Passenger Centric Train Timetabling Problem

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Abstract

The aim of this paper is to analyze and to improve the current planning process of the passenger railway service in light of the recent railway market changes. In order to do so, we introduce the Passenger Centric Train Timetabling Problem. The originality of our approach is that we account for the passenger satisfaction in the design of the timetable. We consider both types of timetable(s): cyclic and non-cyclic. The problem is modeled as a Mixed Integer Linear Programming (MILP) problem with an objective of maximizing the train operating company's profit while maintaining ε level of passenger satisfaction. By solving the model for various values of ε, the Pareto frontier is constructed. The analysis, based on an experiment using realistic data, shows that an improvement of passenger satisfaction while maintaining a low profit loss for the railway company can be achieved. A sensitivity analysis on passenger congestion illustrates a quantitative evidence that the non-cyclic timetables can account better for high density demand in comparison to cyclic timetables.

Keywords: Railway Timetable, Passenger Satisfaction, Profit, Cyclic, Non-cyclic, MILP

1 Introduction

The main product of a Train Operating Company (TOC) is a train timetable, with its consumers being the passengers. It is then important to account for the passengers' preferences during the timetable design, especially with the new legal modifications in the railway markets in Europe (EU Directive 91/440) that allow for competition.

Up to this point, the European national carriers were subsidized by local governments and their purpose was to offer the accessibility and mobility to the public (passengers). With this new market settings, the subsidies are removed and the operators compete not only for the passengers, but also for the infrastructure. The railway network is to be separated from the TOC who has built it (usually the national carrier) and to be handled by an independent Infrastructure Manager (IM). Each TOC will then be obliged to submit its preferred timetable to the IM, who will then adjust the proposed timetables, in order to secure the safety of the network. No common decision rule, on which TOC should gain advantage in case of a conflict, yet exists, but an economic instrument such as bid-auction is expected. Adding everything up, the above facets of the problem put a high pressure on the timetable design and ask for a quantitative method that would provide TOC with an insight about a precise yet functional timetable design that could take into account operational constraints as well as passengers' satisfaction.

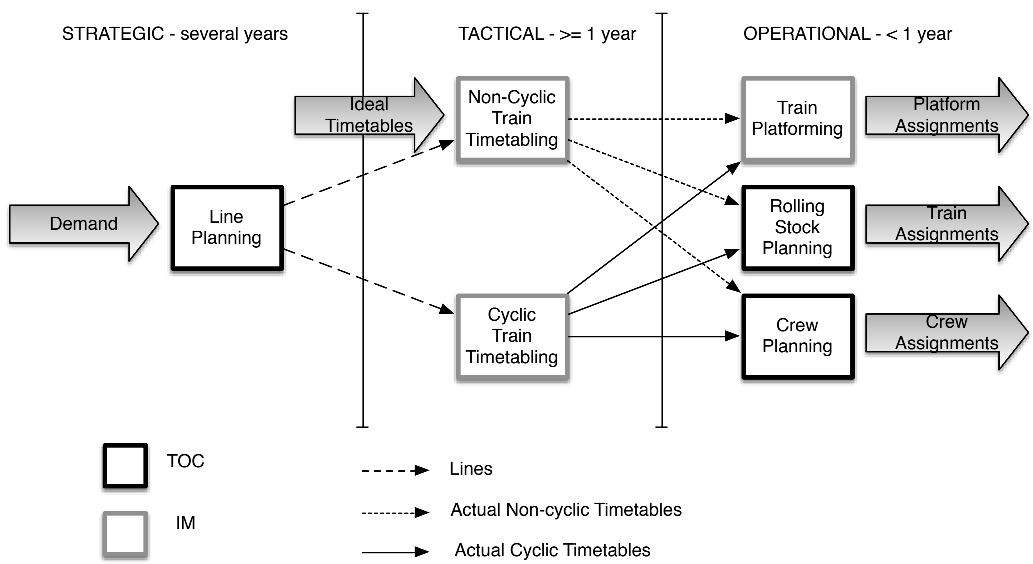


Figure 1: *Planning overview of railway operation*

The new planning horizon as described in Caprara et al. (2007) is visualized in Figure 1. In the previous market settings, all the stages were solved by one TOC, whereas now the interaction between several TOCs and IM is required. The timetable design itself consists of two well-known sequential problems: the Line Planning Problem (LPP) and the Train Timetabling Problem (TTP). In the first stage, each TOC solves the LPP based on a pre-processed pool of potential lines and the estimated aggregated demand between every Origin Destination (OD) pair. The LPP selects the most suitable combination of the lines and their frequencies with an objective of maximizing the number of direct travelers and/or minimizing the operating cost. In the next stage, the IM collects the timetable requests of each TOC and resolves potential conflicts by solving the TTP. As discussed above no universal objective function is defined. Two versions of the TTP exist: non-cyclic and cyclic. In the non-cyclic model, the IM receives ideal timetables as an input. They are defined as the most profitable schedules. The objective of the problem is to maximize the profit of the adjusted conflict-free timetables (Caprara et al. (2002)). The problem does not take into account connections between the trains and thus the timetable adjustments might disconnect the trains and cause discomfort to the passengers. In the cyclic model the focus is on the cyclicity (*i.e.* feasibility) of the proposed timetable rather than on optimizing profit (Caprara et al. (2007)). Although, some user-defined objective functions exist (Peeters (2003)). In the cyclic TTP, the connections between the trains are always secured. However, it is not known if these connections are actually used by the passengers. By the definition of cyclicity, *i.e.* repeating pattern that is easy to remember, the approach is considered passenger oriented. Note that the TTP models in the current literature often deal with the old market settings even though the non-cyclic TTP could be suitable for the IM (as described in Caprara et al. (2007)). All the problems in the above planning horizon are offline, *i.e.* they are solved before the planned operation.

We identify here a gap between the LPP, that focuses on passengers (minimize travel time) and operator (minimize cost), and the timetable problem, that focusses on profit or cyclicity. The inconsistency between the several objectives may be counter productive (as we show later in this paper). The operator's goal is to maximize his profits and the passengers' goal is to receive the best possible service from their origin(s) to their destination(s). The two goals are in competition: the best possible service for passengers may also be the most costly alternative for the operator. Therefore, we propose to address the problem of designing the timetable, taking into account both satisfaction of the passengers and the profit of the operator.

The operator's profit as such is usually well defined. The passenger satisfaction needs specific modeling, based on utility theory. The (dis)utility of traveling for an individual is a function of various features of the trip: the time spent in the train, the time spent waiting, the number of transfers from one train to another and the timeliness at the destination.

In the published literature, passenger satisfaction based on the first three elements already exists and is assessed together using the results of discrete choice models (*i.e.* the passenger perception): the waiting time has a larger relative weight than the in-vehicle-time, the transfer from one train to another is penalized by adding extra in-vehicle-time (Kanai et al. (2011), Sato et al. (2013)). The presented application (Kanai and Sato) is from the delay management, where the passengers are already in the network and the goal is to minimize dissatisfaction. In the offline version of train timetabling, the time of arrival at the origin station is crucial for timetable design. Since people plan their arrival to the destination rather than the origin, the scheduled delay at the destination can be used as a good measure (Small (1982)) to quantify this aspect. Further on, to achieve the full impact of the passenger satisfaction method, the model has to take into account all possible paths between an origin and a destination. In the literature, it is often assumed that the passengers take only their shortest path, which in the end might not be the case as it is dependent on the exact timetable as shown in Schmidt and Schöbel (2015).

In this research, we further extend the concept of the passenger satisfaction by including the scheduled delay that accounts for the preferred arrival time at a passenger's destination. The overall passenger satisfaction may be expressed in the monetary units. Apart from that, we allow the passengers to take any path between their OD pair that is their most satisfying path.

We propose to introduce an additional step in the railway planning process called the Passenger Centric Train Timetabling Problem (PCTTP). The PCTTP is formulated as a Mixed Integer Linear Problem (MILP) and it is using the output of the LPP and serves as an input to the traditional TTP. Hence, it is placed between the two respective problems (Figure 2). The PCTTP is capable of designing both cyclic and non-cyclic timetables. The IM operated TTP is now considered as a one universal problem that should take the base of the non-cyclic TTP and add extra constraints that would secure cyclicity for the operators that wish to run the cyclic timetables (not in the scope of this research).

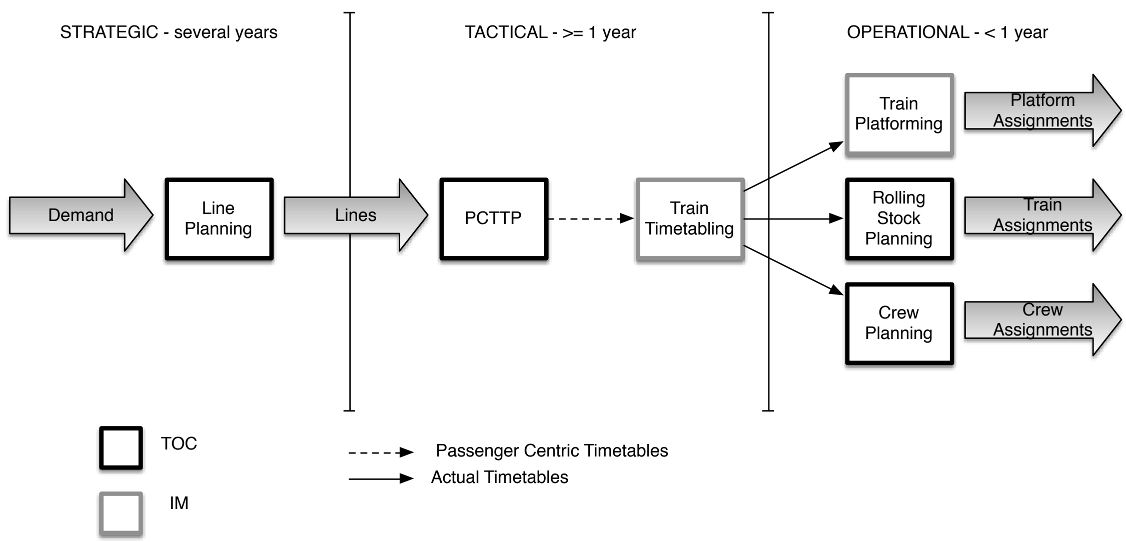


Figure 2: *Modified overview of railway operation*

The structure of the manuscript is as follows: in the literature review (Section 2), we consider different state-of-the-art objectives in the design of timetables. In Section 3, the problem definition, different objectives, mathematical model and procedure to construct the Pareto frontier is given. The case study on which we test the model is shown in Section 4. The paper is finalized by drawing some conclusions and discussion of possible extensions in Section 5.

