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Train Timetable Design Under Elastic Passenger Demand

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Abstract

We integrate a probabilistic demand model in the train timetabling problem. We use a logit model that we calibrate to reflect the known demand elasticities. We further include a competing operator as an opt-out option for the passengers. Subsequently, we integrate the train timetabling problem with a ticket pricing problem. We solve the elastic passenger centric train timetabling problem for various types of timetables using a simulated annealing heuristic on a case study of Israeli Railways. The results of our case study show that the generated revenues can be increased by up to 15% when taking into account the passengers' behavior along with a specific pricing scheme. This study further confirms the advantages of hybrid cyclicity.

Keywords: Passenger Centric Train Timetabling Problem, Choice Modeling, Hybrid Cyclicity,

Ticket Pricing, Revenue

1 Introduction

From economic theory, it is well known that the demand is influenced by the supply. In the railway context: different timetables would attract different passengers, as they provide different levels of services. Therefore, it is important to include the passenger demand and its elasticity in the timetable design. This is particularly crucial nowadays, when the railway market is open for competition. This has lead to a release of the government's subsidies and the operators thus face a pressure to be profitable by increasing their revenues.

The passenger demand itself can be predicted through the choices that the passengers face: mode choice, route choice, operator choice, service choice and departure time choice. At first, the passengers decide on their mode of transport, *i.e.* car, bus, train, *etc.* Subsequently, they decide on their exact path within the selected mode, *e.g.* the exact trains and interchanges. As transportation is offered by several providers, the passengers can select which one to take and in what kind of service, *e.g.* first class with a private operator, having a club card of a specific operator, *etc.* Lastly, the departure time choice of the passengers is affected by the trip purpose (commuting to be on time at work, leisure to be on time for cinema for instance, *etc.*). This choice is the so called time dependency of the demand. The mode choice and the operator choice are especially important when optimizing a service. If not included, the passengers are captive in the system and such optimization would lead to unrealistic performance (*i.e.* departure times leading to a lower ridership than anticipated).

A train timetable is defined as a set of arrival and departure times of each train from each of its stopping stations and it is the output of the Train Timetabling Problem (TTP). Typically, the TTP models use the simplifying assumption that the passengers always take their shortest paths (see Caprara et al. (2002) for instance) and omit the demand from the problem. Recent models relax this assumption and include the demand in the optimization (see Schmidt and Schöbel (2015) for instance). However, these models only increase the attractiveness of a timetable and cannot estimate the realized demand and the underlying revenue. In order to do so, a demand forecasting model is needed. One of such models, integrating the train timetabling and the demand forecast, is presented by Cordone and Redaelli (2011). However, they only consider the mode choice and omit the other choices. Moreover, they consider the timetable only as a frequency of the service rather than the actual departure times. In the model of Espinosa-Aranda et al. (2015), the mode choice is combined with a departure time choice. But their application involves a single high speed railway line and lacks the network dimension (*i.e.* the route choice).

In this paper, we extend and adjust the Passenger Centric Train Timetabling Problem (proposed by Robenek et al. (2016)), that can design a timetable for a whole railway network, with a demand forecasting model. We denote the new model as the Elastic Passenger Centric Train Timetabling Problem (EPCTTP). The objective of the new optimization framework is to maximize the Train Operating Company's (TOC) revenue. The framework is using a discrete choice model to predict the demand throughout the timetable design process. We assume prior solving of the mode choice model and we solve the route choice model along with the departure time choice and the operator choice. The following attributes have influence on the passengers' choices: the travel time, the desired arrival time to their destinations (the time dependency), the ticket fare and the capacity of the trains. We include an universal option of opting out into the passengers' choice set(s), in order to avoid their captivity.

The demand elasticity and other parameters of the discrete choice model are calibrated to known values from the literature and provide a "ready to use" framework. It can design cyclic, non-cyclic and hybrid cyclic timetables. The resulted timetables increase the ridership through accounting for the passengers' wishes and therefore, they increase the operator's revenue as well. We test this approach on a case study of Israeli Railways.

In addition to the above, we further relax the ticket fare to be a decision variable. Such approach leads to a better utilization of the TOC's fleet and to a further increase in its revenue: shift passengers from overcrowded trains to under-used trains, attract back the passengers, who would otherwise opt-out, *etc.* The resulting framework combines pricing problem with an integrated demand-supply timetable design. It differs from the existing literarture, where the pricing and revenue management is performed on a fixed timetable.

The manuscript is structured as follows: Section 2 presents a survey of the literature, depicting the demand interaction in the current timetabling models, along with the literature on railway demand forecasting as well as pricing and revenue management in railways. Subsequently, we propose a demand forecasting model in Section 3, that is further incorporated into a train timetable optimization model in Section 4. The insight, about the methodology of how to solve such integrated demand-supply optimization framework, is given in Section 5. The benefits and the impacts of the framework are shown on a case study of Israeli Railways in Section 6. We finalize the paper by drawing some conclusions and discussion of possible extensions in Section 7.

2 Literature Review

Since the aim of this study is to introduce the demand and its elasticity into the train timetabling, the focus of our literature review is on the passenger demand representation in the railway planning literature. At first, we present the demand representations used in the train timetabling problems (Section 2.1) and continue with the various demand forecasting techniques used in the railway context (Section 2.2). Lastly, we discuss how some of the forecasting techniques are used in the revenue management and ticket pricing within the railways (Section 2.3).

2.1 Demand Representations

The most basic representation of the railway demand is an Origin-Destination (OD) matrix. Such a matrix is typically used in the Line Planning Problem (LPP) to determine the frequency of a train line (Schöbel (2012)).

Once the lines are designed, the Train Timetabling Problem (TTP) assigns a departure time to each train subject to the operational constraints. Two versions of this problem exist: cyclic and non-cyclic. In the cyclic TTP (Peeters (2003)), the main focus is on the cyclicity constraints, whereas in the non-cyclic TTP (Caprara et al. (2002)) the departure times do not have to follow any specific pattern. Traditionally, both problems assume that the passengers follow the shortest path. Therefore, there is no need to include the route choice dimension explicitly and the optimization is performed on the attributes of the shortest paths between the ODs.

Recently, this assumption has been relaxed and the passengers can choose from several paths while minimizing their total travel time: refer to Hoppmann et al. (2015) for the cyclic TTP and Schmidt and Schöbel (2015) for the non-cyclic TTP. However, this approach can be further improved by maximizing the passenger satisfaction instead. The passenger satisfaction better reflects the human behavior. It combines the in-vehicle-time, the waiting time, the number of transfers and the desired arrival time to the destination of a given path, each weighted by the respective human perception. Such approach is called the Passenger Centric Train Timetabling Problem and it can design both: cyclic and non-cyclic timetables (Robenek et al. (2016)) or a recently proposed hybrid cyclic timetable by Robenek et al. (to appear) (for a further description of this timetable refer to Section 4).

A different approach is to use the passenger arrival rates to their origin stations, in order to design a timetable (Luethi et al. (2007), Barrena et al. (2014a,b), Sun et al. (2014), Wang et al. (2015)). However, this method is only suitable for high frequency services such as the public transit.

2.2 Demand Forecasting

In order to forecast the demand, its behavior in form of the decisions needs to be modeled. As mentioned before, the main decisions are: mode choice, route choice, operator choice, service choice and departure time choice. One of the first forecasting models has been introduced by Wardman (1997). He shows a direct demand model taking into account the mode competition (between rail and car) and the service offer (cost and service quality). He further extends this model with additional attributes such as car ownership, car travel time and fuel cost in Wardman (2006). Other models are investigated by Wardman et al. (2007). The main focus is on the generalized cost, *i.e.* service choice. All of these models are in the settings of the British railway market.

Another group of demand forecasting models focuses on high speed rail. The case studies involve Italy (integrated demand model combining demand growth, mode choice and induced demand by Ben-Akiva et al. (2010)), China (hybrid approach combining empirical mode decomposition and gray support vector machine by Jiang et al. (2014)) and Sweden (nested logit model for mode choice by Börjesson (2014)), among others. A recent survey on a high speed railway demand forecasting models is presented by Börjesson (2014).

Remark However, all above models only forecast the demand and do not combine it with a timetabling problem. The supply-demand interaction is a fairly new topic in the timetabling context. Two applications of a train timetabling problem incorporating a mode choice forecast exist: Cordone and Redaelli (2011) and Espinosa-Aranda et al. (2015). Cordone and Redaelli (2011) maximize the demand captured by a cyclic timetable using a logit model as a mode choice and Espinosa-Aranda et al. (2015) maximize the profit of a non-cyclic timetable using a constrained nested multinomial logit model as a mode choice combined with a departure time choice.

2.3 Revenue Management and Pricing

Typically, the demand forecasting models are used in a combination with the Revenue Management (RM) and pricing problems. The aim of these problems is to maximize the revenue while optimizing the prices based on the demand forecasts.

In the pricing problems, the strategy is to adjust the ticket prices based on a given timetable. The key attributes justifying the price variation are space (busy parts of the network vs. under-used parts of the network) and time of the day (peak hours vs. off-peak hours). One of the first frameworks to analyze different pricing policies was introduced by Nuzzolo et al. (2000). They present a nested logit model that takes into account elementary trains, instead of the frequencies. The model combines route choice, departure time choice and service choice. However, an empirical study of the effects of pricing policies and demand elasticities on a service choice in the context of Dutch Railways (van Vuuren (2002)) suggests that the peak hour demand is rather inelastic. This makes the strategy of higher prices during peak hours more effective than a price reduction during the off-peaks (Whelan and Johnson (2004)). This conclusion was found in the context of avoiding train overcrowding using an incremental logit model to do the forecasts. The model is predicting route choice, departure time choice and service choice. A similar goal, of flattening the demand throughout the service, is pursued by Li et al. (2006). They propose several pricing schemes combining spatial and time attributes. The passenger behavior is mimicked through activity-based microsimulation that takes into account mode choice, route choice and service choice.

