A Multidimensional Decisions Modelling Framework for Built Space Supply

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Abstract

The spatial and temporal distribution of built space supply plays an important role in shaping urban form and thus the general travel pattern in an urban area. Within an integrated framework, we are interested in modeling the decisions of a builder in terms of when, where, what type, and how much built-space to build. We present a multidimensional discrete-continuous model formulation for the built space supply decisions that are based on expected profit maximization. The framework is applied to estimate a model for supply of new office space in the Greater Toronto Area (GTA) for the duration of 1986 to 2006. To our knowledge, this work is the first that models the where, when, how much, and what type of office space to supply in a single econometric framework at a fairly disaggregate spatial zoning system. The results indicate a risk taker behaviour on the builders' part, while market conditions and supply of resources (labour, construction cost etc.) are also found to be important factors in decision making.

1. Introduction

Understanding the factors that affect built space[1] location/relocation decisions plays an extremely important role in our greater understanding of travel behaviour in urban areas. Since

[1] *Built space* is a generic term, used throughout this paper, to represent various types of spaces in an urban area that have a physical structure and associated monetary value; can be identified as individual quasi-unique units (based on

activity location, especially residential and work related, strongly influences the spatial distribution of morning and afternoon peak-period travel, where households and firms choose to locate greatly influences short-term individual-level decisions such as mode of transportation, and long-term household-level decisions such as residential/work relocation. Conditions in the built space market affect households/firms' location and relocation decisions, and hence influence the general travel patterns in an urban area. An important dimension of decision making in the context of built space markets, is the supply decisions. Builders in the built space market respond to market conditions, availability of land and other resources (e.g. capital, construction material etc.), and regional economic conditions by adjusting the supply of different categories of built space. In case of brownfield redevelopment (De Sousa, 2002), builders transform an existing built space into a new space of the same category or some different category. In terms of the total inventory of built space in the urban area, due to these rebuild decisions by the builders, the stock of one category of built space may decrease and the stock of other's may increase. Brownfield development usually requires changes in the zoning bylaws that are related to the land parcel. In the outer limits of an urban area, land owners sell land to developers. In the context of the Greater Toronto Area (GTA), most of the time, developers will buy cheap agricultural land surrounding the city many years ahead of time and keep it until the local municipality designates it for development. Developers then help shape the final zoning of the land and develop it into parcels. Builders, based on zoning and the expected demand, build different categories of built space on these parcels. A developer and a builder can be the same or different agents. An analysis of the InfoCanada[2] dataset for businesses operating in the GTA for

their attributes and location); and provide opportunities for various activities. These spaces include: dwelling units, office spaces, retail spaces, industrial spaces, etc.

[2] InfoCanada is a marketing consultant firm that conduct surveys related to business firms in various major cities of the world, including Toronto.

year 2006 shows that there are less than 500 builders operating the GTA. Buzzelli and Harris (2003) suggest that the number of active developers is even less than the number of builders. Developers usually work with the same builders and in very few cases the developer of land also builds the space. This supports the argument that the built space supply market in the GTA is a well-connected oligopoly. The analysis of InfoCanada dataset on builders shows that in 2006 the sales volume of 13% of the builders was less than 1 million dollars, 70% of the builders had sales between 1 and 10 million, and 17% of them had sales more than 10 million dollars. The building industry is thus dominated numerically by small- to medium-sized builders, but at the same time there is a significant presence of heavy-weight players in the industry. Buzzelli and Harris (2003) similarly report that the building industry in Ontario has a high number of small- to medium-sized builders. The total volume of the sales by the building industry in 2006 was approximately 5 billion dollars, with small to medium builders contributing 800 million dollars of this total. The large-sized builders contributed 4.2 billion dollars, which is more than five times of what was contributed by the small- and medium-sized (83% numerically) builders. This shows that the large-sized builders play a dominating role in the building industry. Another interesting fact about the building industry is that the number of employees for about 95% of the builders is less than 25. This is because builders do not perform the construction job in-house. Instead, they heavily rely on contractors and sub-contractors to actually do the job for them and their employees are usually only managing the project. Buzzelli and Harris (2003) reported that this relation between the builders and contractors is spatially localized and long-term.

The building project has various identifiable stages (Somerville, 2001) (figure 1). In the first stage, a builder applies for a permit to construct a certain quantity of built space, seeks any required zoning changes, and acquires financial backing. Once approved, the builder may start the construction of the entire or some quantity of the built space it is permitted to build. The time to start the construction may vary, depending on market and regional economic conditions, but the latest time to start is dictated by the terms and conditions of the loans. The completion time of the projects may also vary depending on these conditions. The introduction of space within the market may vary both in terms of time and quantity. Moreover, the whole project construction process may vary for different categories of built space.