2 Literature Review

In this study, we focus on the design of the passenger centric timetables. The literature review reports on different definitions of the quality of a timetable. In other terms, we investigate the various objective functions that have been used in the operations research literature on timetabling. The main aim of the Train Timetabling Problem is to remove any potential track occupation conflicts. We organize the literature based on the point-of-view: operator, passenger or integrating both.

2.1 Operator

In the literature, operator oriented objective functions can be found only for the non-cyclic version of the TTP. Two formulations of this model exist: Mixed Integer Linear Programming (MILP) or Integer Linear Programming (ILP). The MILP model uses continuous time, whereas the ILP model discretizes the time. The ILP models use the same objective function, whereas MILP models consider different objectives.

2.1.1 ILP Formulations of the non-cyclic TTP

In this formulation, it is assumed that the ideal timetables with the maximum profit are known a priori. Even though there is no available formal definition or methodology of creating such timetables in the literature, Caprara et al. (2002, 2006, 2007) mention that these timetables are defined as being the most profitable for the train operators. The objective then is to look for a conflict free (also called the actual) timetable for a whole railway network by minimizing the profit loss induced by changes made to the ideal timetables.

The model allows for two modifications of the ideal timetables: shifting of the departure time and shifting of the arrival time. Since a train can be kept waiting at a train station, the shift in the departure time is done by adding an extra waiting time at a station. On the other hand the arrival time is dependent on the departure from the previous station (already covered by the departure time change) and the running time between the stations. Thus the arrival time adjustment is done by stretching the running time, *i.e.* by lowering the speed of a train. Since the problem does not have access to the original profit function, the adjustments are actually addition of an extra waiting time/delay to the ideal timetable instead of adjustments of the real profit. However, there is a need for a more sophisticated method that would be used by the IM in the future.

One of the first TTP papers is Brannlund et al. (1998). The authors discretize the time and solve the problem using lagrangian relaxation of the track capacity constraints. The lagrangian relaxation of the same constraints is used as well in Caprara et al. (2002, 2006), Fischer et al. (2008) and Cacchiani et al. (2012). On the other hand, in Cacchiani et al. (2008), a column generation approach is applied. This approach tends to find better bounds than the lagrangian relaxation. Subsequently, several ILP re-formulations are introduced and compared in Cacchiani et al. (2010a). In Cacchiani et al. (2010b), the ILP formulation is adjusted, in order to be able to schedule extra freight trains, whilst keeping the timetables of the passengers' trains fixed. A dynamic programming approach, to solve the clique constraints, is used in Cacchiani et al. (2013).

2.1.2 MILP Formulations of the non-cyclic TTP

The MILP model has received less attention in the literature. A common assumption is the knowledge of the originally planned timetable constructed by the operator without stating its properties (unlike for the ILP models). Due to the complexity of the model, different heuristics are common solution method.

In Carey and Lockwood (1995), the objective is to minimize the overall cost associated with allocating a train path consisting of a trip time, waiting time, dwell time, and arrival and departure time.

A heuristic, that considers one train at a time and solves the MILP, based on the already scheduled trains, is introduced. Several more heuristics to solve the MILP model are presented in Higgins et al. (1997), where minimization of a total weighted travel time is used as an objective. Another MILP formulation minimizing the delays on arrivals and departures is given in Harrod (2012).

Subsequently, in Oliveira and Smith (2000) and Burdett and Kozan (2010), a job-shop scheduling reformulations are presented. In Oliveira and Smith (2000), the objective is to minimize the deviation from the originally planned timetable that is expressed as a delay. In Burdett and Kozan (2010), on the other hand, the objective is to minimize the feasibility violations by penalization.

2.2 Passenger

As mentioned in the introduction, the cyclic version of the TTP is considered passenger oriented. The reasoning behind it is that if a train line leaves from a station at the same time in every cycle, it is then easy for the passengers to remember the timetable and thus use the railways more often.

All of the cyclic models are based on the Periodic Event Scheduling Problem (PESP), which was first defined by Serafini and Ukovich (1989). The aim of the PESP is to schedule events in evenly spaced intervals/periods. In the early years of the PESP models the focus was mainly on finding a feasible solution and these problems did not have an objective function per se (Odijk (1996)), *i.e.* an arbitrary feasible solution was selected. When the problem is too complex or when no feasible solution exists, a minimization of a constraint violation can be used as an objective (Peeters (2003)). Given the complexity of the timetabling problems some simplifications, such as the assumption that passengers would travel only along shortest paths, have been introduced into the models. This assumption reduces the objective of minimizing the total travel time into minimizing the waiting time only (Nachtigall (1996), Peeters (2003)). This approach can be further extended by exploiting different waiting times and their values (Vansteenwegen and Oudheusden (2006, 2007)). In the current publication of Schmidt and Schöbel (2015), the shortest path assumption has been relaxed and thus the passenger routing and train timetabling is an integrated decision based on the travel time minimization. The considered model is non-cyclic.

Another way of securing a higher comfort to the passengers is to maximize the robustness of a timetable by pulling the trains apart of each other – maximizing the departure time difference (Peeters (2003)) or to minimize the average weighted delay using simulation (Kroon et al. (2008)) or by designing robust transfers (Vansteenwegen and Oudheusden (2007)).

All of the above examples take as input deterministic demand. However the demand depends on the supply. A way to account for this phenomenon is to use discrete choice model to estimate the number of passengers for different versions of a timetable and select the one with the highest amount of passengers using the train as their mode of transport (Cordone and Redaelli (2011)).

A "simple" approach to account for the passengers is to weight the delay of the trains by the amount of passengers on board (Espinosa-Aranda and Garcia-Rodenas (2013)). However, a more sophisticated approach is to model the passenger satisfaction affected by various elements (waiting time, running time, number of transfers) and try to minimize it (Sato et al. (2013)). This can be further improved by taking into account passenger congestion on board of the trains (Kanai et al. (2011)).

2.3 Integrated

Since the cyclicity constraints reduce the solution space significantly, the cyclic model has become more suitable for exploring of advanced objectives that can combine both operator and passenger point-of-view. One such example is a minimization of a weighted combination of the following objectives: required number of train compositions, passenger connection times, sum of running times and dwell times as shown in Kroon et al. (2014).

In order to further increase the potential gains in the objectives an integration with the LPP could be suitable. This methodology with an objective of minimization of weighted sum of the total engine time and the total passenger journey time has been tested by Kaspi and Raviv (2013).

2.4 Remarks

From our Literature Review, we have learnt that the main part of the publications is rather focused on the operational aspect of the problem (network safety) than the search of the most suitable departure times for the passengers. However, it can be noticed that a new direction of passenger point-of-view problems is emerging. This class of problems aims at using passenger satisfaction as a quantitative measure. This attribute is based upon human behavior, mathematically represented by discrete choice (utility) theory. In our research, we further extend this concept.

3 The Model

In this section, we present a mixed integer programming formulation for the Passenger Centric Train Timetabling Problem (PCTTP). It is a bi-objective optimization problem. We decide to treat the operator objective function as the main objective, and to include the passenger satisfaction as an ε-constraint (Ngatchou et al. (2005)).

3.1 Data and Variables

The input of the PCTTP is the demand that takes the form of the number of passengers n_i^t that want to travel between OD pair $i \in I$ and that want to arrive to their destination at their preferred arrival time $t \in T_i$. The actual value of the preferred arrival time is stored in the parameter a_i^t and the set T_i is used for indexing, *i.e.* each pair i has several preferred arrival times (indexed by t) each of them different. Subsequently, there is a pool of lines $\ell \in L$. Each line ℓ can be further decomposed into an ordered set of segments S^ℓ, a segment is a part of the line between two stations, where the train does not stop. The segments are unique part of an infrastructure irrespective of the lines. Each line has an assigned frequency expressed as the available trains $v \in V^\ell$ (lines, segments and frequencies are the output of the LPP). Based on the pool of lines, the set of paths between every OD pair $p \in P_i$ can be generated. Each path p between an origin and a destination consists of several attributes: a sequence of lines in order that they are being traversed L^p, running time from the origin of the line to the origin of the OD pair $b_i^{p\ell}$ (where $\ell = 1$), the running time from an origin of the OD pair to a transferring point between two lines $r_i^{p\ell}$ (where $\ell = 1$), the running time from the origin of the line to the transferring point in the path $b_i^{p\ell}$ (where $\ell > 1$ and $\ell < |L^p|$), the running time from one transferring point to another $r_i^{p\ell}$ (where $\ell > 1$ and $\ell < |L^p|$) and the running time from the last transferring point to a destination of the OD pair $r_i^{p\ell}$ (where $\ell = |L^p|$). Note different lines using the same track might have different running times. For more explanations of what a path is, refer to the Appendix B.

Part of the PCTTP is the routing of the passengers through the railway network. Using a decision variable x_i^{tp}, we secure that each passenger (combination of indices (i, t)) can use at most one path. If there is no path assigned to a given passenger (due to the limited capacity of the trains), it is assumed that the passenger would travel outside of the planning horizon h. In such a case, the revenue this passenger would generate, is not accounted for in the objective function and different satisfaction function will be applied (detailed explanation further below).

Within the path itself, passenger can use exactly one train on every line in the path (decision variable $y_i^{tp\ell v}$). These decision variables, among others, allow us to backtrace the exact itinerary of every passenger. The timetable is understood as a set of departures for every train on every line (values of d_v^ℓ). The timetable can take form of a non-cyclic or a cyclic version (depending if the cyclicity constraints are active, see below).

Since we know the exact itinerary of every passenger, we can measure the train occupation ω_{vs}^ℓ of every train v of every line ℓ on each of its segment s. Derived from the occupation, number of train units μ_v^ℓ is assigned to each train. This value can be equal to zero, which means that the train is not running and the frequency of the line can be reduced. The model also keeps track of the number of train drivers α_v^ℓ needed to realize the timetable.

The list of all inputs can be found in Table 1. The used acronyms are: min for minutes, CHF for Swiss Franc (any other currency can be used).