The RM models, on the other hand, focus more on the price adjustments related to the demographics of the population. The key attributes justifying the price variation are the time of the purchase, which is related to the trip purpose (leisure travelers buy their tickets early vs. business travelers, who buy last minute tickets), class (1st class vs. 2nd class), and others. For a more detailed overview discussing the principal differences in RM of airlines and railways refer to Armstrong and Meissner (2010).

Overall, the literature on RM in railways is scarce and focuses solely on the price variation over the time of the purchase. A quantitative analysis on the pricing suggestions for Indian Railways is shown by Bharill and Rangaraj (2008) and in Hetrakul and Cirillo (2014), a multinomial logit and latent class models are tested within the pricing optimization framework. Both focus on service choice.

2.4 Summary

From our literature review, we have learned that several railway passenger demand forecasting techniques exist, but that they are mainly used within the revenue management and the ticket pricing over a fixed timetable. Only a few models combine timetable design and demand forecasts and no model combining the demand forecasting, timetable design and ticket pricing exists.

3 Demand Model

At first, we present the demand forecasting model. We assume that a mode choice model has been already used to predict the railway mode users and that these users now face a route choice decision within their selected mode of transport (as shown in Figure 1). The route choice decision is further combined with departure time choice and operator choice.

A passenger is characterized by an Origin-Destination (OD) pair $i \in I$ and a desired arrival time

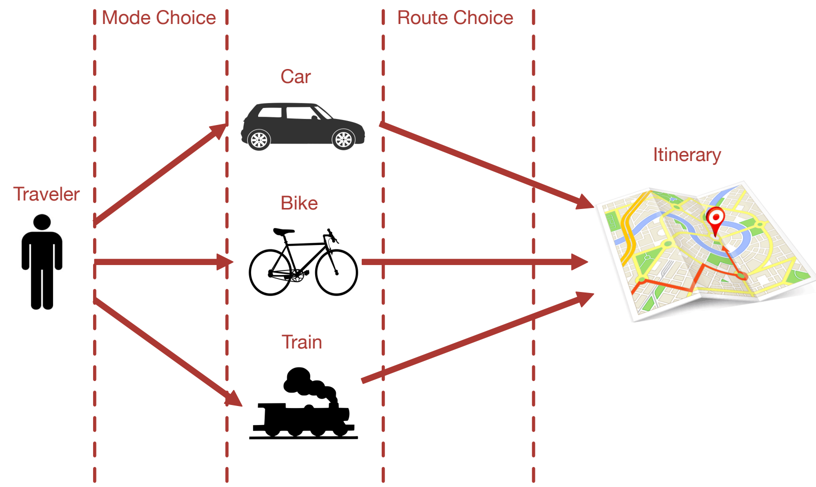


Figure 1: *Decision process of a traveler*

$t \in T_i$ at destination. The actual value of their desired arrival time is represented by the parameter a_{it} and the set T_i is only used for indexing purposes. In other words, each OD pair i is having several passenger groups with different desired arrival times (indexed by t). Therefore, the combination of indices (i, t) forms a unique group of passengers of size n_{it}. Note, that the time is discretized into minutes.

Each passenger group is presented with a set of available paths $p \in P_i$ to get from its origin to its destination. Given the operational constraints of the model this set might be reduced for specific groups: $P_{it} \subseteq P_i$. A train is defined by its line $\ell \in L$ and its position within the set of available trains $v \in V^\ell$ per line ℓ. Any line $\ell \in L$ is an ordered sequence of stopping stations, that includes the dwell times at the stations and the travel times in between the stations. Each line also consists in a set of segments $s \in S^\ell$. A segment is that part of the infrastructure, where a train (line) does not stop. Therefore, the passengers can enter/exit a train only at the stopping stations. Any path p is then an itinerary consisting in an ordered sequence of $j \in J^p$ trains (*e.g.* $j = 1$ is some combination of indices (ℓ, v), *etc.*) and the segments on which they are being used (known by a binary parameter $x^{ps}_{\ell v}$ – 1 if the passengers on the path p are using the train (ℓ, v) on the segment s, 0 – otherwise).

To make the total demand elastic, we include an opt-out option in the set of paths P_i. Otherwise the passengers are captive in the system, *i.e.* the total demand would not change whatever changes in the timetable and levels of fares. Since we assume a prior solving of the mode choice model, a passenger can exit the system only by usage of another TOC. If a competing operator exists, it is sufficient to add his itineraries inside of the set $p \in P_i$. However, since the existence of a competition is not yet wide spread, we propose the following assumption: given the nature of the railway competition (fighting for the same finite demand), the OD pairs served, the departure times and the prices of the competing operators are typically very similar, *i.e.* the opt-out option for every passenger group is the current shortest path (that has the lowest value of the in-vehicle time and has zero waiting time in the transfers and arrives to the passengers' destination exactly at their desired arrival time) and its underlying ticket fare. Such an assumption is realistic and allows to explore the "worst" case scenario.

When faced with a set of of discrete path alternatives, it is assumed that a passenger chooses the one that yields her maximum level of utility (Ben-Akiva and Lerman (1985)). For a traveler in group (i, t), utility $\mathcal{U}$ for a path $p \in P_{it}$ is defined as:

$$\mathcal{U}_{it}^{p} = \mathcal{V}_{it}^{p} + \varepsilon_{it}^{p} \tag{1}$$

The deterministic component $\mathcal{V}$ of the utility function is a function of observable attributes, *i.e.* relevant variables that describe a choice alternative and which levels cause the expected level of utility. Note that we don't consider any source of individual heterogeneity outside of the error term ε. In our framework, only attributes of alternatives enter the utility function. For a group (i, t), all passengers face the same levels of these attributes: $\mathcal{V}^{p}$ is only function of (i, t). We also assume that $\mathcal{V}_{it}^{p}$ is defined as a linear combination of the following travel attributes:

- **in-vehicle-time** $\sum_{j \in J^p} r_i^{pj}$ **–** is the time spent on board of each train j in path p.
- **waiting time** w_{it}^{p} **–** is the total time passengers spend waiting in transferring stations.
- **number of transfers** $|J^p| - 1$
- **schedule passenger delay –** indicates the time dependency of passenger demand and drives the departure time choice. Given the desired arrival time a_{it} to the destination of passenger group (i, t), it can either be on time (equal to zero), early or late ($\bar{a}_{it}$ being the actual arrival time).
  - **early –** $\delta_{it}^{p} = max(a_{it} - \bar{a}_{it}, 0)$
  - **late –** $\gamma_{it}^{p} = max(0, \bar{a}_{it} - a_{it})$

The deterministic component is therefore defined as follows:

$$\mathcal{V}_{it}^{p} = -\beta_V' \cdot \sum_{j \in J^p} r_i^{p\ell} + \beta_W' \cdot w_{it}^{p} + \beta_T' \cdot (|J^p| - 1) + \beta_E' \cdot \delta_{it}^{p} + \beta_L' \cdot \gamma_{it}^{p} \tag{2}$$

The parameters β weigh the different attributes. They model sensitivity of the passengers to the change in the related attributes. Note that they are considered to be the same across individuals (homogeneity assumptions). The values of βs have to be estimated from data and have the following signs: $\beta_V' > 0$, $\beta_W' < 0$, $\beta_T' < 0$, $\beta_E' < 0$, and $\beta_L' < 0$. In this research, we use values of βs from the literature. To do so, we divide the utility function by $-\beta_V'$ in order to convert the utility into travel time units. Indeed, parameters themselves are not directly transferable, since they include the scale parameter of the error term. The scale parameter cancels out when ratios of parameters are considered, so the values of these ratios are comparable from one model to the next. The values of the parameters are as follows:

- $\beta_{\mathbf{W}} = \beta_W'/\beta_V'$ is the substitution rate between waiting time and in-vehicle-time per minute. Its value is equal to -2.5 (Wardman (2004)).

- $\beta_{\mathbf{T}} = \beta'_T/\beta'_V$ is the penalty for having to change a train, expressed as an additional in-vehicle-time per transfer. Its value is equal to -10 minutes (as used by Dutch Railways (de Keizer et al. (2012))).

- $\beta_{\mathbf{E}} = \beta'_E/\beta'_V$ is the willingness to arrive to the destination earlier then the desired arrival time, in order to reduce the in-vehicle-time. As shown in Small (1982), the travelers are willing to shift their arrival time by 1 to 2 minutes earlier, if it would save them 1 minute of the in-vehicle-time. We consider here the highest value and set β_E to -0.5.

- $\beta_{\mathbf{L}} = \beta'_L/\beta'_V$ is the willingness to arrive to the destination later then the desired arrival time, in order to reduce the in-vehicle-time. As shown in Small (1982), the travelers are willing to shift their arrival time by 1/3 to 1 minute later, if it would save them 1 minute of in-vehicle-time. We consider here the highest value and set β_L to -1.

