whether trip frequency or trip accessibility is more important to a sustained transit accessibility over time.

While traditional measures of accessibility assume that users will adjust their departure times to meet the schedule provided by a transportation system, a central benefit of a CDAF-based approach to accessibility is that accessibility information is available for any desired departure time. The CDAF enables detailed investigation of how accessibility at a given location varies over time, and how it is influenced by the time distribution and properties of trips departing from that location. When trip schedule and route information is available, it becomes possible to plot accessibility over a time period of interest and compute statistical properties such as mean and median accessibility, both of which can be calculated for any stop on a transit network. The range and variation of accessibility during a time period can also be calculated. These measures provide a better representation of the accessibility experienced by users. Several examples are presented as proof of concept,
and the relationships are investigated between accessibility measures provided using a CDAF to traditional time-point based accessibility measures.

## 2 Methodology

Construction of the CDAF for a specific location begins with the identification of all trip departures in a time period of interest. For each trip departure, the departure time and a vector $T$ representing the travel times provided by that trip to all reachable destinations are retained. A vector $O$ is also established which provides the number of opportunities at each reachable destination:

$$ T = < t_1, t_2, ..., t_n > $$
$$ O = < o_1, o_2, ..., o_n > $$

Given these sets of travel times and opportunity counts, it is possible to implement a wide range of accessibility metrics. Most contemporary implementations can be traced at least back to Hansen (1959), who proposes a measure where potential destinations are weighted by a function of their access cost and then summed:

$$ A_i = \sum_j o_j f(t_{ij}) \quad (1) $$

- $A_i = \text{accessibility for zone } i$
- $o_j = \text{number of opportunities in zone } j$
- $t_{ij} = \text{time cost of travel from } i \text{ to } j$
- $f(C_{ij}) = \text{weighting function}$

Any accessibility function satisfying these requirements can be used; this study focuses on cumulative opportunities measures of accessibility, which use a binary weighting function based on a travel time threshold:

$$ f(C_{ij}) = \begin{cases} 
1 & \text{if } C_{ij} \leq t \\
0 & \text{if } C_{ij} > t 
\end{cases} \quad (2) $$

- $t = \text{travel time threshold}$

Accessibility is calculated for specific time thresholds and the result is a simple count of destinations that are reachable within that threshold. This approach involves both advantages and disadvantages. Both calculation and interpretation of the accessibility measure are dramatically simplified. But accessibility must be reported separately for each time threshold of interest, and the model cannot be finely calibrated to account for varying user preferences, values of time, etc.
Using the selected accessibility function, the accessibility provided at the departure time for each trip is calculated. A time sampling interval is then selected and the calculation moves backwards through time from each trip departure time, applying the same accessibility function to the next trip’s travel time vector with the current time offset subtracted from each element. When the departure time for the previous trip is reached, the process is restarted using the travel time vector for that trip if the accessibility provided by the previous trip is greater than the accessibility provided by waiting for the next trip:

\[
C = \begin{cases} 
AT_1 + \Delta t_1, O & \text{if } t \leq d_1 \\
AT_2 + \Delta t_2, O & \text{if } d_1 < t \leq d_2 \\
... \\
AT_n + \Delta t_n, O & \text{if } d_{n-1} < t \leq d_n 
\end{cases}
\]

The computations for these measures are performed using a schedule-based travel time calculator developed by the authors (for Anderson et al. (2012)) that is compatible with the General Transit Feed Specification (GTFS), a common publicly-available schedule format for North American cities. The calculation method follows Krizek et al. (2007), with the modification that pedestrian and bike accessibility are not considered (i.e. accessibility is measured from the transit stop, rather than the true origin of the trip). Travel times are combined with land use information collected and calculated as part of the Access to Destinations research project undertaken by the Nexus Research Group at the University of Minnesota.

\section{Data}

In this paper, four examples of a CDAF are presented. These stop locations were selected to demonstrate four situations that are common in many transit systems worldwide. The schedule information for these comes from Metro Transit’s March 2012 schedule definition, while 2010 land use and year 2000 TAZ definitions were produced by the Metropolitan Council.

Example 1 is an \textbf{Urban Local}. The stop, located at Como Avenue and 19th Avenue SE in Minneapolis, is served by one local bus route. Headways are less than 15 minutes throughout the day.

Example 2 is an \textbf{Major Transfer Point}. The stop, located at Nicollet Avenue and 5th Street in downtown Minneapolis, is served by seven local bus routes. Headways are less than 5 minutes throughout most of the day.

Example 3 is a \textbf{Suburban Local}. The stop, located at 85th Avenue NW and Norway Avenue NW in Coon Rapids, is served by one bus route. Headways are typically 1 hour. This is distinguished from Example 1 by the longer headways and fewer transfer opportunities.

Example 4 is a \textbf{Suburban Express}. The stop, the 95th Avenue Park & Ride in Blaine, is served by three express routes. The stop is only served during peak periods.
4 Analysis

Plots and analyses of a stop’s CDAF reveal the marginal accessibility associated with specific scheduled trips. In the following figures, vertical dashed lines indicate trip departure times. Not all trips provide equal marginal accessibility, particularly in cases where a route has several termini or where multiple routes serve the same stop. Marginal accessibility is also sensitive to trip headways. Comparisons of the CDAFs provided by a variety of trip schedules can indicate the optimal distribution of trips over time with the goal of maximizing mean accessibility over a time period. This information can guide a transit provider’s decisions regarding adding, removing, or shifting trips to a given route.