Name	Description	Units	Purpose
I	set of origin-destination pairs	–	set
T_i	set of preferred arrival times for OD pair i	–	set
P_i	set of possible paths for OD pair i	–	set
L	set of operated lines	–	set
L^p	set of lines in the path p	–	set
V^ℓ	set of available trains for the line ℓ (frequency)	–	set
S^ℓ	set of segments on line ℓ	–	set
h	end of the planning horizon	min	parameter
M	sufficiently large number (can take the value of h)	–	parameter
m	minimum transfer time	min	parameter
c	cycle	min	parameter
a_i^t	preferred arrival time of a passenger (i, t) to her destination	min	parameter
$r_i^{p\ell}$	running time for OD pair i on path p using line ℓ	min	parameter
$b_i^{p\ell}$	time to arrive from the starting station of the line ℓ to the origin/transferring point of the OD pair i in the path p	min	parameter
n_i^t	number of passengers wishing to travel between OD pair i at time t	–	parameter
VOT	value of (in-vehicle-) time (VOT)	CHF/min	parameter

β_W	value of the waiting time in the relation to the VOT	–	parameter
β_L	coefficient of being late in the relation to the VOT	–	parameter
β_E	coefficient of being early in the relation to the VOT	–	parameter
β_T	penalty for having a train transfer	min	parameter
e_s	ticket price of a segment s	CHF	parameter
f	cost of a train driver	CHF/train-km	parameter
o	operating cost of a single train unit	CHF/km	parameter
q	capacity of a single train unit	–	parameter
g	maximum length of the train	train units	parameter
k^ℓ	length of the line ℓ	km	parameter
ε	current minimum level of passenger satisfaction	CHF	parameter
r_i^1	in-vehicle-time of the shortest path between OD pair i	min	parameter
u_i	number of transfers in the shortest path between OD pair i	–	parameter
$\mathcal{V}_i^t$	generalized time when not serving passenger (i, t) within h	CHF	parameter
$\mathcal{T}_i^t$	generalized time of a passenger (i, t) when served within h	min	decision
$\mathcal{T}_i^{tp}$	generalized time of a passenger (i, t) using path p when served within h	min	decision
δ_i^{tp}	the scheduled delay of being early in path p of a passenger (i, t)	min	decision
γ_i^{tp}	the scheduled delay of being late in path p of a passenger (i, t)	min	decision
x_i^{tp}	1 – if passenger (i, t) chooses path p; 0 – otherwise	binary	decision
$y_i^{tp\ell\nu}$	1 – if a passenger (i, t) on the path p takes train ν of the line ℓ; 0 – otherwise	binary	decision
d_ν^ℓ	the departure time of a train ν on the line ℓ (from its first station)	min	decision
z_ν^ℓ	dummy variable to help modeling the cyclicity corresponding to a train ν on the line ℓ	$\mathbb{N}$	decision
$\omega_{\nu s}^\ell$	train occupation of a train ν of the line ℓ on a segment s	–	decision
μ_ν^ℓ	number of train units of a train ν on the line ℓ	–	decision
α_ν^ℓ	1 – if a train ν on the line ℓ is being operated; 0 – otherwise	binary	decision
w_i^{tp}	waiting time of a passenger (i, t) using path p	min	decision
$w_i^{tp\ell}$	waiting time of a passenger (i, t) using path p on a line ℓ	min	decision

Table 1: Inputs of the PCTTP model

3.2 Objective Functions

The aim of this section is to define an objective function for the PCTTP. Since the operator and the passengers have different goals, we define two objectives: profit maximization for the operator and satisfaction maximization for the passengers. In both cases, we assume that a forecasted demand consisting of the location of the origin, the location of the destination, and the preferred arrival time at the destination is known for each trip. In this research, we do not take into account the elasticity of the demand, but rather aim at exploration of different trade-offs.

3.2.1 Operator

Given the free market settings, the goal of the operator is to maximize the profit. The profit itself is a well defined function: revenue minus the operating cost. In our formulation, we assume a basic fare structure: the price of a ticket e_s on a segment s is irrespective of a line. To assess the total ticket price between OD pair i it is sufficient to add up the prices of every segment s in the path p traversed via every line in the path (L^p). We denote the train occupation ω^{ℓ}_{vs} for every line ℓ, train v and segment s. By multiplying this value with the respective fares, we can estimate the revenue (first term of the Equation 1).

$$\max \sum_{\ell \in L} \sum_{v \in V^{\ell}} \sum_{s \in S^{\ell}} \omega^{\ell}_{vs} \cdot e_s - \sum_{\ell \in L} \sum_{v \in V^{\ell}} \left(\alpha^{\ell}_{v} \cdot f \cdot k^{\ell} + \mu^{\ell}_{v} \cdot o \cdot k^{\ell} \right) \tag{1}$$

The operating cost (second term of the Equation 1) can be split into two parts: the cost of a driver and the cost of the rolling stock operations. The train driver salary f is given in monetary units per kilometer (standardized to our case study, different formulations for different countries might apply), k^{ℓ} is the length of the line ℓ in kilometers. α^{ℓ}_{v} is a binary decision that is equal to one if the train is being operated or to zero where the train is being dropped from the timetable.

As for the rolling stock itself, we consider a homogenous fleet of train units and each train can consist of several units. The operating cost for a single train unit o is in monetary units per kilometer. The number of units used per train v on a line ℓ is given by the integer variable μ^{ℓ}_{v}. The overall profit maximization objective function is given by the Equation 1.

3.2.2 Passenger

Each passenger is traveling between her origin and destination. She also knows the point in the time at which she would like to arrive at her destination. Suppose, we have a given timetable, how can we assess which exact trains would she take?

A widely used method to predict travel demand is discrete choice analysis (Ben-Akiva and Lerman (1985)). The basic assumption in discrete choice models is that each traveler maximizes her own utility function. Given a set of available alternatives, the traveler chooses the one with the highest utility related to her. The utility function of each alternative depends on various attributes of the alternative, such as travel time and travel cost, and characteristics of the traveler, such as income, gender or trip purpose. We denote the utility function $\mathcal{U}^{tp}_{i}$ for each passenger (i, t) with different alternatives being different paths P_i to get from an origin to a destination. In this research, we assume the passengers to be homogenous. On the other hand, the alternatives/paths differ in the attributes as follows:

- **in-vehicle-time** (weighted by β'_V): (total) time in minutes that passengers spend within the various trains along the path.

- **waiting time** (weighted by β'_W): time in minutes that passengers spend waiting between two consecutive trains in their respective transfer points.

- **number of transfer(s)** (weighted by β'_T): how many times passengers have to change from one train to another.

- **scheduled delay (early)** (weighted by β'_E): time in minutes indicating how early a passenger has arrived to her destination as compared to her preferred arrival time, if she is on time or late, it is then equal to zero.

- **scheduled delay (late)** (weighted by β'_L): time in minutes indicating how late a passenger has arrived to her destination as compared to her preferred arrival time, if she is on time or early, it is then equal to zero.

We have omitted the price of a ticket from the utility function (*i.e.* from the passenger point-of-view), as in our case study the prices among the paths for same ODs do not differ and thus no difference among alternatives in the utility function can be achieved. Note that the utility theory deals only with marginal differences among alternatives. Given the above information, we can now assess the utility function of a single passenger (i, t) for a given path (alternative) p as follows:

$$\mathcal{U}_i^{tp} = -\beta'_V \cdot \sum_{\ell} r_i^{p\ell} + \beta'_W \cdot w_i^{tp} + \beta'_T \cdot (|L^p| - 1) + \beta'_E \cdot \delta_{ip}^t + \beta'_L \cdot \gamma_{ip}^t \tag{2}$$

The above utility function, which is to be maximized is unitless. The values of βs are unknown for the moment and have the following signes: $\beta'_V > 0$, $\beta'_W < 0$, $\beta'_T < 0$, $\beta'_E < 0$, and $\beta'_L < 0$. The first variable of the equation is the sum of the in-vehicle-time spent on board of each of the trains. Its coefficient has a negative value, since the higher the time the less attractive the path is. The second variable is the total waiting time w_i^{tp} of a path p. Constraints **(19)** in Section 3.3 show, how to calculate its value. The third variable is the number of transfers (*i.e.* the number of lines in a path p minus one). The fourth and the fifth variables are the scheduled delay. The scheduled delay for being early is calculated by constraints **(17)** and for being late by constraints **(18)**, both in Section 3.3.

We can divide the whole utility function by $-\beta'_V$ and thus obtain a quantity in minutes: the generalized time. As we divide by a negative number, this quantity has the opposite sign as the utility, and is therefore minimized by the passengers.

$$\mathcal{U}_i^{tp} = \sum_{\ell} r_i^{p\ell} - (\beta'_W/\beta'_V) \cdot w_i^{tp} - (\beta'_T/\beta'_V) \cdot (|L^p| - 1) - (\beta'_E/\beta'_V) \cdot \delta_{ip}^t - (\beta'_L/\beta'_V) \cdot \gamma_{ip}^t \text{ [min]} \tag{3}$$

In order to obtain the values of βs that can be understood as a weight of an attribute an estimation using real choice data is needed. We have surveyed the published literature and found the already estimated values of the βs:

- $\beta'_W/\beta'_V = \beta_W = -2.5$ is the perception related to the waiting time and is evaluated as double and a half of the in-vehicle-time (see Wardman (2004)).

- $\beta'_T/\beta'_V = \beta_T = -10$ is the penalty for having to change a train, expressed as an additional in-vehicle-time. The value of ten minutes is used by Dutch Railways (de Keizer et al. (2012)).

- $\beta'_E/\beta'_V = \beta_E = -0.5$ is the willingness to arrive to the destination earlier then the preferred arrival time, in order to reduce the in-vehicle-time. As shown in Small (1982), the travelers are willing to shift their arrival time by 1 to 2 minutes earlier, if it would save them 1 minute of in-vehicle-time.

- $\beta'_L/\beta'_V = \beta_L = -1$ is the willingness to arrive to the destination later then the preferred arrival time, in order to reduce the in-vehicle-time. As shown in Small (1982), the travelers are willing to shift their arrival time by 1/3 to 1 minute later, if it would save them 1 minute of in-vehicle-time.