By replacing the original βs in Equation 2 with the estimated ones (*i.e.* division by $-\beta'_V$), we obtain function expressed in minutes. This concept is also known as the generalized time $\mathcal{T}$, that is supposed to be minimized by the travelers:

$$\mathcal{T}^p_{it} = \sum_{j \in J^p} r^{pj}_i - \beta_W \cdot w^p_{it} - \beta_T \cdot (|J^p| - 1) - \beta_E \cdot \delta^p_{it} - \beta_L \cdot \gamma^p_{it} \quad [min] \tag{3}$$

Since different paths between an origin and a destination may result in different fares, we further monetarize the utility, before we add the fare attribute. The monetarization is done by multiplying the utility by the Value-Of-Time (VOT). The VOT is the willingness-to-pay for travel time savings. The VOT of commuters in Israel (for instance) as of the year 2012 is 21.12 New Israeli Shekel (NIS)/hour (the updated value was given to us by the author of Shiftan et al. (2008)). It basically means that the Israeli travelers are willing to pay in average 21.12 NIS to save one hour of travel time. Converted into the minutes, we get 0.352 NIS/min. By using the VOT, we obtain the generalized cost $\mathcal{C}$ in monetary units and we can simply add the fare f^p associated to the path p into its formula:

$$\mathcal{C}^p_{it} = VOT \cdot \mathcal{T}^p_{it} + f^p \quad [monetary] \tag{4}$$

Since in the utility theory, the travelers are maximizing their utility and all of the attributes in the generalized cost are making the alternatives less attractive, we need to multiply the Equation 4 by minus one. Indeed the higher the generalized cost is, the lower the value of the utility is. This concept is also known as the passenger satisfaction $\mathcal{S}$:

$$\mathcal{S}^p_{it} = (-1) \cdot \mathcal{C}^p_{it} \quad [monetary] \tag{5}$$

The passenger satisfaction is now the observable part of the utility function ($\mathcal{V} = \mathcal{S}$). We now assume that the unobservable part of the utility function ϵ^p_{it}, is independently and identically distributed according to the Extreme Value distribution, the probability Pr^p_{it} of a passenger group (i, t) selecting a path p can be reformulated as:

$$Pr^p_{it}(w^p_{it}, \delta^p_{it}, \gamma^p_{it}, f^p | d_{\ell v}) = \frac{e^{\mu \mathcal{S}^p_{it}}}{\sum_{p' \in P_{it}} e^{\mu \mathcal{S}^{p'}_{it}}} \tag{6}$$

The probability Pr is a function of endogenous and exogenous variables. The endogenous ones being the waiting time, the schedule passenger delay and the fare, and the exogenous variable being the timetable (modeled through decisions on departure times $d_{\ell\nu}$). μ is a scale parameter. This parameter is related to the variance of ε in Equation 1 and it indirectly controls the elasticity of the passenger demand. A last step in calibrating the parameters of the choice probabilities is to find appropriate value of the scale parameter μ. It can be done by computing the elasticity of the total demand. The aggregate direct point elasticity related to the changes in the generalized cost is given as:

$$E = \frac{\sum_{it} \sum_{p \in P_{it}} \left(\frac{\partial \mathrm{Pr}_{it}^{p}}{\partial \mathcal{S}_{it}^{p}} \cdot \frac{\mathcal{S}_{it}^{p}}{\mathrm{Pr}_{it}^{p}} \cdot \mathrm{Pr}_{it}^{p} \cdot n_{it} \right)}{\sum_{it} \sum_{p' \in P_{it}} \mathrm{Pr}_{it}^{p'} \cdot n_{it}} \tag{7}$$

Upon solving of the derivatives, this transforms into:

$$E = \frac{\sum_{it} \sum_{p \in P_{it}} (1 - \mathrm{Pr}_{it}^{p}) \cdot \mu \cdot \mathrm{Pr}_{it}^{p} \cdot n_{it}}{\sum_{it} \sum_{p' \in P_{it}} \mathrm{Pr}_{it}^{p'} \cdot n_{it}} \tag{8}$$

According to Whelan and Johnson (2004), the value of the elasticity for the railway passenger demand with respect to the fare and the generalized cost is $E = -0.58$. Therefore, the value of μ can be calibrated by solving Equation 8 for any case study. In our application, we have calibrated the value of μ to 0.06.

Remark In our application, we use a simple operational demand model that is designed to illustrate the framework. More advanced models, such as path-size logit or nested logit models, could be used instead if available.

4 Problem Definition and Mathematical Formulation

In this section, we present the Elastic Passenger Centric Train Timetabling Problem (EPCTTP) which is an extension of the PCTTP model defined by Robenek et al. (2016). The aim of the EPCTTP is to design timetable maximizing the TOC's revenue accounting for the passenger forecasts. Whereas the aim of the original PCTTP was to design timetable maximizing the overall passenger satisfaction (without the fare) and thus securing the social optimum (the actual choices of the passengers were unknown). We compare the performance of the two models in Section 6.3. For the reader's convenience, we organize all the mathematical notation in Table 1.

A train is defined by its line $\ell \in L$, *i.e.* the set of stations that it serves. Each line ℓ has a train frequency and therefore, a specific train is given by a combination of indices (ℓ, ν), where $\nu \in V^{\ell}$ is the train's position within the set of available trains of the line ℓ. Both lines and frequencies are given by the Line Planning Problem (LPP). A timetable is defined as a set of arrival and departure times of each train (ℓ, ν). Since the travel times r_i^{pj} consisting in dwell times and running times in between stations are deterministic, it is sufficient to decide only on the departure time $d_{\ell\nu}$ of each train (ℓ, ν) from its origin station. The model can design three types of timetable: non-cyclic (by default), cyclic (imposing cyclicity constraints) and hybrid cyclic (imposing hybrid cyclicity constraints). The model does not take care of the conflicts among trains.

Based on the set of trains, the set of paths P_i for each OD pair i is given. Each passenger group is having its own set of available paths based on the original one: $P_{it} \subseteq P_i$. Any path can consist in several consecutive trains $j \in J^p$ (limited to 3 in this study), in order that they are being used, *i.e.* a passenger takes the first train and then transfers to its second train, *etc.* If a transfer is not possible (destination train departs before the arrival of the origin train), the path is removed from the set P_{it}. For a notation convenience, we add the additional index j to a train that expresses its position within the path, *i.e.* (ℓ_j, v_j). The set of all paths is pre-processed and can be created with an algorithm described in Appendix B. When making a transfer from one train to another, a minimum transfer time m is always secured. Any additional time spent in the transferring stations is counted as a waiting time (w_{it}^{pj}, where $j = 1$ is the waiting time for a transfer between the first and the second train and $j = 2$ is the waiting time for a transfer between the second and the third train). The set of paths contains the opt-out option which is equal to the shortest path, where the passengers do not experience any waiting time and arrive to their destinations exactly on their desired arrival time a_{it}.

The passengers are assigned to trains by using the demand forecasting model presented in the previous section. The probability of a passenger group (i, t) selecting a path p is given by Equation 6. The groups of passengers cannot be split. Since the passengers can enter or leave a train at any of its stopping stations, each train is further decomposed into segments $s \in S^\ell$. The occupation $o_{\ell v}^s$ of any train (ℓ, v) on any of its segments s cannot exceed its capacity C. The model assumes a homogenous fleet of trains, but it can be easily adapted to a heterogenous one. If a train is full on some of its segments, all the paths for any of the passengers consisting in the given segment are removed from the respective passengers' choice sets.

The revenue of a TOC is calculated as a function of the collected fares $f_{\ell v}^s$ for each train on each of its segments (multiplied by the train occupation). The fare itself is a decision variable and serves as an instrument to either flatten the demand, increase the revenue or to attract further passengers. Overall, the EPCTTP is formulated as:

$$\max \sum_{\ell \in L} \sum_{v \in V^\ell} \sum_{s \in S^\ell} o_{\ell v}^s \cdot f_{\ell v}^s, \tag{9}$$

$$o_{\ell v}^s = \sum_{i \in I} \sum_{t \in T_i} \sum_{p \in P_{it}} x_{\ell v}^{ps} \cdot n_{it} \cdot Pr_{it}^p(w_{it}^p, \delta_{it}^p, \gamma_{it}^p, f^p | d_{\ell v}), \qquad \forall \ell \in L, \forall v \in V^\ell, \forall s \in S^\ell, \tag{10}$$

$$o_{\ell v}^s \leq C, \qquad \forall \ell \in L, \forall v \in V^\ell, \forall s \in S^\ell, \tag{11}$$

$$P_{it} = \bigcup_{p=1}^{|P_i|} p \cdot \tau_{it}^p, \qquad \forall i \in I, \forall t \in T_i, \tag{12}$$

$$f^p = \sum_{j \in J^p} \sum_{s \in S^{\ell_j}} f_{\ell_j v_j}^s, \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{13}$$

$$d_{\ell v} \leq d_{\ell v+1} - 1, \qquad \forall \ell \in L, \forall v \in V^\ell : v < |V^\ell|, \tag{14}$$

$$d_{\ell v} \in \mathbb{N}, \qquad \forall \ell \in L, \forall v \in V^\ell, \tag{15}$$

$$f^s_{\ell\nu} \geq 0, \qquad \forall \ell \in L, \forall \nu \in V^\ell, \forall s \in S^\ell, \tag{16}$$
$$f^p \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{17}$$
$$o^s_{\ell\nu} \geq 0, \qquad \forall \ell \in L, \forall \nu \in V^\ell, \forall s \in S^\ell, \tag{18}$$
$$\tau^p_{it} \in (0,1), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{19}$$
$$Pr^p_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{20}$$

Ojective function (**9**) is maximizing the TOC's revenue that is given as the train occupation $o^s_{\ell\nu}$ of each train (ℓ, ν) on each of its segments multiplied by the fare cost $f^s_{\ell\nu}$ of train (ℓ, ν) on segment s. Constraints (**10**) use the logit model to estimate the train occupation. The parameter $x^{ps}_{\ell\nu}$ indicates whether path p is using train (ℓ, ν) on segment s (equals to 1) or not (equals to 0). Constraints (**11**) secure that the capacity of a train is not exceeded at any point of its travel. If a train is full at some of the segments, all the paths of any additional passenger using that train at that segment are removed from their choice sets inside of the logit model. Note, that a simple multiplication by the decision variable τ^p_{it} would create a proper behavior within the optimization model, but would lead to wrong predictions of the probabilities. Therefore, constraints (**12**) explicitly define those paths that are to be included in the set P_{it}. The decision itself, on which passenger should be restricted, is driven by the objective function. Constraints (**13**) calculate the fare to be payed for usage of each path p. Constraints (**14**) remove the symmetrical solutions, *i.e.* trains are ordered according to their departure times (ascending). The constraints (**15**)-(**20**) are the domain constraints.