Figure 1 provides an example of a CDAF calculated over one hour at a single transit stop. Peaks in this chart represent trip departures; these points have the highest accessibility because users experience no wait time. At all other times, however, the time cost of waiting results in lower accessibility. The shape of the curves between trip departure peaks is determined by the specific method used to compute accessibility. In this case, accessibility is calculated using a cumulative opportunity model, where all opportunities available within the selected time threshold are simply added. Time is shown in seconds after midnight. This figure also shows the mean accessibility over the time period, which is significantly lower than the peak accessibility.

Figure 5 shows the same location with the CDAF calculated over an entire day. The maximum accessibility remains the same, as this occurs in the 7-8am period selected previously, but the mean drops by about 55%. Some of this decline can be attributed to the lack of transit service between midnight and 6am, but this 24 hour plot also demonstrates a variance in the accessibility of each trip across the day. The peak accessibility is achieved only once, in the morning peak period, while the accessibility of each trip appears to be lowest in the middle of the day.

Figure 2 represents the one hour plot for Example 2. In contrast to Example 1, there is relatively little variation in accessibility between trips, even though these trip arrivals represent several different bus routes. The headways are also small enough in this selection that accessibility does not significantly drop off between trips. In Figure 6, the all day plot for this location, it can be seen that service is very frequent throughout the day, and trip accessibility is relatively consistent across small time intervals. This means that, with the exception of the beginning and end of the transit service period, a moving average of the CDAF would be comparable with the local maximum. This is observed in Figure 2, where the mean:max ratio for the hour is 0.85.

Figure 3 represents the one hour plot for Example 3. This demonstrates what a CDAF plot looks like with large headways between trips. The two trips in this plot are equal, and in fact Figure 7 shows that almost all trips at this stop are equivalent. This result suggests that the bus route serving this stop does not have any transfer possibilities within 20 minutes. The accessibility between trips declines to zero due to the selection of a 20-minute cumulative opportunity model. However, it is apparent that there are few opportunities located near this stop because the CDAF is nearly zero between 20 and 12 minutes before a trip arrival.

Figure 4 is the one hour plot for Example 4. Although accessibility is very low in this time period, one interesting feature is that trip frequencies early in the hour are sufficient to maintain a constant accessibility. This is not present on any of the other examples. The all-day plot for this location, Figure 8, shows that this one hour selection excludes the morning peak of accessibility, which happens in the 6-7am hour. Knowing that this location is served only by express routes, this result may suggest that the 20 minute cumulative opportunity model is not a good choice here.
because the major destinations served by the 7-8am trips are more than 20 minutes away. A look at the route schedules confirms this. The routes serving this stop reach their destinations in downtown Minneapolis and the University of Minnesota in 22 and 23 minutes, respectively.
Figure 1: Continuous Accessibility Over One Hour for Example 1

Cumulative Opportunities Accessibility
Threshold = 20

Max accessibility: 190495
Mean accessibility: 64086
Mean/Max: 0.34

Figure 2: Continuous Accessibility Over One Hour for Example 2

Cumulative Opportunities Accessibility
Threshold = 20

Max accessibility: 240067
Mean accessibility: 203801
Mean/Max: 0.85
Figure 3: Continuous Accessibility Over One Hour for Example 3

Figure 4: Continuous Accessibility Over One Hour for Example 4
Figure 5: Continuous Accessibility Over 24 Hours for Example 1

Figure 6: Continuous Accessibility Over 24 Hours for Example 2
Figure 7: Continuous Accessibility Over 24 Hours for Example 3

Figure 8: Continuous Accessibility Over 24 Hours for Example 4
Figure 9: Gravity-Based Accessibility Over 24 Hours for Example 4
5 Conclusion

After establishing the methodology used to determine CDAFs and demonstrating their application, potential extensions to this work emerge. Accessibility can be combined with information regarding trip capacities to compute the maximum potential person-weighted accessibility offered by each scheduled trip. Similarly, accessibility can be combined with the inverse of trip cost to investigate how the affordability of accessibility varies over time.

The cumulative opportunity model of accessibility is used here, but there are many other functions which would also work in this context. A 24 hour plot for Example 4, gravity-based accessibility instead of cumulative opportunities, is shown in Figure 9. A negative exponential function is another possibility. The advantage of the cumulative opportunity model is that its results are easy to interpret (i.e. 100,000 jobs are accessible from this stop at 7am). However, the choice of threshold is a judgment of what travel times should be considered valuable. Other accessibility functions, gravity-based and negative exponential included, are able to include all reachable destinations and determine the overall value by weighting travel times. As can be seen in Figure 9, a gravity-based function produces a CDAF that is more “generous”, in a way, because the accessibility never reaches zero. This function embodies a principle that all accessibility has some value, which may be useful for certain types of analysis. It represents the way transit systems are used by captive riders, those who have no other modes to choose from. On the other end of the spectrum, a negative exponential function would result in a very spiky CDAF that declines quickly from the peaks at trip arrivals. This would favor higher frequency schedules if used in an analysis. Ultimately, the choice and calibration of specific accessibility weighting functions is context-specific, and should be approached with an understanding of the characteristics of each function.

Levine et al. (2012) concluded that density is more important to accessibility than faster travel. Here, high trip frequency, as seen in Example 2, is also more important to accessibility than high trip accessibility, demonstrated by Example 3. In other words, trip density is more important than the speed at which a trip reaches destinations and transfer opportunities.

Our investigation uses metropolitan transit systems as a case study and proof-of-concept. However, the methods described here can be applied to any scheduled transportation system where travel times to destinations of interest are known. Applications to inter-city air and rail networks are possible. With air networks, minimum access and transfer times would need to be added to the calculation steps. Challenges include incorporating information about trip capacities and demand (because schedules are designed for flights to operate full) and managing delays. Inter-city rail networks operate more like metropolitan transit, but challenges might include incorporating train and track capacities.

References