Note that there is no estimate for the in-vehicle-time. Indeed there is no need for such estimate as all the other βs where estimated in relation to the in-vehicle-time. Also note that all of the estimates have negative value, as expected. Since both β'_V and β'_W were 1/min, the β_W has then no unit (this rule applies to all of the βs). Replacing the ratios by the βs from the literature, we get the final form of the generalized time function to be minimized:

$$\mathcal{T}_i^{tp} = \sum_{\ell} r_i^{p\ell} + \beta_W \cdot w_i^{tp} + \beta_T \cdot (|L^p| - 1) + \beta_E \cdot \delta_{ip}^t + \beta_L \cdot \gamma_{ip}^t \text{ [min]} \tag{4}$$

The resulting quantity is in minutes. We can also monetarize it by multiplication with the Value Of Time (VOT). The VOT is the willingness-to-pay for travel time savings. The average VOT for commuters using public transport in Switzerland is 27.81 Swiss francs per hour (Axhausen et al. (2008)), it basically means that the swiss travelers are willing to pay in average 27.81 CHF to save one hour of travel time. Using the VOT, we can obtain the generalized cost:

$$\mathcal{C}_i^{tp} = \mathsf{VOT} \cdot \mathcal{T}_i^{tp} \tag{5}$$

By calculating it for all passengers, we get the following objective function:

$$\min \sum_{i \in I} \sum_{t \in T_i} n_i^t \cdot \mathcal{C}_i^t \tag{6}$$

Since a passenger uses only one path (one with the highest value – constraints (**8**)), the path index is omitted from the objective function. The minimization of the generalized cost is equivalent to the maximization of its opposite quantity - passenger satisfaction. Thus from now on, we will refer to the passengers' objective as to the maximization of the passenger satisfaction (with an opposite sign of its value).

3.3 Constraints

Feasibility Constraints The PCTTP model can be decomposed into 2 parts: feasibility constraints and satisfaction estimation. The first takes care of the feasibility of the solution, whereas satisfaction estimation takes care of the passenger satisfaction attributes. At first, we present the Feasibility Constraints:

$$-\mathrm{VOT} \cdot \sum_{i \in I} \sum_{t \in T_i} n_i^t \cdot \mathcal{T}_i^t \geq \varepsilon, \tag{7}$$

$$\mathcal{T}_i^t \geq \mathcal{T}_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{8}$$

$$\mathcal{T}_i^t \geq \mathcal{V}_i^t \cdot \left(1 - \sum_{p \in P_i} x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \tag{9}$$

$$\sum_{p \in P_i} x_i^{tp} \leq 1, \qquad \forall i \in I, \forall t \in T_i, \tag{10}$$

$$\sum_{v \in V^\ell} y_i^{tp\ell v} = x_i^{tp}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall \ell \in L^p, \tag{11}$$

$$\left(d_v^\ell - d_{v-1}^\ell\right) = c \cdot z_v^\ell, \qquad \forall \ell \in L, \forall v \in V^\ell : v > 1, \tag{12}$$

$$d_v^\ell \leq d_{v+1}^\ell - 1, \qquad \forall \ell \in L, \forall v \in V^\ell : v < |V^\ell|, \tag{13}$$

$$\omega_{vs}^\ell = \sum_{i \in I} \sum_{t \in T_i} \sum_{p \in P_i} y_i^{tp\ell v} \cdot n_i^t, \qquad \forall \ell \in L, \forall v \in V^\ell, \forall s \in S^\ell, \tag{14}$$

$$\mu_v^\ell \cdot q \geq o_{vs}^\ell, \qquad \forall \ell \in L, \forall v \in V^\ell, \forall s \in S^\ell, \tag{15}$$

$$\alpha_v^\ell \cdot g \geq \mu_v^\ell, \qquad \forall \ell \in L, \forall v \in V^\ell, \tag{16}$$

The constraints (7) assure that a certain level of the total passenger satisfaction (ε) will be maintained. The constraints (8) select the path with the best generalized time for each passenger (i, t). Due to the capacity constraints (14), some passengers might not be served within the planning horizon h. If such a case happens, first possible shortest path outside of the planning horizon is being offered (constraints 9). Since the passenger will have to wait for the whole cycle, the trains in the shortest path are assumed to be perfectly connected as the prospective waiting time would be in any case marginal to the cycle and there might be earlier path offered as lines in the railway networks often overlap. Constraints (10) secure that every passenger is using at most one path to get from her origin to her destination. Similarly constraints (11) make sure that every passenger takes exactly one train on each of the lines in her path, if this path is being used. Constraints (12) model the cyclicity using integer division. When solving the non-cyclic version of the problem, these constraints have to be removed. Constraints (13) aim at reducing the solution space in terms of the departure time combinations, *i.e.* the departures of the trains are in ascending order with a difference of at least one minute. Constraints (14) keep track of a train occupation. Note that if $y_i^{tp\ell v}$ is equal to zero (no path assigned to a passenger) then this passenger is not accounted for in the profit of the operator and she has a different satisfaction function (constraints 23). Constraints (15) verify that the train capacity is not exceeded on every stretch/segment of the line. Constraints (16) assign train drivers, *i.e.* if a train v on the line ℓ is being operated or not.

Satisfaction Estimation To make the PCTTP complete, we need to expand the Feasibility Constraints with the satisfaction estimation. We will add the constraints related to the passenger satisfaction in blocks of attributes that constitute it.

$$\delta_i^{tp} \geq \left(a_i^t - \left(d_v^{|L^p|} + b_i^p|L^p| + r_i^{p|L^p|}\right)\right) \\ -M \cdot \left(1 - y_i^{tp|L^p|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L^p|}, \tag{17}$$

$$\gamma_i^{tp} \geq \left(\left(d_v^{|L^p|} + b_i^p|L^p| + r_i^{p|L^p|}\right) - a_i^t\right) \\ -M \cdot \left(1 - y_i^{tp|L^p|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L^p|}, \tag{18}$$

The first block of constraints takes care of the **scheduled delay**. The constraints **(17)** model the earliness of the passengers and constraints **(18)** model the lateness. If a train arrives early to the destination then the earliness has a positive value and the lateness has a negative value (and thus forced to be zero by the positivity domain constraint), vice versa in case the train arrives late. Since scheduled delay is related to the last line in the path, we use the size of the path $|L^p|$ to get the last index.

$$w_i^{tp} = \sum_{\ell \in L^p \setminus 1} w_i^{tp\ell}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{19}$$

$$w_i^{tp\ell_1} \geq \left(\left(d_{v_1}^{\ell_1} + b_i^{p\ell_1}\right) - \left(d_{v_2}^{\ell_2} + b_i^{p\ell_2} + r_i^{p\ell_2} + m\right)\right) \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall \ell_1 \in L^p : \\ -M \cdot \left(1 - y_i^{tp\ell_2 v_2}\right) - M \cdot \left(1 - y_i^{tp\ell_1 v_1}\right), \qquad \ell_1 > 1, \ell_2 = \ell_1 - 1, \forall v_1 \in V^{\ell_1}, \forall v_2 \in V^{\ell_2}, \tag{20}$$

$$w_i^{tp\ell_1} \leq \left(\left(d_{v_1}^{\ell_1} + b_i^{p\ell_1}\right) - \left(d_{v_2}^{\ell_2} + b_i^{p\ell_2} + r_i^{p\ell_2} + m\right)\right) \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall \ell_1 \in L^p : \\ +M \cdot \left(1 - y_i^{tp\ell_2 v_2}\right) + M \cdot \left(1 - y_i^{tp\ell_1 v_1}\right), \qquad \ell_1 > 1, \ell_2 = \ell_1 - 1, \forall v \in V^{\ell_1}, \forall v_2 \in V^{\ell_2}, \tag{21}$$

The second block of constraints is modeling the **waiting time**. There are 2 types of waiting time: the total waiting time of every path w_i^{tp} and the waiting time at every transferring point in every path $w_i^{tp\ell}$. The constraints **(20)** and **(21)** are complementary constraints that model the waiting time in the transferring points in every path. In other words, these two constraints find the two best connected trains in the two train lines in the passengers' path. Thus the connections between the trains will be passenger imposed. Constraints **(19)** add up all the waiting times in one path to estimate the total waiting time in a given path.

$$\mathcal{T}_i^{tp} = \sum_{\ell \in L^p} r_i^{p\ell} + \beta_W \cdot w_i^{tp} + \beta_T \cdot (|L^p| - 1) + \beta_E \cdot \delta_i^{tp} + \beta_L \cdot \gamma_i^{tp} \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{22}$$

$$\mathcal{V}_i^t = r_i^1 + \beta_W \cdot c + \beta_T \cdot u_i + \beta_L \cdot \left(h + c + r_i^1 + m \cdot u_i - a_i^t\right), \qquad \forall i \in I, \forall t \in T_i. \tag{23}$$

At last, constraints **(22)** combine all the attributes together into a generalized time of a passenger (i, t) for a path p. Constraints **(23)** do not involve any decision variable and are included in the model only to show, how the generalized time of a passenger that can not be served within the planned horizon is calculated.

Note that in our optimization model, we use the amount of available trains from the LPP as the upper bound and allow the model to reduce the number of trains as needed (mostly due to the low occupation). Since the goal of the PCTTP is to look for the best timetables from both point-of-views, it leaves the safety issues to the IM operated TTP. The PCTTP formulation is an original work with only cyclicity constraints taken from the existing literature.

3.4 Pareto Frontier

In order to construct the Pareto Frontier, the model needs to be solved for several levels of ε under the objective of maximizing the TOC's profit (Equation 1). At first, two extreme points are being solved: $\varepsilon = 0$, where the constraint (7) is removed from the model (thus pure profit maximization, *i.e.* objective (1) and constraints (10)-(23)), and $\varepsilon = 100$, where the passenger satisfaction level is set to the best possible value minus one (as the satisfaction is a real number and it would have been difficult to set it to the exact value, when using warm starts of CPLEX; objective (1) and constraints (7)-(23)) upon solving the model with objective of pure passenger satisfaction maximization (objective (6) and constraints (10)-(23)) noted as $\varepsilon = 100$*. All other εs (20,40,60,80) are equally spaced intervals with passenger satisfaction expressed as a percentage of the difference between the best and the worst passenger satisfaction possible (objective (1) and constraints (7)-(23)), *i.e.* $\varepsilon = 20$ meaning that passenger satisfaction is being set to the worst possible value plus 20% of the gap between the worst and the best satisfaction level.

4 Case Study

In order to test the PCTTP model, we have selected the network of S-trains in canton Vaud, Switzerland during the morning peak hours as our case study. We aim at comparing the currently operated cyclic timetable of Swiss Federal Railways' (SBB) with the PCTTP designed cyclic and non-cyclic timetable. The exact procedures, assumptions and further information about the data can be found in Appendix A.