Additional constraints, modeling some of the attributes used in the observable part of the utility function (Equation 5), are needed:

$$w^p_{it} = \sum_{j \in J^p : j < |J^p|} w^{pj}_{it}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{21}$$
$$\begin{aligned} w^{pj}_{it} = \left(d_{\ell_{j+1},\nu_{j+1}} + b^{pj+1}_i\right) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \\ - \left(d_{\ell_j \nu_j} + b^{pj}_i + r^{pj}_i + m\right), \qquad & \forall j \in J^p : j < |J^p|, \end{aligned} \tag{22}$$
$$\delta^p_{it} \geq a_{it} - \bar{a}^p_{it}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{23}$$
$$\gamma^p_{it} \geq \bar{a}^p_{it} - a_{it}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{24}$$
$$\bar{a}^p_{it} = \left(d_{\ell_{|J^p|}\nu_{|J^p|}} + b^{p|J^p|}_i + r^{p|J^p|}_i\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{25}$$
$$w^p_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{26}$$
$$w^{pj}_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \forall j \in J^p : j < |J^p|, \tag{27}$$
$$\delta^p_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it} \tag{28}$$
$$\gamma^p_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}, \tag{29}$$
$$\bar{a}^p_{it} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_{it}. \tag{30}$$

Constraints (**21**) add up the waiting times in all of the transfer stations in path p. Subsequently, constraints (**22**) define the value of the waiting time in the transferring stations. These constraints subtract the arrival time (departure time from its origin station plus the time to get to the boarding station of the passenger plus the time to get to the transfer station) of the origin train at the transferring station plus the necessary minimum transfer time m, from the departure time of the destination train at the transferring station (departure time from its origin station plus the time to get to the transfer station). Note, that the paths where a transfer is not possible will be removed by constraints **(12)**. Constraints **(23)** calculate the early schedule passenger delay and the constraints **(24)** calculate the late schedule passenger delay. The actual arrival time of a passenger group to its destination is given by constraints **(25)**. Constraints **(26)**-**(30)** are the domain constraints.

The above model would produce a non-cyclic timetable. However, by adding some additional constraints, the model can produce any type of a timetable. In this manuscript, we consider two additional timetable types: cyclic and hybrid cyclic.

Cyclic Timetable The cyclicity originates from the Periodic Event Scheduling Problem (PESP) first defined by Serafini and Ukovich (1989). The PESP secures that a set of events is scheduled in equally spaced intervals. In the railway context, the departure times of the trains on the same line are scheduled in cycles of size c. Typical value of c is 60 minutes leading to a clock-faced timetable. Such a measure leads to a better memorability of a timetable by the passengers. To obtain a cyclic timetable, the below constraints need to be included in the EPCTTP:

$$d_{\ell v} - d_{\ell v-1} = c \cdot z_{\ell v}, \qquad \forall \ell \in L, \forall v \in V^{\ell} : v > 1, \tag{31}$$

$$z_{\ell v} \in \mathbb{N} \backslash \{0\}, \qquad \forall \ell \in L, \forall v \in V^{\ell}. \tag{32}$$

Constraints **(31)** secure that the departure time of two consecutive trains v and $v-1$ of each line ℓ are spaced in the multiples $z_{\ell v}$ of cycles c. The domain constraints **(32)** verify that these multiples are non-negative natural numbers.

Hybrid Cyclic Timetable The cyclic timetable, when compared to other types of timetable, might be too restrictive and in fact it may lead to worse solutions in terms of the passenger satisfaction. Instead of using a fully cyclic timetable, Robenek et al. (to appear) propose a novel hybrid cyclic timetable combining the benefits of both cyclic and non-cyclic timetables. They show that the novel timetable can achieve similar performance as the non-cyclic one in terms of the passenger satisfaction.

Overall, the aim of the hybrid cyclic timetable is to decide on the ratio between the cyclic and the non-cyclic trains. It discretizes the planning horizon H into a set of unique cycles $k \in K : K = H/c$. The cyclic and non-cyclic trains have to follow the below rules for each cycle k:

- Any cycle k can contain no train
- Any cycle k can contain at maximum one cyclic train
- Any cycle k can contain one or several non-cyclic trains, only if there is a cyclic train present inside the kth cycle

To obtain a hybrid cyclic timetable, the below constraints need to be added to EPCTTP:

$$
\begin{aligned}
& q_{\ell v} \cdot q_{\ell v'} \cdot d_{\ell v} - q_{\ell v} \cdot q_{\ell v'} \cdot d_{\ell v'} \\
& \qquad = q_{\ell v} \cdot q_{\ell v'} \cdot \left(c \cdot z_{\ell v}^{v'} \right), && \forall \ell \in L, \forall v, v' \in V^{\ell} : v > 1, v \neq v', && (33) \\
& y_k^{\ell} \leq \sum_{v \in V^{\ell} : d_{\ell v}/c = k} q_{\ell v}, && \forall k \in K, \forall \ell \in L, && (34) \\
& (1 - q_{\ell v}) \cdot d_{\ell v} \leq y^{\ell}_{d_{\ell v}/c} \cdot H, && \forall \ell \in L, \forall v \in V^{\ell}, && (35) \\
& q_{\ell v} \in (0, 1), && \forall \ell \in L, \forall v \in V^{\ell}, && (36) \\
& z_{\ell v}^{v'} \in \mathbb{N} \backslash \{0\}, && \forall \ell \in L, \forall v, v' \in V^{\ell} : v > 1, v \neq v', && (37) \\
& y_k^{\ell} \in (0, 1), && \forall k \in K, \forall \ell \in L. && (38)
\end{aligned}
$$

In the above formulation, a train is a cyclic one if the decision variable $q_{\ell v} = 1$, otherwise it is a non-cyclic train. Similarly as for the cyclic timetable, constraints (33) secure the cyclic pattern only among the cyclic trains. Constraints (34) propagate the information, if there is a cyclic train present in the kth cycle of the line ℓ, onto a binary decision variable y_k^{ℓ}. This variable equals to one, only if there is a cyclic train present in kth cycle. Lastly, constraints (35) allow a non-cyclic train to exist, only if there is a cyclic train present in its scheduled cycle k. Constraints (36)-(38) are the domain constraints.

Name	**Description**	**Units**	**Type**
L	set of operated train lines given by the LPP	–	set
V^{ℓ}	set of available trains for the line ℓ (frequency)	–	set
(ℓ, v)	a train v serving the line ℓ	–	index
I	set of origin-destination pairs	–	set
T_i	set of preferred arrival times for OD pair i	–	set
(i, t)	a passenger group traveling between OD pair i with a preferred arrival time t	–	index
P_i	set of possible paths between OD pair i	–	set
P_{it}	set of available paths for a group (i, t)	–	set
J^p	set of trains in the path p	–	set
K	set of cycles within the planning horizon	–	set
S	set of segments in the network	–	set
S^{ℓ}	ordered set of segments crossed by the line ℓ to get from its origin station to its destination station	–	set
H	duration of the planning horizon	min	parameter
n_{it}	number of passengers wishing to travel between OD pair i at time t	–	parameter
a_{it}	desired arrival time of a passenger group (i, t) to its destination	min	parameter
m	minimum transfer time from one train to another	min	parameter

r_i^{pj}	travel time for OD pair i on path p using train j	min	parameter
b_i^{pj}	time to get from the origin station of the jth train in path p to a boarding station of the passengers traveling between OD pair i	min	parameter
$x_{\ell\nu}^{ps}$	1 - if the path p uses train (ℓ, ν) on the segment s, 0 - otherwise	binary	parameter
C	maximum capacity of any train on any of its segment	–	parameter
$\|set\|$	size of the set	–	parameter
$(\|J^p\| - 1)$	is the number of transfers in the path p	–	parameter
c	size of the cycle	min	parameter
$\bar{a}_{it}^p$	actual arrival time of passenger group (i, t) to its destination using path p	min	decision
$o_{\ell\nu}^s$	occupation of a train (ℓ, ν) on a segment s	–	decision
$f_{\ell\nu}^s$	ticket fare of traveling on segment s with a train (ℓ, ν)	monetary	decision
f^p	ticket fare of using path p	monetary	decision
$Pr_{it}^p(w, \delta, \gamma, f\|d)$	probability of the passenger group (i, t) to select the path p as a linear function of w, δ, γ on the current timetable d	–	decision
$d_{\ell\nu}$	the departure time of a train ν on the line ℓ (from its first station)	min	decision
w_{it}^p	waiting time of a passenger group (i, t) using path p	min	decision
w_{it}^{pj}	waiting time of a passenger group (i, t) using path p while transferring from a jth train to a $j + 1$th train	min	decision
δ_{it}^p	the schedule passenger delay of being early in path p of a passenger group (i, t)	min	decision
γ_{it}^p	the schedule passenger delay of being late in path p of a passenger group (i, t)	min	decision
$z_{\ell\nu}$	dummy variable to help modeling the cyclicity	$\mathbb{N}\backslash\{0\}$	decision
$q_{\ell\nu}$	1 - if a train (ℓ, ν) has a cyclic departure time, 0 - otherwise	binary	decision
y_k^ℓ	1 - if there is a cyclic train scheduled in the cycle k on the line ℓ, 0 - otherwise	binary	decision
$z_{\ell\nu}^{\nu'}$	dummy variable to help modeling the cyclicity, its value expresses the number of cycles between the departure time of (ℓ, ν) and (ℓ, ν')	$\mathbb{N}\backslash\{0\}$	decision

Table 1: Mathematical notations

5 Solution Algorithm

Since the EPCTTP formulation is non-linear and that we want to solve the problem for large instances, we propose to use a heuristic. Given that the original formulation of the PCTTP has been solved successfully with a Simulated Annealing (SA) heuristic by Robenek et al. (to appear), we use the same algorithm with some changes to reflect the specific features of the EPCTTP formulation. The changes are related to the value function (revenue maximization based on probabilities instead of deterministic passenger satisfaction maximization), the passenger assignment (user optimum to reflect better passenger behavior instead of system optimum) and new neighborhood moves (for the pricing of the tickets). Since the heuristic is well defined and has been calibrated and validated (by Robenek et al. (to appear)), we provide only its general description and focus more on the specifics of our current application.