The reduced network of S-trains is represented in Figure 3 (as of timetable 2014). We consider only the main stations in the network (in total 13 stations). The network consists of 14 bidirectional lines (S1, S2, S3, S4, S11, S21 and S31). The SBB operated timetable of the lines can be seen in Table 2. The table presents the 7 lines that run in both directions. Each combination of a line and its direction has its unique ID number. Column "from" marks the origin station of the line and column "to" marks its destination. The columns "departures" report the currently operated timetable (*i.e.* departures from the origin of the line) in the morning peak hour (5 a.m. to 9 a.m.), which is the time horizon used in our study. Trains that did not follow the cycle (marked with a star *) were set to a cycle value, in order to enforce the cyclicity constraints (the timetables in Switzerland are cyclic with a cycle of one hour).

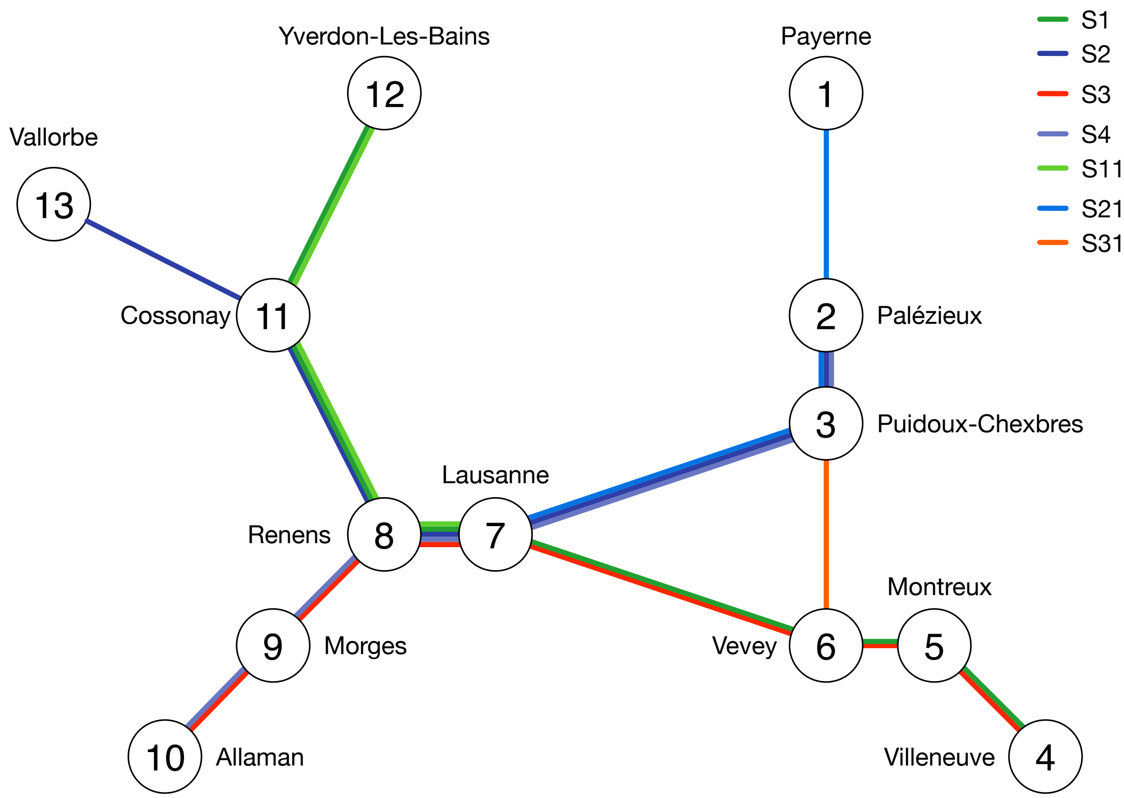


Figure 3: *Network of S-trains in canton Vaud, Switzerland*

We consider three instances of the model. The instance labeled “SBB 2014” is designed to mimic an existing timetable. The decision variables d_v^ℓ are set to the values presented in Table 2. The instance labeled “cyclic” considers the departure time d_v^ℓ as decision variables, and includes the cyclicity constraints (12). Finally, the instance labeled “non cyclic” is the same as the cyclic one, without the constraints (12). Furthermore, in this specific case study, we can reformulate the cyclicity constraints (12) in the following manner:

$$d_v^\ell - d_{v-1}^\ell = 60 \cdot z_{v1}^\ell + 120 \cdot z_{v2}^\ell, \qquad \forall \ell \in L, \forall v \in V^\ell : v > 1, \tag{24}$$

$$z_{v1}^\ell + z_{v2}^\ell = 1, \qquad \forall \ell \in L, \forall v \in V^\ell : v > 1. \tag{25}$$

As we can notice in Table 2, we either have 4 trains over 4 hours horizon (60 minutes difference between all consecutive trains) or 3 trains over 4 hours horizon (one train will have 120 minutes time distance from the next train, whereas the other trains will keep the 60 minutes difference). This attribute is modeled by adding an extra index to the cyclicity variable z, stating if the difference between two consecutive trains is 60 or 120 minutes.

Line	ID	From	To	Departures			
S1	1	Yverdon-les-Bains	Villeneuve	–	6:19	7:19	8:19
	2	Villeneuve	Yverdon-les-Bains	5:24	6:24	7:24	8:24
S2	3	Vallorbe	Palézieux	5:43	6:43	7:43	8:43
	4	Palézieux	Vallorbe	–	6:08	7:08	8:08
S3	5	Allaman	Villeneuve	–	6:08	7:08	8:08
	6	Villeneuve	Allaman	–	6:53	7:53	8:53
S4	7	Allaman	Palézieux	5:41	6:41	7:41	8:41
	8	Palézieux	Allaman	–	6:35	7:35	8:35
S11	9	Yverdon-les-Bains	Lausanne	5:26*	6:34	7:34	8:34
	10	Lausanne	Yverdon-les-Bains	5:55	6:55	7:55	8:55
S21	11	Payerne	Lausanne	5:39	6:39	7:38*	8:39
	12	Lausanne	Payerne	5:24	6:24	7:24	8:24
S31	13	Vevey	Puidoux-Chexbres	–	6:09	7:09	8:09
	14	Puidoux-Chexbres	Vevey	–	6:31*	7:36	8:36

Table 2: *List of S-train lines in canton Vaud, Switzerland*

All of the tested instances have been run in CPLEX Interactive Optimizer (CPLEX version 12.5.1) on a Unix server with 8 cores of 3.33 GHz and 62 GiB RAM. The CPLEX time limit has been set to 2 hours as most of the gap improvement is covered in this horizon (we have tested several models over a 6 hour horizon, the gap improvement in the additional 4 hours was less than 1 percent). In order to speed up CPLEX, we would first solve the SBB 2014 timetable and give its solution as a warm start to the cyclic model and solve it. Further along, we would give the solution of the cyclic model as a warm start to the non-cyclic model.

4.1 Results

In this section, we present the results of our case study. The detailed numerical results can be found in Tables 3, 4 and 5. The first row of the tables represents the level of the passenger satisfaction in percentage with respect to the gap between the best and the worst level of the satisfaction. The row cs provides the consumer surplus, *i.e.* how many Swiss francs can be gained in passenger satisfaction per one franc of profit loss as compared to the ε at level 0. The row lb provides lower bound on satisfaction for the case 100* and lower bound on profit for all other cases. We report the gap in percentage and in Swiss francs. Note that the value and the sign of the satisfaction has no meaning as such and should be used only for comparison across instances. Following rows state number of needed drivers, rolling stock and the percentage of served passengers.

From the tables, the cyclic and the non-cyclic timetables yield lesser profit than the SBB 2014 timetable for ε between 20 and 100. However, since the SBB 2014 timetable is cyclic, its result can be a feasible solution for the other two cases as well (cyclic timetable is a feasible solution for the non-cyclic timetable) and thus, if CPLEX is given infinite time, it would find at least as good solution

ε [%]	0	20	40	60	80	100	100*
profit [CHF]	53 067	52 926	50 730	49 564	13 826	4 211	-27 168
satisfaction [CHF]	-588 934	-505 899	-422 864	-339 828	-256 793	-173 759	-173 758
cs [CHF]	–	589	71	71	8	8	–
lb [CHF]	54 046	54 598	54 776	54 394	54 600	51 195	168 016
gap [%]	1.84	3.16	7.98	9.74	294.91	1115.74	3.30
gap [CHF]	979	1 672	4 046	4 830	40 774	46 984	5 742
drivers [-]	17	17	22	22	46	48	49
rolling stock [-]	32	32	32	32	46	55	98
covered [%]	99.35	99.34	100.00	100.00	100.00	100.00	100.00

Table 3: *Computational results of the SBB 2014 timetable*

ε [%]	0	20	40	60	80	100	100*
profit [CHF]	53 145	41 565	15 543	13 833	4 917	-27 200	-27,200
satisfaction [CHF]	-523 367	-452 808	-382 249	-311 691	-241 132	-170 574	-170,573
cs [CHF]	–	6	4	5	6	4	–
lb [CHF]	54 805	55 026	54 513	54 759	54 555	54 470	129 258
gap [%]	3.12	32.39	250.72	295.86	1009.52	300.26	24.22
gap [CHF]	1 660	13 461	38 970	40 926	49 638	81 670	41 315
drivers [-]	17	25	47	47	48	49	49
rolling stock [-]	32	36	47	47	54	98	98
covered [%]	99.35	97.42	99.91	100.00	100.00	100.00	100.00

Table 4: *Computational results of the cylic timetable*

ε [%]	0	20	40	60	80	100	100*
profit [CHF]	53 159	17 791	12 573	11 871	6 584	4 479	-27 257
satisfaction [CHF]	-516 011	-462 005	-376 000	-305 994	-235 989	-165 984	-165 983
cs [CHF]	–	2	3	5	6	7	–
lb [CHF]	54 465	54 955	54 927	54 862	54 824	54 682	124 693
gap [%]	2.46	208.89	336.86	362.15	732.69	1120.85	24.88
gap [CHF]	1 306	37 164	42 354	42 991	48 240	50 203	41 290
drivers [-]	17	46	48	48	48	48	49
rolling stock [-]	32	48	49	49	52	54	98
covered [%]	99.35	100.00	100.00	100.00	100.00	100.00	100.00

Table 5: *Computational results of the non-cylic timetable*

for the cyclic and non-cyclic timetable. At the $\varepsilon = 0$, the timetables give similar profit. Indeed almost all of the passengers get served while using the same number of trains. In case of the $\varepsilon = 100$*, we can see a difference for the passenger satisfaction between the models: the cyclic model achieves 3 185 CHF of savings and the non-cyclic timetable improves this value further by 4 590 CHF. Moreover, if we consider the maximum profit while securing the minimum passenger satisfaction ($\varepsilon = 100$), the non-cyclic timetable needs one driver and 43 train units less for operation.

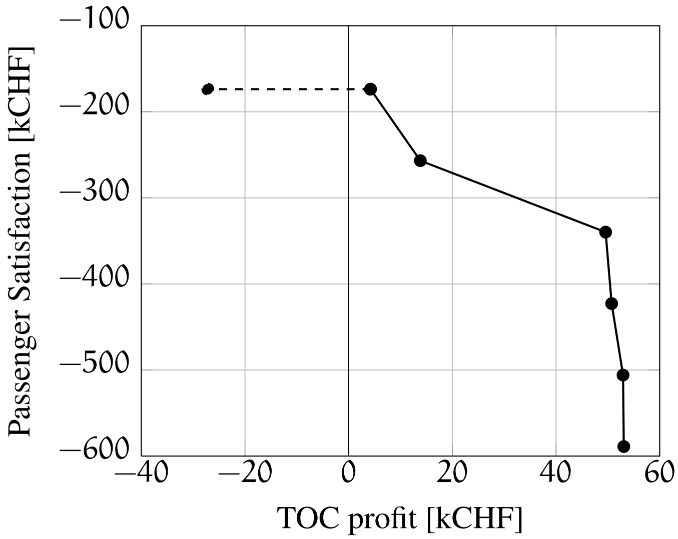


Figure 4: *Pareto frontier of the SBB 2014 timetable*

Lastly, we plot the trade-off between the profit and the satisfaction as a Pareto frontier (Figure 4) for the SBB 2014 timetable. In this Figure, we can spot an almost vertical line consisting of four points ($\varepsilon = 0, 20, 40, 60$). This means that large increase in passenger satisfaction can be achieved without having a significant change in the TOC profit. However the TOC might choose even the points with lower profit as the passenger satisfaction difference between points 60 and 100 is 166 069 CHF. Since the difference is quite large, it can justify running of the $\varepsilon = 100$ percent solution for public operators or to give an incentive on how to increase the ticket prices to private operators. Running of the whole fleet ($\varepsilon = 100$*), on the other hand, would not make sense as the current demand can be operated efficiently with less resources and achieve at least a small profit. It is worth mentioning that the consumer surplus has the highest value at $\varepsilon = 20$. It can be seen in Figure 4 that there is almost vertical line between the points $\varepsilon = 0$ and $\varepsilon = 20$, *i.e.* almost no profit loss and large satisfaction gain.

Overall, the insignificant difference among the three instances has two causes: low level of passenger demand and historical evolution of timetables. By the historical evolution, we mean that the operators have been offering their service for decades and thus learned how to match their supply with demand by trial and error.

4.2 Impacts of Passenger Congestion

In this section, we further investigate what would be the effect of a passenger congestion on the results from both points-of-view: operator's and passengers'. In order to do that, we gradually increase the demand (by multiplying the arrival rates by the same factor (Appendix A)) up until the point where the coverage decreases to a level of 70%. The passenger coverage as a function of the demand for the SBB 2014 timetable (the coverage is more or less the same for the other two timetables) can be found in Figure 5(a). As it can be seen, the congestion starts at the amount of cca. 27 000 passengers and that the coverage goes down almost linearly. All of the discussed sensitivity analysis results can be found in Figure 5.

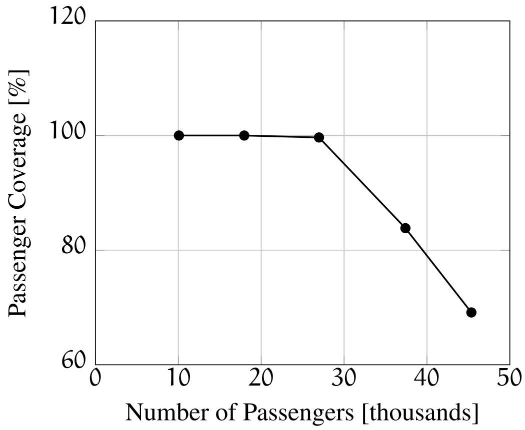
4.2.1 Impacts on Operator(s)

At first, we consider the operator point-of-view, *i.e.* how increase in demand affects the profit. The profit as a function of demand can be seen in Figure 5(c). We plot the results for the SBB 2014 timetable only (as from the previous section this timetable gave the smallest gap given its restrictions on the decision variables). Both minimum ($\varepsilon = 100$) and maximum ($\varepsilon = 0$) profit tend to grow (as expected). The difference between the two respective profit functions tends to decrease with the size of the demand (Figures 5(c) and 5(d)). The function becomes less steep when the network is experiencing the congestion. This seems logical accounted to the fact that when the trains become completely full (*i.e.* equally occupied), the profit should stabilize on the same values for both cases of ε. Note that if a passenger can not realize her complete journey due to the capacity limits, she would abandon the whole trip, thus when the congestion starts, some trains are not yet at full capacity. Moreover, the model changes its behavior under the congestion: in the profit maximization, passengers who travel longer distances and thus pay more are given advantage, whereas in the satisfaction maximization, large groups of passengers are given advantage. This fact allows to increase the profit difference at the beginning of the congestion (Figure 5(d)). A more realistic model for the selection of passengers during congestion would reduce this bias.

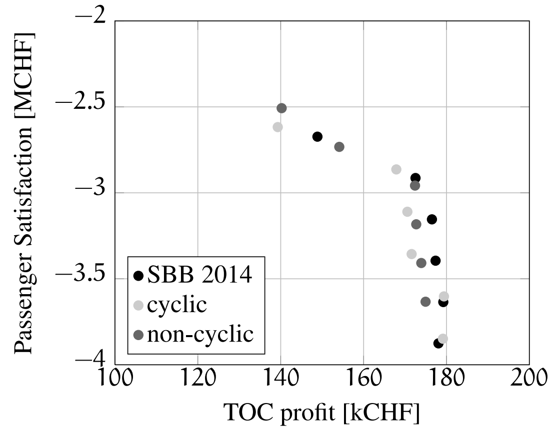
4.2.2 Impacts on Passengers

Subsequently, we consider the effects of congestion on the passenger satisfaction. In all of the plots, we consider the satisfaction at the $\varepsilon = 100^*$ as these are the best possible values. The satisfaction of the passengers, in the SBB 2014 timetable, as a function of the demand can be found in Figure 5(e). This function for the other two timetables yields similar result. In opposition to the profit, the passenger satisfaction decreases rather exponentially and its function can be split into two linear parts: non-congested (gradual slope) and congested (steep slope). This might be useful for practitioners as it would allow them to predict the passenger satisfaction.

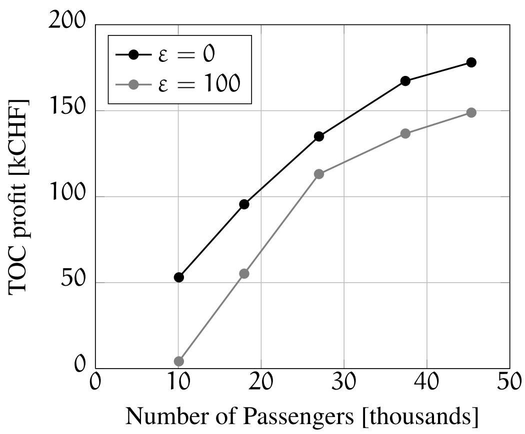
Subsequently, we plot the relative difference of cyclic and non-cyclic timetables as opposed to the SBB 2014 timetable in Figure 5(f). In general, the cyclic timetable tends to find slightly better timetables than the SBB 2014 model (in the congested cases the benefit even dramatically increases). The non-cyclic timetable, on the other hand, is more flexible and achieves significantly higher satisfaction. This is due to the fact that the trains do not have to follow the cyclic frequency and thus



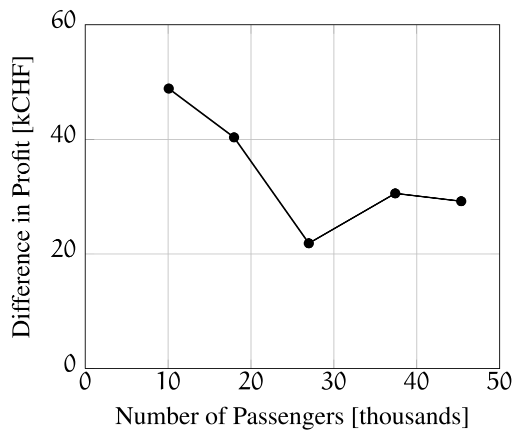
(a) *Passenger coverage as a function of the demand for the SBB 2014 timetable*



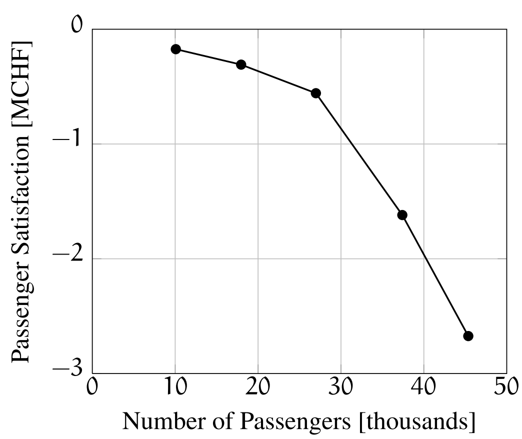
(b) *Pareto frontiers of the most congested case*



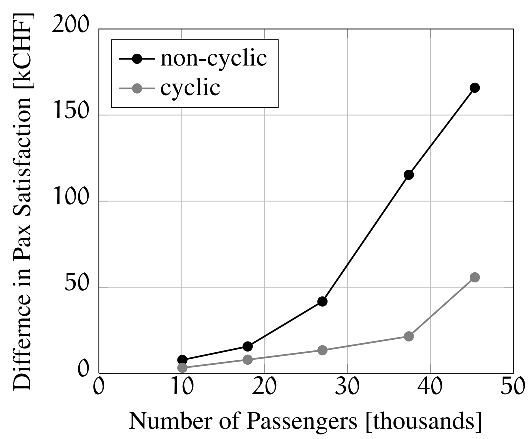
(c) *Profit as a function of the demand for the SBB 2014 timetable*



(d) *Difference in profit as a function of the demand for the SBB 2014 timetable*



(e) *Passenger satisfaction as a function of the demand for the SBB 2014 timetable at ε 100*%*



(f) *The difference in passenger satisfaction of the cyclic and non-cyclic timetable as compared to the SBB 2014 timetable at ε 100*%*

Figure 5: *Impacts of Passenger Congestion*

are more densely scheduled, for instance in the most congested case (cca. 45 000 passengers), the average headway between two consecutive trains on same lines is 22.6 minutes, with minimum value of 1 minute and maximum value of 238 minutes.