Algorithm 1: Simulated Annealing

Data: d_c, N, ρ, T_c, T_f
Result: Best timetable d^*
begin
  Initialize
  repeat
    reset operators
    for $n \in N$ **do**
      select a neighborhood move
      impose this move on d_c and obtain d_c'
      perform passenger assignment on d_c'
      apply acceptance criterion on d_c'
      update weights ρ
    cooling of the T_c
  until $T_c \leq T_f$

The Algorithm 1 shows the general pseudo-code of the heuristic. The SA heuristic was first defined by Kirkpatrick et al. (1983). The algorithm takes as an input an initial timetable d_0 that consists in the departure times of each train. Since we solve this heuristic for several types of timetables (cyclic, hybrid cyclic and non-cyclic), an initial feasible solution for any type of these timetables is any cyclic timetable. In our case study, we use the currently operated (cyclic) timetable of Israeli Railways. The heuristic starts off with an initial temperature T_0 that has been previously calibrated to be a function of the value of the initial solution $f(d_0)$ as follows: $T_0 = 10^{-5} \cdot f(d_0)$. This temperature is gradually cooled down up until the final temperature T_f is reached. The cooling scheme has been calibrated to be 5% and a standard value of the final temperature of 0 has been previously used.

For each temperature, the weights ρ of the neighborhood operators are reset to one and the algorithm performs $N = 1000$ iterations (previous calibration). For each iteration n, a neighborhood move is selected using a roulette wheel mechanism based on the current weights ρ. The overview of the timetable specific moves can be found in Table 2. The columns represent the moves per timetable type and the rows represent a stage of a move. Only the cells with a grey background constitute a move. The last column represents the underlying distributions from which to draw randomly, in order to obtain the respective attribute's value.

A move is imposed on the current solution d_c in 3 steps: selection, modification and application.

		N.	Cyclic		Hybrid Cyclic			Distribution
Select	(ℓ, v)	■		■	■		■	$\mathcal{U}(1, \lvert L \rvert), \mathcal{U}(1, \lvert V^\ell \rvert)$
	ℓ		■			■		$\mathcal{U}(1, \lvert L \rvert)$
	$q_{\ell v} = 0$				■			
	$q_{\ell v} = 1$						■	
Modify	$d_{\ell v}$	■			■			$\mathcal{U}(0, H-1)$
	$d_{\ell v} \bmod c$		■			■		$\mathcal{U}(0, c-1)$
	k			■			■	$\mathcal{U}(0, H/c-1)$
Apply	$\forall v \in V^\ell : q_{\ell v} = 1$					■		
	$y^\ell_{k/c} = 0$			■			■	
	$y^\ell_{d_{\ell v}/c} = 1$				■			

Table 2: *Overview of the neighborhood moves by the type of a timetable*

The selection step serves to identify that part of the current solution d_c that is to be modified. The modification step serves to identify new value(s) for the selected part of the solution. The application step effectuates the change. The detailed description of how the neighborhood moves function, can be found in Appendix B.

Apart from the timetable specific neighborhood moves, we introduce a new pricing scheme move. This move selects any train randomly (according to the distributions in the first part of Table 2) and subsequently, it selects a random segment s of this train from a uniform distribution $\mathcal{U}(1, |S^\ell|)$. In the modification stage, it first decides if the price has to be increased or decreased (with a uniform probability of 50:50%) and subsequently, by how much it will be changed ($\mathcal{U}(1, 5)$ in monetary units). As this move is valid for any train, it is not further conditioned in the application stage. At the beginning of the algorithm, the prices are set to the values used by Israeli Railways.

Once a new solution x'_c is obtained (by applying a selected move), its revenue is calculated using the passenger assignment in Algorithm 2. In order to model the passenger behavior as realistically as possible, the assignment is using a First Come First Serve (FCFS) policy. At the beginning, the probabilities of all paths of all passengers are predicted. The passengers are then assigned to the trains in their paths with their probabilities in a non-capacitated fashion. Afterwards, the algorithm enters a while loop that serves to resolve any capacity issues. It iterates through all overflowing trains (*i.e.* those exceeding their capacity) that have been sorted according to their descending occupation. If a train exceeds its capacity on any of its segments, the passengers that have entered this train on this specific segment are sorted ascending according to the overall revenue that they bring to the company (the FCFS policy). Even though the passengers do not behave this way (they would sort themselves randomly), this policy is in line with the objective of revenue maximization. Afterwards, the algorithm removes these passengers one by one until the occupation of this train on this segment does not exceed its capacity. For each of the removed passengers, the path that they were using to travel on the segment of the concerned train is removed from their choice set and the probabilities of their remaining paths are recalculated. Once all the conflicts (of this iteration of the while loop) are

Algorithm 2: Passenger Assignment

Data: C, I, L, n_{it}, P_{it}, S^{ℓ}, T_i, V^{ℓ}

Result: revenue

```
begin
    for i ∈ I do
        for t ∈ T_i do
            for p ∈ P_it do
                calculate Pr_it^p(w_it^p, δ_it^p, γ_it^p, f^p | d_c')
                assign n_it · Pr_it^p passengers to the trains in the path p
    run ← true
    while run do
        run ← false
        sort overflowing trains (ℓ, v) according to their occupation descending
        for ℓ ∈ L do
            for v ∈ V^ℓ do
                for s ∈ S^ℓ do
                    if o_ℓv^s > C then
                        sort passengers entering the train on the segment s by revenue ascending
                        for these passengers do
                            remove (i, t) from the current path
                            update its probabilities
                            run ← true
                            if o_ℓv^s ≤ C then
                                break
        if run then
            re-assign passengers
    calculate revenue
```

resolved, the algorithm again re-assigns all the passengers to the trains in a non-capacitated fashion from scratch (the new probabilities of the removed passengers affect other passengers and trains). The algorithm terminates when there are no capacity issues found in a one single iteration of the while loop. Lastly, it provides as a result the overall revenue calculated according to Equation 9.

Once the value of the new solution $d_{c'}$ is known, the SA heuristic applies its acceptance criteria and updates the weight ρ of the used move j:

- If $f(d_{c'}) \geq f(d^*) \rightarrow \rho_j = \rho_j + 3$
- If $f(d_{c'}) \geq f(d_c) \rightarrow \rho_j = \rho_j + 2$
- If $f(d_{c'}) < f(d_c)$ and the new solution is accepted with a probability $r < exp(-(d_c - d_{c'})/T_c)$, where r is drawn from $\mathcal{U}(0, 1)$, then $\rho_j = \rho_j + 1$

The heuristic terminates, when the final temperature T_f is reached.

6 Case Study

As our case study, we have selected the network of Israeli Railways (in Figure 2). The aim is to evaluate the effect of different timetables on the revenue and what is the potential increase in the revenue, when the timetable design is integrated with ticket pricing. The exact procedures, assumptions and information about the data can be found in Appendix B.

We consider two instances in our study: the *2008* situation and the *2014* situation. The 2008 situation is built from the ticket selling machines' data of an average working day (from 6 a.m. to 1 a.m.) in 2008 in Israel. This data was kindly provided to us by Mor Kaspi and Tal Raviv, who have used it in their study Kaspi and Raviv (2013). In 2008, there are 126 036 passengers. The 2014 scenarios assume that the structure of the demand is the same as in 2008, with a uniform inflation of 1.6, resulting in a total number of 193 886 passengers (based on the newspaper article in Globes (2015)).

We consider the network layout of 2008: there were 47 stations and the red line in Figure 2 was operated only between *Hod HaSharon* and *Tel Aviv - HaHagana*. We do not consider the night line (blue with black bordering) as it runs mainly in the period for which we do not have the demand data (*i.e.* between 1 a.m. and 6 a.m.). Even though there are only 18 unidirectional lines visualized in Figure 2, in reality there are 34 unidirectional lines in the timetable (some lines are operated with higher frequency of 2 or more cyclic times, *e.g.* every xx:15 and xx:45). Since some of the trains follow different stopping patterns within a line, we have taken a union of stopping stations for each line (but same colored lines with different cyclic times may operate different stopping patterns). Since the OD matrix is given for an average working day, we have removed the trains that operate only during the holidays.

The timetable operated in Israel is cyclic with a cycle of 60 minutes. We use the timetable of 2013/14[1] (it was the latest published timetable at the time of the implementation) where 6 out of 388

[1]Unlike Europe, the timetable change in Israel happens during the summer period, *i.e.* the naming 13/14.

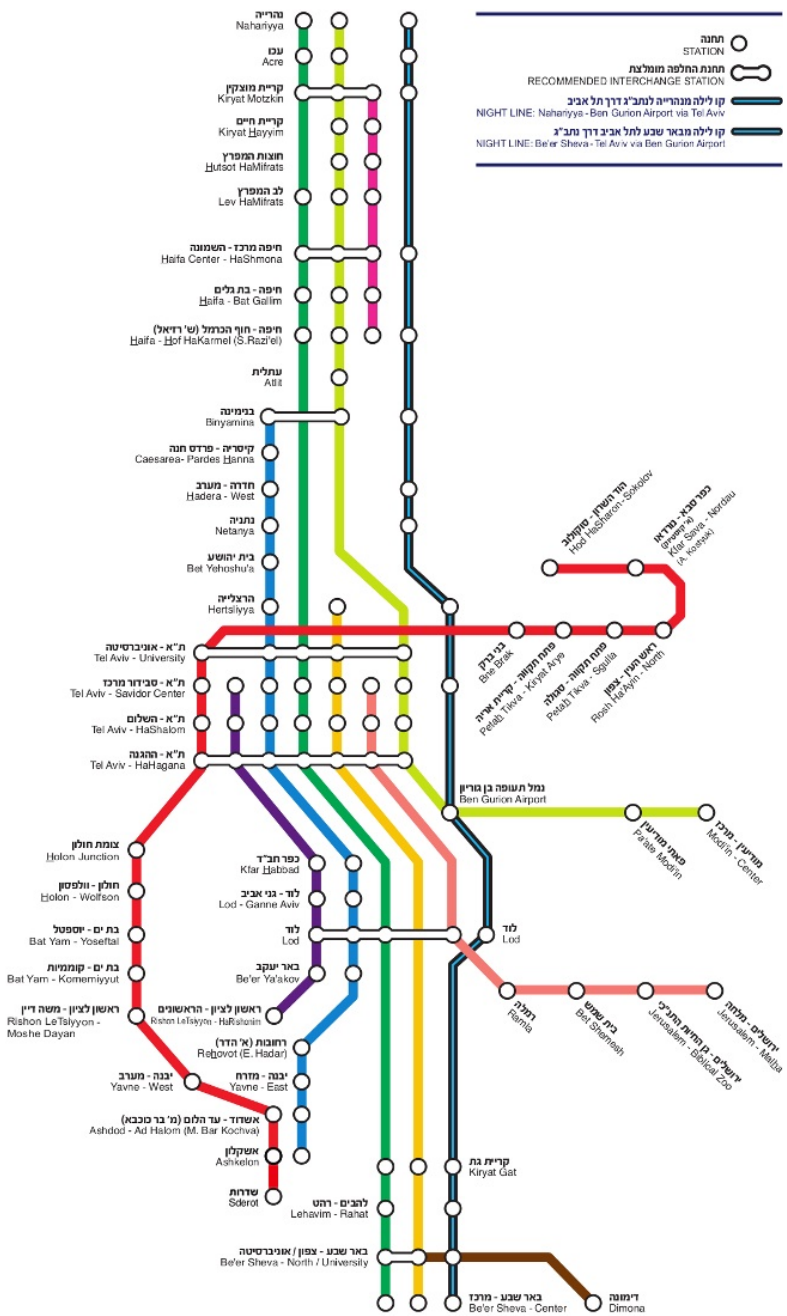


Figure 2: *Network of Israeli Railways (www.rail.co.il)*

trains have non-cyclic departure times. The planning horizon H of this case study is one day. Even though the demand is between 6 a.m. and 1 a.m., we allow the PCTTP model to schedule trains during any time of the day. We solve the PCTTP using the SA heuristic for the following timetables:

- **IR 13/14 –** the departure times are fixed to the ones of IR timetable of 13/14. Since the departure times are fixed, it serves as the benchmark for comparison of the performance of the other timetables.

- **Non-Cyclic –** no specific rule on the departure times is enforced.
- **Cyclic –** the departure times have to be cyclic according to constraints **(31) - (32)**.
- **Hybrid Cyclic –** the departure times have to comply with constraints **(33) - (38)**.

The revenues of each type of timetable are given in NIS. At first, the problem is solved without pricing and subsequently, it is solved with pricing. Each time, the heuristic is given as an initial solution the previously found timetables. All of the tested instances have been run in Java on a Unix server with up to 24 cores of 3.33 GHz and 62 GiB RAM.

6.1 Results without pricing

The results of the EPCTTP model without pricing can be found in Tables 3 and 4. Therefore, the prices in the model are fixed to those used by Israeli Railways (obtained from IR's website *www.rail.co.il*). The columns represent the different types of timetables (as described above) and the rows show the values of various attributes. The concerned attributes are: the revenue, the revenue gain as compared to the initial IR 13/14 timetable, the market share (percentage of passengers transported by IR), the number of passengers transported by IR, the average and the median train occupation and the solution time of the heuristic.

	IR 13/14	cyclic	hybrid cyclic	non-cyclic
revenue [NIS]	4 865 777	5 083 828	5 299 618	5 269 661
revenue gain [NIS]	–	+218 051	+433 841	+ 403 884
market share [%]	73.61	75.47	77.96	–
# transported passengers	92 779	95 127	98 267	–
avg. train occupation	116	121	126	–
median train occupation	84	90	98	–
solution time [sec]	7	174 897	164 828	167 939

Table 3: *Computational results of the various timetables for 2008 situation without pricing*

Both tables show that taking into account passenger behavior leads to a better performing timetables (in all of the concerned attributes). The EPCTTP based timetables were able to attract back some passengers from the competition and thus increase the IR's market share. The average revenue increase is 5% and 10% for the cyclic and hybrid cyclic timetable respectively.

We used the hybrid cyclic timetable as the initial solution for the non-cyclic one in the 2008 situation. However, it has failed to find a better solution than the hybrid cyclic one. In the 2014 situation, it was able to improve the solution only marginally. The marginal differences are caused by the randomness of the heuristic approach. The similarity, in the solutions of the two types of timetables, shows that the concept of the hybrid cyclic timetable can diminish any impact of the

	IR 13/14	cyclic	hybrid cyclic	non-cyclic
revenue [NIS]	7 150 254	7 490 054	7 896 806	7 905 615
revenue gain [NIS]	–	+339 800	+746 552	+755 361
market share [%]	70.63	72.78	75.56	75.59
# transported passengers	136 981	141 143	146 535	146 606
avg. train occupation	172	180	189	189
median train occupation	130	141	159	157
solution time [sec]	11	287 531	295 501	298 114

Table 4: *Computational results of the various timetables for 2014 situation without pricing*

cyclicity constraints on the performance of the timetable (the impact being the differences between a cyclic and a non-cyclic solution).

The average ratio between the cyclic and the non-cyclic trains, in the hybrid cycle timetable, is around 50:50%. The solution time of the heuristics was on average 2 (for 2008) and 3 (for 2014) days.

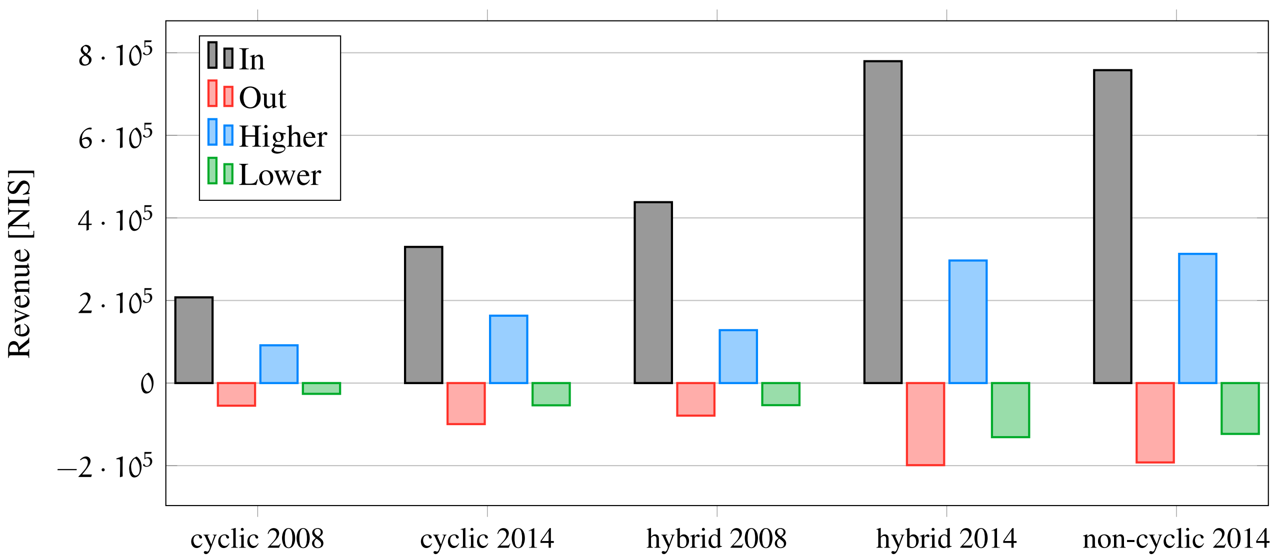


Figure 3: *Difference in composition of passengers for the various timetables*

In order to better understand, what is the cause of the additional revenue, we plot the difference in composition of passengers per timetable as compared to the IR 13/14 timetable in Figure 3. This Figure shows the revenues generated by 4 different groups of passengers: *in* – the ones who were not using IR in the original timetable, *out* – the ones who were using IR in the original timetable and left, *higher* – the passengers who stay and pay more, *lower* – the passengers who stay and pay less. The sum of these groups per timetable provides the value of revenue gain. It can be seen that the main source of the additional revenue across timetables is the new coming passengers. However, we can see that in the hybrid cyclic and non-cyclic timetables for the situation in 2014, the strategy also includes preference to the more paying passengers.

6.2 Results with pricing

The results of the EPCTTP model with pricing can be found in Table 5. The prices are now decision variables with their initial values set to those of IR. The revenue gain is now the additional revenue as compared to the solution of the same timetable without pricing. The overall revenue shows the additional profit as compared to the original IR 13/14 timetable.