It could be objected that the relative difference is rather marginal compared to the total satisfaction level (in the best case 165 000 CHF against 2.5 million CHF), however we still consider it to be a significant value, especially since the network is quite dense (the demand goes towards Lausanne area and most of the trains pass through Lausanne and overlap in the network). In a less dense network the difference between the cyclic and the non-cyclic timetable is expected to be much larger.

4.2.3 Analysis of Trade-off(s)

The final step, in our sensitivity analysis, is the exploitation of the pareto frontiers in the most congested case (Figure 5(b)). The three frontiers are vertically spaced as they yield different levels of satisfaction (previous paragraphs). Shape-wise, the SBB 2014 timetable, as compared to the base demand scenario, has a frontier with one base point ($\varepsilon = 100$) and direct transition to an almost vertical line of solutions, which now contains one more point ($\varepsilon = 80$). The cyclic timetable, on the other hand, has a transition period consisting of $\varepsilon = 80 - 60 - 40$, before moving to the same solutions as the SBB 2014 timetable. This means that ε other than 100, 20 or 0 is not a good trade-off solution as it would lead to a worse profit than the SBB 2014 solutions. Lastly, frontier of the non-cyclic timetable has a longer transition period than the other two frontiers and it stops at a higher level of passenger satisfaction ($\varepsilon = 0$ of the non-cyclic timetable is at the same level as $\varepsilon = 20$ of the other two timetables), however with a slightly worse profit.

The best trade-off between the two objectives lies with the region of cyclic timetable at $\varepsilon = 20$, SBB 2014 timetable at $\varepsilon = 20$ and non-cyclic timetable at $\varepsilon = 40$. Solutions in this area lead to approx. 1 million CHF of passenger satisfaction increase with a profit loss of cca. 15 000 CHF.

5 Conclusions and Future Work

In this research, we introduce the passenger centric train timetabling problem. In this problem, we take into account both the TOC(s) and the passengers. The passengers' interests are modeled as a passenger satisfaction using utility theory. We consider all the possible paths between passenger's origin and destination. The connections between two trains are then passenger induced. The resulted timetables are trade-offs between the two stakeholders. The operator can select, if the designed timetables are to be cyclic or non-cyclic. We have tested the model using an experiment on a realistic railway network. The model is able to handle 1000 passenger groups (combination of OD pairs and preferred arrival time to the destination).

The results show that it is possible to achieve large improvements in passenger satisfaction, while keeping a low profit loss for the operator. We have shown that in a dense railway network with low volume of passengers, running cyclic or non-cyclic timetable has marginal impacts on the passengers. However, when the volume of passengers is high, the non-cyclic timetable outperforms the cyclic one. In a less dense networks, the impacts are expected to be larger.

In order to assess the benefits of the cyclic timetable properly, we would need to include in the model the fact that a cyclic timetable is easy to remember. This would need to be quantified in the context of utility theory, like the other variables discussed above. To the best of our knowledge, such a quantification has not been performed yet, and we could not include this attribute in the utility function. Another interesting future extension would be accounting for the elasticity of the total demand. In this study, we assumed that the passenger would use the railway as his mode of transport, no matter the value of his satisfaction. In the real world however, the passenger would switch to another TOC (if available) or to a different mode. This could be investigated by introducing a discrete choice model into the PCTTP. Similarly, a discrete choice model taking into account several competing TOCs could be yet another interesting extension. Lastly, since we have omitted the solution methodology side of the problem and rather focused on the concept, a future work should aim at the efficient solving of the problem that could allow larger time horizons (preferably whole day) and larger networks.

6 Bibliography

Axhausen, K. W., Hess, S., König, A., Abay, G., Bates, J. J. and Bierlaire, M. (2008). Income and distance elasticities of values of travel time savings: New swiss results, *Transport Policy* **15**(3): 173 – 185.

Ben-Akiva, M. and Lerman, S. (1985). *Discrete Choice Analysis*, The MIT Press, Cambridge Massachusetts.

Brannlund, U., Lindberg, P. O., Nou, A. and Nilsson, J. E. (1998). Railway timetabling using lagrangian relaxation, *Transportation Science* **32**(4): 358–369.

Burdett, R. and Kozan, E. (2010). A sequencing approach for creating new train timetables, *OR Spectrum* **32**(1): 163–193.

Cacchiani, V., Caprara, A. and Fischetti, M. (2012). A lagrangian heuristic for robustness, with an application to train timetabling, *Transportation Science* **46**(1): 124–133.

Cacchiani, V., Caprara, A. and Toth, P. (2008). A column generation approach to train timetabling on a corridor, *4OR* **6**(2): 125–142.

Cacchiani, V., Caprara, A. and Toth, P. (2010a). Non-cyclic train timetabling and comparability graphs, *Operations Research Letters* **38**(3): 179 – 184.

Cacchiani, V., Caprara, A. and Toth, P. (2010b). Scheduling extra freight trains on railway networks, *Transportation Research Part B: Methodological* **44**(2): 215 – 231.

Cacchiani, V., Caprara, A. and Toth, P. (2013). Finding cliques of maximum weight on a generalization of permutation graphs, *Optimization Letters* **7**(2): 289–296.

Caprara, A., Fischetti, M. and Toth, P. (2002). Modeling and solving the train timetabling problem, *Operations Research* **50**(5): 851–861.

Caprara, A., Kroon, L. G., Monaci, M., Peeters, M. and Toth, P. (2007). Passenger railway optimization, *in* C. Barnhart and G. Laporte (eds), *Handbooks in Operations Research and Management Science*, Vol. 14, Elsevier, chapter 3, pp. 129–187.

Caprara, A., Monaci, M., Toth, P. and Guida, P. L. (2006). A lagrangian heuristic algorithm for a real-world train timetabling problem, *Discrete Applied Mathematics* **154**(5): 738 – 753. IV ALIO/EURO Workshop on Applied Combinatorial Optimization.

Carey, M. and Lockwood, D. (1995). A model, algorithms and strategy for train pathing, *The Journal of the Operational Research Society* **46**(8): pp. 988–1005.

Cordone, R. and Redaelli, F. (2011). Optimizing the demand captured by a railway system with a regular timetable, *Transportation Research Part B: Methodological* **45**(2): 430 – 446.

de Keizer, B., Geurs, K. and Haarsman, G. (2012). Interchanges in timetable design of railways: A closer look at customer resistance to interchange between trains., *in* AET (ed.), *Proceedings of the European Transport Conference, Glasgow, 8-10 October 2012 (online)*, AET.

Espinosa-Aranda, J. L. and Garcia-Rodenas, R. (2013). A demand-based weighted train delay approach for rescheduling railway networks in real time, *Journal of Rail Transport Planning & Management* **3**: 1 – 13.

Fischer, F., Helmberg, C., Janšen, J. and Krostitz, B. (2008). Towards solving very large scale train timetabling problems by lagrangian relaxation, *in* M. Fischetti and P. Widmayer (eds), *ATMOS 2008 - 8th Workshop on Algorithmic Approaches for Transportation Modeling, Optimization, and Systems*, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany, Dagstuhl, Germany.

Harrod, S. S. (2012). A tutorial on fundamental model structures for railway timetable optimization, *Surveys in Operations Research and Management Science* **17**(2): 85 – 96.

Higgins, A., Kozan, E. and Ferreira, L. (1997). Heuristic techniques for single line train scheduling, *Journal of Heuristics* **3**(1): 43–62.

Kanai, S., Shiina, K., Harada, S. and Tomii, N. (2011). An optimal delay management algorithm from passengers' viewpoints considering the whole railway network, *Journal of Rail Transport Planning & Management* **1**(1): 25 – 37. Robust Modelling of Capacity, Delays and Rescheduling in Regional Networks.

Kaspi, M. and Raviv, T. (2013). Service-oriented line planning and timetabling for passenger trains, *Transportation Science* **47**(3): 295–311.

Kroon, L. G., Peeters, L. W. P., Wagenaar, J. C. and Zuidwijk, R. A. (2014). Flexible connections in pesp models for cyclic passenger railway timetabling, *Transportation Science* **48**(1): 136–154.

Kroon, L., Maróti, G., Helmrich, M. R., Vromans, M. and Dekker, R. (2008). Stochastic improvement of cyclic railway timetables, *Transportation Research Part B: Methodological* **42**(6): 553 – 570.

Nachtigall, K. (1996). Periodic network optimization with different arc frequencies, *Discrete Applied Mathematics* **69**: 1 – 17.

Ngatchou, P., Zarei, A. and El-Sharkawi, M. (2005). Pareto multi objective optimization, *Intelligent Systems Application to Power Systems, 2005. Proceedings of the 13th International Conference on*, pp. 84–91.

Odijk, M. A. (1996). A constraint generation algorithm for the construction of periodic railway timetables, *Transportation Research Part B: Methodological* **30**(6): 455 – 464.

Oliveira, E. and Smith, B. M. (2000). A job-shop scheduling model for the single-track railway scheduling problem, *Technical report*, University of Leeds.

Peeters, L. (2003). *Cyclic Railway Timetable Optimization*, ERIM Ph.D. series Research in Management, Erasmus Research inst. of Management (ERIM).

Sato, K., Tamura, K. and Tomii, N. (2013). A mip-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers, *Journal of Rail Transport Planning & Management* **3**(3): 38 – 53. Robust Rescheduling and Capacity Use.

Schmidt, M. and Schöbel, A. (2015). Timetabling with passenger routing, *OR Spectrum* **37**(1): 75–97.

Serafini, P. and Ukovich, W. (1989). A mathematical model for periodic scheduling problems, *SIAM J. Discret. Math.* **2**(4): 550–581.

Small, K. A. (1982). The scheduling of consumer activities: Work trips, *The American Economic Review* **72**(3): pp. 467–479.

Swiss Federal Railways (2013). SBB: Facts and Figures 2013, *Technical report*.