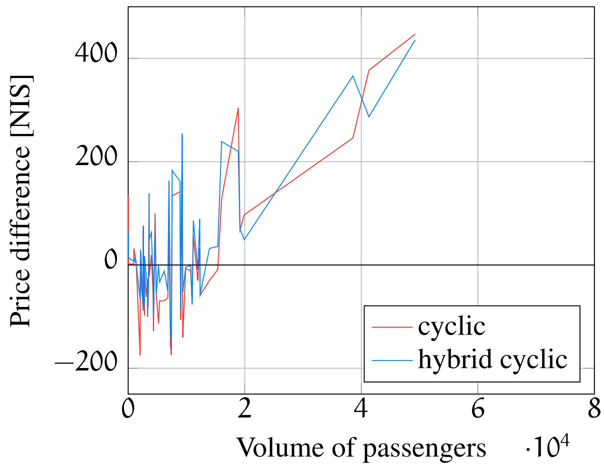
	cyclic		hybrid cyclic		non-cyclic	
	2008	2014	2008	2014	2008	2014
revenue [NIS]	5 256 959	7 767 140	5 461 559	8 229 227	–	8 259 557
revenue gain [NIS]	+173 132	+277 087	+161 942	+332 421	–	+353 942
market share [%]	76.31	74.05	76.87	75.99	–	76.32
# transported passengers	93 848	139 451	96 886	147 373	–	148 017
avg. train occupation	127	188	128	194	–	196
median train occupation	95	148	99	156	–	157
solution time [sec]	97 105	185 560	89 044	197 695	–	182 810
overall revenue gain [NIS]	+391 183	+616 887	+595 783	+1 078 973	–	+1 109 303

Table 5: *Computational results of the various timetables with pricing*

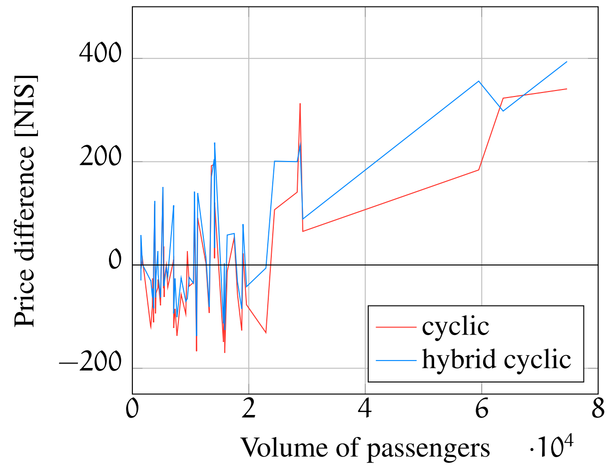
The pricing was able to further improve the revenue of the timetables. All timetables increased the ridership with exception of the hybrid cyclic one for the 2008 situation. However, it has still managed to achieve a higher profit. This is due to the fact, that the passengers who left were generating lesser profit than the ones who came in (Figure 4(e)). The additional increase in the revenues by pricing is up to 5%. We used the hybrid cyclic timetable as the initial solution for the non-cyclic one in the 2008 situation. However, it has failed to find a better solution than the hybrid cyclic one.

In order to analyze the pricing scheme, we plot various graphs in Figure 4. At first, we look at the pricing variations by location – Figures 4(a) and 4(b). Both plots show the overall increase/decrease in price per segment of the network as compared to the original prices set by Israeli Railways. The x-axis represents the segments by volumes of passengers traveling on them. For low demand segments the pricing can go either way, whereas for the high demand segments the prices go strictly up. The boundary between the two pricing strategies is around 20 000 passengers per segment.

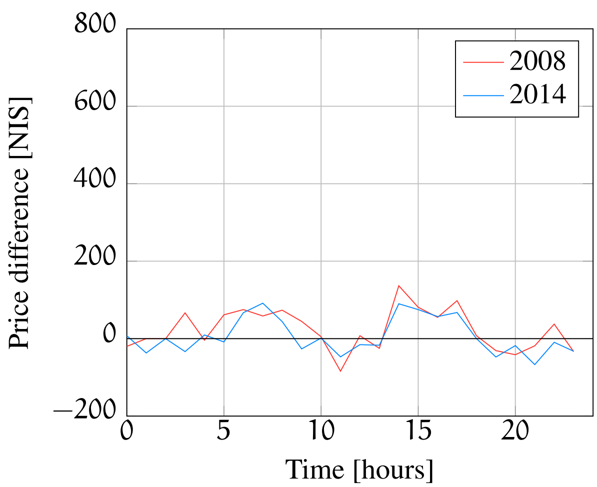
Subsequently, we look at the price variations based on the time of the day – Figures 4(c) and 4(d). Both plots show the overall increase/decrease in price per hour in the network as compared to the original prices set by Israeli Railways. The figures exhibit clear patterns on how to adjust the pricing: increase the price for high demand density periods and decrease the price for low demand density periods. Since the morning peak is more dense, its prices are higher than of the evening peak (the evening demand is more spread). The price decrease, as compared to the increase in peak hours, is lower. This might be due to the fact that during the off-peak hours also the supply is more scarce, *i.e.* the prices can be higher. The cyclic timetable is having less extreme changes than the hybrid cyclic one. This is due to the fact that the cyclicity makes the trains more spread. Therefore, the results in Figure 4(c) are somewhat more aggregate (the hours contain a larger mix of trains with increased and



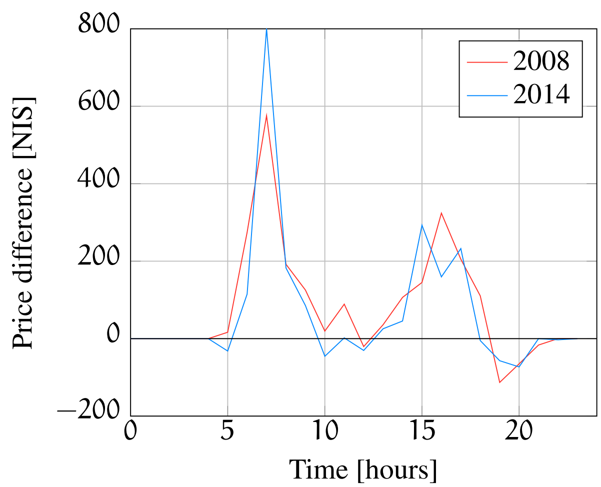
(a) *By location – 2008*



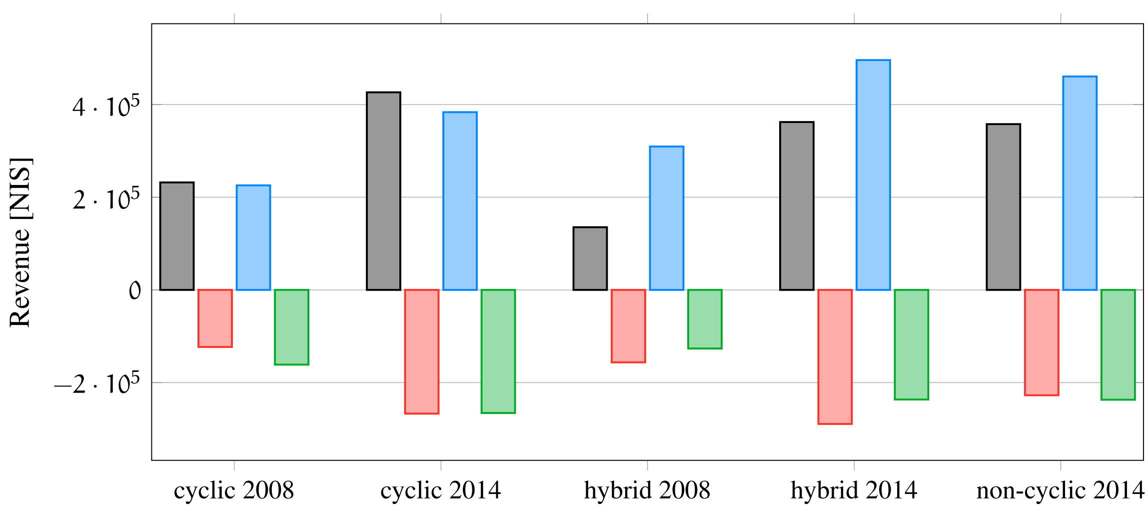
(b) *By location – 2014*



(c) *By time – Cyclic*



(d) *By time – Hybrid cyclic*



(e) *Difference in composition of passengers for the various timetables (same legend as in Figure 3)*

Figure 4: *Pricing analysis*

decreased prices).

Finally, to understand the changes in the served passengers, we plot the differences in the composition of the passengers for various timetables as compared to their previous versions without pricing in Figure 4(e). The legend is the same as in Figure 3. It can be seen, that as compared to the timetabling without pricing, the higher paying passengers are now having a larger contribution to the revenue gain. In the hybrid and non-cyclic timetables, the share is even larger than of the new coming passengers. The revenue loss of leaving and less paying passengers is now larger. These passengers are most likely left out or less paying, in order to free the space in the trains desired by the higher paying passengers. This behavior is in line with the objective of maximizing revenue, but might not be desired by public owned operators. Therefore, we further investigate what is the impact of this approach on the passengers' satisfaction, in the next Section.

6.3 Maximize satisfaction or maximize revenue?

Maximizing revenues is a natural objective in a competitive environment. However, in many countries, particularly in Europe, there is little or no competition. Moreover, many railway operators are subsidized by public money, and must focus on public services as well. Therefore, the revenue maximization might be considered controversial. In order to analyze the situation, two approaches are evaluated against each other: one timetable designed to maximize the satisfaction of the passengers (Robenek et al. (to appear)), and one timetable designed to maximize revenues, as described in this paper. In order to do so, the timetables (*i.e.* the decision variables $d_{\ell v}$) created by Robenek et al. (to appear) (through the passenger satisfaction maximization) are given as a fixed input to the probabilistic model of EPCTTP. The comparison consists in the resulting revenue, satisfaction and number of transported passengers per timetable in Table 6.

In this Table, *(s)* denotes satisfaction maximization approach, *(r)* denotes the revenue maximization approach and *(p)* denotes the presence of prices generated by the pricing problem. If *(p)* is missing, the original prices given by IR are used. Note that in the 2008 situation, the hybrid cyclic timetable was not designed using the satisfaction approach (due to the insignificant differences in satisfaction among the cyclic and non-cyclic timetables in 2008 (Robenek et al. (to appear))). The non-cyclic timetable in 2008 with pricing is not included as it did not improve the revenue and its fares were never reported.

The main observation from the results is that even though the objectives were different, the resulting passenger satisfaction is more or less similar. Indeed, when maximizing the revenue, the passengers need to be satisfied, in order to take the train. The main difference, between the two approaches, is the generated revenue. This is again in line with the objectives: the objective of revenue maximization takes care of both revenue and satisfaction, whereas the other approach only cares about the satisfaction. Notable to mention, that the presence of pricing did not distort the average values of the satisfaction. Its values are similar with and without pricing. The cause of it is most likely the reduced fare for some of the passengers.

It could be objected that the satisfaction is now similar across conceptually different timetables and that it is caused by the opt-out option of the competitor. However, given that the number of passengers is more or less the same between the two concepts of the design, it proves that the somehow

	revenue [NIS]	satisfaction [NIS]	# transported passengers
cyclic 2008 (s)	4 950 614	-10 264 108	93 580
cyclic 2008 (r)	5 083 827	-10 314 244	95 127
hybrid 2008 (r)	5 299 618	-10 396 741	98 267
non-cyclic 2008 (s)	5 043 234	-10 281 466	95 438
cyclic 2014 (s)	7 287 127	-15 630 858	138 778
cyclic 2014 (r)	7 490 054	-15 680 300	141 143
hybrid 2014 (s)	7 526 215	-15 647 618	142 228
hybrid 2014 (r)	7 896 806	-15 920 860	146 535
non-cyclic 2014 (s)	7 452 564	-15 627 362	141 918
non-cyclic 2014 (r)	7 905 615	-15 952 986	146 606
cyclic 2008 (p)(s)	5 142 573	-10 311 044	95 396
cyclic 2008 (p)(r)	5 256 959	-10 399 474	96 196
hybrid 2008 (p)(r)	5 461 560	-10 564 349	96 886
cyclic 2014 (p)(s)	7 596 212	-15 686 365	143 135
cyclic 2014 (p)(r)	7 767 140	-15 955 037	143 612
hybrid 2014 (p)(s)	7 821 338	-15 855 025	143 345
hybrid 2014 (p)(r)	8 229 227	-16 174 935	147 373
non-cyclic 2014 (p)(s)	7 749 755	-15 803 502	143 227
non-cyclic 2014 (p)(r)	8 259 557	-16 166 223	148 017

Table 6: *Differences between passenger satisfaction (s) and revenue (r) oriented timetable designs*

"capitalist" concept of revenue maximization is not anti-passenger and that it is suitable for the public operators as well. This should encourage operators to consider the new approach proposed in this research to design their timetable.

7 Conclusion

In this research, we further extend the passenger centric train timetabling approach with the concept of the competitive market by accounting for elasticity of the passenger demand. We include a competing operator into the market, so that the passengers are not captive in the system. This framework allows for a prediction of the realized demand, which subsequently allows to calculate the revenue of the operator. In the second phase, we integrate the proposed timetabling problem with the ticket pricing problem.

The results of our case study show that accounting for the behavioral dimension of the problem has a significant impact on the generated revenue. We show on our case study that the passenger oriented timetabling can be combined with the ticket pricing. The resulting pricing strategies suggest higher prices for large volumes of demand by location and time. The two together can further increase the revenue up to additional 15%. Given that we used the best possible competitor, in our case study, the gain is expected to be even larger, when this is not the case. Lastly, we provide empiricial evidence that the concept of revenue maximization is in line with passengers' objective of their satisfaction maximization. Therefore, this concept is suitable for public operators as well.

In this research, we have considered the passengers to be homogeneous. However, in the real world, this is not the case. Each person has a different behavior. This behavior is incorporated in the discrete choice model through the taste parameters beta. These parameters may vary across the population. We propose to treat this as a possible future extension. Subsequently, this would allow for an introduction of different class fares and to further improve the revenue of the operator.

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A Neighborhood Moves

As mentioned before, each move is decomposed into 3 parts: selection, modification and application. Two entities can be selected: a specific train (ℓ, v) or the whole line ℓ. The selection of a specific train might be conditioned by its type $(q_{\ell v})$. If no condition is specified, any train can be selected.

The aim of the modification is to replace the value of one of the 4 attributes with a new one. The only not yet defined attribute is $d_{\ell v} \bmod c$. This attribute represents the modulo time within any cycle. For instance, when the cycle c is equal to one hour and the departure time of a train is 5:45, then the modulo time is 45 minutes. This time is the same among all cyclic trains of the same line.

The application of the modification is always performed on the pre-selected entity. However, its application might be conditioned. Some applications are made only to the cyclic trains of the given line ($\forall q_{\ell\nu} = 1$). Other applications are conditioned that the new cycle k does not already contain a cyclic train ($y^{\ell}_{k/c} = 0$) or that the new departure time $d_{\ell\nu}$ is in a cycle that does already contain a cyclic train ($y^{\ell}_{d_{\ell\nu})/c} = 1$). If the newly generated values of attributes fail to fulfill the last two conditions (when they are required), the modification is repeated (until they do).

For illustration, consider the first move of the hybrid cyclic timetable: at first a train is selected by drawing the line ℓ from $\mathcal{U}(1, |L|)$ and subsequently drawing the ν from $\mathcal{U}(1, |V^{\ell}|)$. The drawing from the distribution(s) is repeated until the condition $q_{\ell\nu} = 0$ is satisfied. Hence, a non-cyclic train is selected. In the second phase, a new departure time $d_{\ell\nu}$ of this train is drawn from $\mathcal{U}(0, h-1)$. In the third phase, this new departure time is applied given that it is in the cycle where there is a cyclic train scheduled ($y^{\ell}_{d_{\ell\nu}/c} = 1$). Otherwise the modification phase is repeated until this condition is complied.

B Data Description

The data used in this study were obtained from the IR's website (*www.rail.co.il/EN*) and from other studies concerning the IR's network (Kaspi and Raviv (2013)). An algorithm in Java was coded, in order to find a set of all possible paths between every OD pair. The algorithm allows a maximum of 3 consecutive lines to get from an origin to a destination. The algorithm iterates through all OD pairs, where at first it considers the paths that consist of a single line and then the paths that would transfer from the currently selected line up to two other lines. The transfer from one line to another can be made only at one of the designated transfer points (there are 7 recommended interchange stations in the network of Israel – Figure 2). Note that the fact, that a transfer is actually possible depends on the operated timetable. Therefore, some paths might be eliminated later on by the EPCTTP model itself. When all the possible paths are generated, the algorithm removes the paths that a passenger would not consider (note that these rules are related to the network layout of Israel and might differ for other case studies):

- paths that consist of several lines including a direct line between the given OD pair, where both options travel on the same infrastructure (*i.e.* the passenger would rather stay on the direct line instead of transferring to another line).
- paths that consist of several lines, where two of them can reach the given destination. Changing one train to another, when both of them are going to the same destination would not make sense (the same does not happen for the origin).
- paths that take 25% longer generalized time (sum of in-vehicle-times and transfer penalties) than the shortest possible path.
- paths that consist of redundant transfers, *i.e.* transferring from one line to another line that covers the same stations.

The traveling times have been extracted from the IR's website along with the dwell times at stations that remain fixed (as of the timetable 2013/14). The minimum transfer time is set to 4 minutes as in Kaspi and Raviv (2013). The Value Of Time of commuters in Israel as of the year 2012 is

21.12 New Israeli Shekel (NIS)/hour (the updated value was given to us by the author of Shiftan et al. (2008)). The β parameters and their values are as follows: $\beta_W = -2.5$ (Wardman (2004)), $\beta_T = -10$ (de Keizer et al. (2012)), $\beta_E = -0.5$ and $\beta_L = -1$ (Small (1982)).

B.1 Passenger

The OD flows were kindly provided by Mor Kaspi and Tal Raviv, who have cleaned the ticket selling machines' data for the year 2008 and produced the flows of an average working day in Israel. They have used this data in their study Kaspi and Raviv (2013). The OD matrix consists of hourly passenger rates between 6 a.m. and 1 a.m. The flows were smoothed into minutes by using non-homogenous Poisson process, where the hourly flows per OD pair were used as the arrival rate variable. Since the schedule passenger delay is related to the destination, we have added the time that it takes to get from an origin to a destination using the shortest path (if the path consisted of transfers, we assumed the perfect connection, *i.e.* only the minimum transfer time without any additional waiting at the transfer station). In total there are 1 505 out of 2 162 OD pairs with 126 036 passengers.

B.2 Operator

As no information about the rolling stock fleet of IR is available, we have introduced the following assumptions:

- The fleet is homogenous
- A train unit has a passenger capacity of 250
- Each train consist of 2 train units

In order to verify that the assumed train capacity is reasonable, we have solved the un-capacitated PCTTP of the IR 13/14 timetable under the 2008 demand, where the average train occupation was 172 passengers per train per segment (pptps), minimum occupation was 0 pptps, maximum occupation 1188 pptps and median was 124 pptps. Thus the capacity of 250 passengers per train unit offers a good level of service.