Vansteenwegen, P. and Oudheusden, D. V. (2006). Developing railway timetables which guarantee a better service, *European Journal of Operational Research* **173**(1): 337 – 350.

Vansteenwegen, P. and Oudheusden, D. V. (2007). Decreasing the passenger waiting time for an intercity rail network, *Transportation Research Part B: Methodological* **41**(4): 478 – 492.

Wardman, M. (2004). Public transport values of time, *Transport Policy* **11**(4): 363 – 377.

A Data Description

The data used in this case study were taken directly from the SBB's website (www.sbb.ch) and from the SBB's annual report for 2013 (Swiss Federal Railways (2013)). However, not all of the data required to solve the PCTTP were available and thus some assumptions and approximation techniques have been used. The synthetic data consist of the passenger demand (OD flows estimated based on demographic data and observation, ideal departure times estimated based on the SBB report and statistical theory, volume of the passengers estimated on demographics and statistical theory), the ticket fare distribution (several fare reduction schemes exist in Switzerland, distributed using simple assumption), maximum number of train units per train (based on observation) and the operator's cost structure (based on the aggregated value from SBB report and broken down by external knowledge).

An algorithm in Java has been coded, in order to find a set of possible paths between every OD pair. The algorithm allowed maximum of 3 consecutive lines to get from an origin to a destination. The traveling times have been extracted from the SBB website and they include stopping times at the stations that are not part of the case study, thus the distinction between the slow and fast services remains part of the problem. The minimum transfer time between two trains has been set to 4 minutes. The β estimates used are as described in Section 3.

A.1 Operator

The SBB is operating the Stadler Flirt train units on the lines S1, S2, S3 and S4. In our case study, we have homogenized the fleet and thus use this type of train also for the rest of the lines. The capacity of this unit is 160 seats and 220 standing people. The operating cost has been taken from the SBB's annual report for 2013 (Swiss Federal Railways (2013)), where a regional service has a cost of 30 CHF of a train per kilometer. Since no further details have been provided, we had to break down the cost using external knowledge. From a project for a Swiss public transport operator, we know that the driver's salary makes up more than half of the cost, *i.e.* in the "worst" case it is equal to a half (the higher the cost of the driver the cheaper the operating cost of additional train units thus the worst case). This leads to an operating cost of 15 CHF per train unit per km. The length of the lines in kilometers has been estimated using Google Maps. The maximum amount of train units per train is 2 (as SBB never uses more units). The amount of train units per train remains the same along the line, but it might change at the end stations (we don't go into further details as this is the task of the Rolling Stock Problem).

The ticket prices have been taken directly from the SBB website. In Switzerland, many people have a so called General Abonnement (GA) or a half-fare card. With GA, you pay yearly fee and get a free access to almost all public transportation in Switzerland. The half-fare card also involves a yearly fee (significantly smaller than GA) and gives access to a half price tickets for public transportation. In our case study, we have applied the half-fare prices to all of the passengers. This approach will balance the prices between GA users and normal users (normal user does not posses GA or half-fare card and thus pays the full price).

A.2 Passengers

The total amount of passengers in the network has been estimated in the following manner: the population of Switzerland is 8 211 700 habitants and the population of Canton Vaud is 755 369 habitants, which leads to a rough ratio of 1:10. Applying this ratio to a reported amount of passenger journeys per day by Swiss Federal Railways (2013) (in total one million for the whole SBB network), we arrive to a demand volume of 100 000 passenger journeys per day in canton Vaud. However not all of these journeys are being realized using S-trains. Since almost all of the trains in Canton Vaud have to pass through its capital city Lausanne, we can derive the ratio, between the S-trains and other class trains passing through Lausanne, of 40:60 percent, which leaves us with a 40 000 passenger journeys per day using S-trains in Canton Vaud. Furthermore, the SBB report provides hourly distribution of passengers on a regional services from Monday to Friday. According to this report 25 percent of the journeys are being realized in the morning peak hour, which gives us cca. 10 000 passenger journeys in the morning peak hour for the S-train network of Canton Vaud.

In order to ease the size of the generated lp file(s), the passengers have been split into 1 000 passenger groups (indices $(\mathtt{i}, \mathtt{t})$) of varying sizes. These groups have been divided into hourly rates (Figure 6) according to the SBB report and smoothed into minutes using non-homogenous Poisson process. Since we use concept of an ideal arrival time to the destination, the generated arrival time at the origin has been shifted, by adding up the shortest path travel time between the OD pair, to the destination of the passengers.

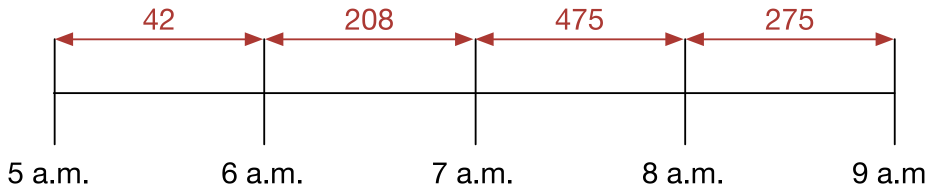


Figure 6: *The hourly distribution of the passenger groups*

In order to generate realistic OD flows (index i), we consider the following probabilities:

$p(D = 7) = 0.5$	–	probability of a destination being Lausanne
$p(D = 8) = 0.2$	–	probability of a destination being Renens
$p(D = other) = 0.3$	–	probability of a destination being other than Lausanne or Renens
$p(O = any) = 1/12$	–	probability of an origin being any station (except the already selected destination)

Since Lausanne is the biggest city in the Canton with all the lines, except the line 13 and 14, passing through it, it has the largest probability of being a destination (many people also use Lausanne as a transfer point to higher class trains). The city with the second highest probability is Renens, because it is the closest station to one of the biggest universities in Switzerland and from the network diagram (Figure 3), we can see at the most of the lines stop there, which suggests high demand. The rest of the stations have equal probability of being a destination (0.3/11), which is rather small as in the morning peak hour people travel towards their work/school in big cities. On the other hand, the probability of being an origin is uniformly distributed and dependent on its destination (origin can not be the same as a destination). The final probability $p(O = o, D = d)$ for every OD pair can be seen in Table 6.

In order to reach the total demand, the average size of a group should be ρ = the total demand divided by the number of groups. In the current scenario $\rho = 10\,000/1\,000 = 10$. In our study, we use 3 different classes of groups: small, medium and large. The size of the small group is drawn from the uniform distribution $\mathcal{U}(1, 0.6\rho)$ and applied to ODs with a probability $p(O = o, D = d) \in [0, 1.5)\,\%$. The size of the medium group follows $\mathcal{U}(0.6\rho + 1, \rho)$ and is applied to ODs with a probability $p(O = o, D = d) \in [1.5, 3)\,\%$. The largest group size follows a distribution $\mathcal{U}(\rho + 1, 2\rho)$ and is applied to a probability $p(O = o, D = d) \in [3, 4.5)\,\%$.

In total there are 10 077 passengers in the network for the current situation (the deviation from the estimated value of 10 000 is due to the randomness).

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
2	0.002272727	0	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
3	0.002272727	0.002272727	0	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
4	0.002272727	0.002272727	0.002272727	0	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
5	0.002272727	0.002272727	0.002272727	0.002272727	0	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
6	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
7	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
8	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
9	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0	0.002272727	0.002272727	0.002272727	0.002272727
10	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0	0.002272727	0.002272727	0.002272727
11	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0	0.002272727	0.002272727
12	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0	0.002272727
13	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0

Table 6: *Origin – Destination distributions*

B Path Representation

In order to better understand what a path is, consider the following example of going from Allaman (Station 10) to Villeneuve (Station 4) in our case study network (Figure 3). The set P_i (where i is the index of the given OD pair, in our case i=134) consists of 4 paths p. Each path p consists of the lines (L^p) ordered as they are traversed. The sets are as follows (numbered as in Table 2):

$$
\begin{aligned}
L^1 &= \{5\} \\
L^2 &= \{7, 1\} \\
L^3 &= \{7, 1\} \\
L^4 &= \{7, 14, 1\}
\end{aligned}
$$

There are two ways of transferring between lines 7 and 1: either the transfer is made in Renens or in Lausanne. The information of where the transfer is made can be extracted either from parameter $r_i^{p\ell}$ or parameter $b_i^{p\ell}$. The parameter $r_i^{p\ell}$ is the in-vehicle time of every line in the path. Note that the line id is now value in the set and that the indexing is within a set, thus $\ell \in L$ is different from $\ell \in L^p$.

$$
\begin{aligned}
r_{134}^1 &= \{58\} \\
r_{134}^2 &= \{14, 49\} \\
r_{134}^3 &= \{21, 42\} \\
r_{134}^4 &= \{37, 12, 17\}
\end{aligned}
$$

Since the decision of departure times of each train d_v^ℓ is related to the starting station of each line, we also need to know the traveling time from the origin of the line to the embarking station of a passenger $b_i^{p\ell}$.

$$
\begin{aligned}
b_{134}^1 &= \{0\} \\
r_{134}^2 &= \{0, 30\} \\
r_{134}^3 &= \{0, 37\} \\
r_{134}^4 &= \{0, 0, 62\}
\end{aligned}
$$

Each path p can be realized with different trains that have different departure times. The decision $y_i^{tp\ell v}$ relates the exact trains a passenger is taking in her path.

In order to verify that the capacity of a train is not exceeded, we need to measure train occupancy between any two stopping stations. This is done by using segments. The Table 7 shows all the segments in our case study. Each line then consists of a set of segments S^ℓ. For instance line 1 (from Yverdon-les-Bains to Villeneuve) has the following segments $S^1 = \{12, 11, 8, 6, 5, 4\}$.

Segment	Station 1	Station 2
1	Payerne (1)	Palézieux (2)
2	Palézieux (2)	Puidoux-Chexbres (3)
3	Puidoux-Chexbres (3)	Vevey (6)
4	Villeneuve (4)	Montreux (5)
5	Montreux (5)	Vevey (6)
6	Vevey (6)	Lausanne (7)
7	Puidoux-Chexbres (3)	Lausanne (7)
8	Lausanne (7)	Renens (8)
9	Renens (8)	Morges (9)
10	Morges (9)	Allaman (10)
11	Renens (8)	Cossonay (11)
12	Cossonay (11)	Yverdon-les-Bains (12)
13	Cossonay (11)	Vallorbe (13)

Table 7: *List of segments used in the case study*