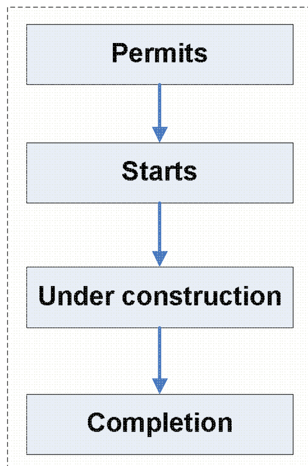

Figure 1
Various Stages of Construction

Supply of new built space is a very complex process in which there are markets (land, development, and building market) various stages, different types/levels of finished product (land, developed parcels, and built space), and various types of decision making agents (land owners, developers, and builder) (figure 2). One approach to deal with this complexity is the grid cell concept (Waddell *et al.*, 2008; Hunt *et al.*, 2007). The urban area is divided into fixed grid cells that act as evolving cellular automata. The grid cell is a very rough abstraction of the developer and builder agents that maintains its own inventory and decides on the built space supply decisions (land is the decision maker). It seems to be an over simplification of the process. We think that the different markets, finished products and agents should be identified separately with inter- and intra-type interaction. Moreover, the parcels evolve by merging and splitting (especially in the case of brownfield redevelopment). The concept of a fixed dimension grid cell cannot represent this evolution. Martinez and Roy (2004) within the equilibrium framework modelled the supply process as a chain of market processes in which landowners, developers, and builders interact. Their modelling approach is a better representation of the various markets, finished products, and agents involved in the built space supply. But the strong equilibrium assumption fails to capture the complex interactions occurring among the various agents within these markets.

Realizing the highly complex nature of the supply process that results from the behaviour and interactions of the involved agents, in this paper, we only focus on the decision making behaviour of one agent, the *builder*. In the context of new built space supply, a single builder is faced with various dimensions of decisions. It has to decide when, where, what type, and how much to build so that it can maximize its expected profit. In this context, we assume that the builder:

- Has access to financial and other resources required to build the amount of space it decides to build.
- Can acquire land parcels anywhere in the urban area.
- Can get a permit for the construction of the quantity it desires to build.
- Has a fixed time of construction.
- Introduces the entire quantity of built space at once to the built space market, at the end of the project.

- Is an expected profit maximizer, which it computes at the start of the project by speculating about the future revenues from sale, rent, and lease and various costs associated with the project.

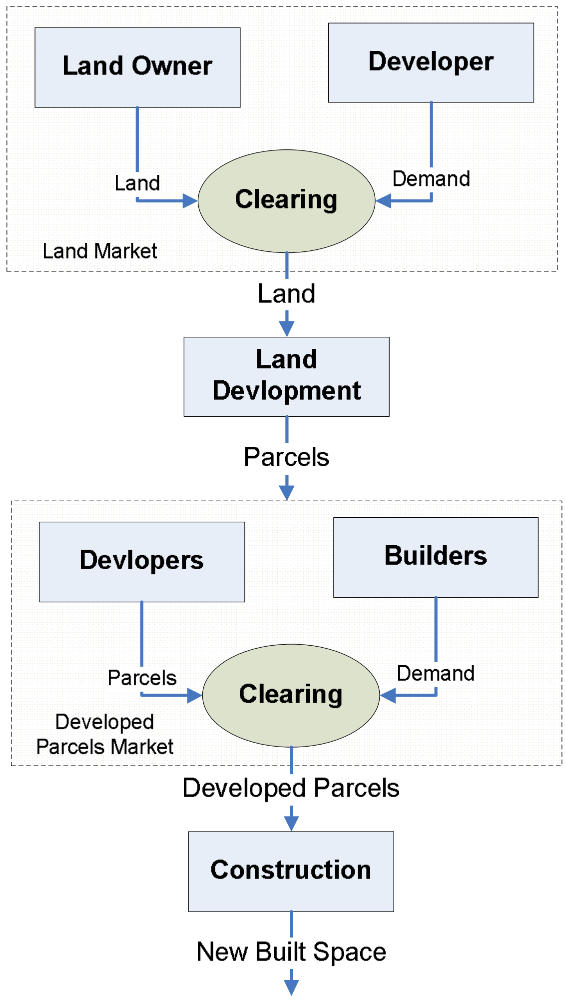

Figure 2
Various Markets and Agents involved in the Built Space Supply

Under these assumptions we propose a novel approach that explicitly ties the time, location, quality, and quantity decisions for new built space supply together into a single dynamic framework that is based on expected profit maximization. As an application, we then estimate a model for new office space development at a fairly disaggregate spatial resolution. We treat the problem of building new built space as a situation in which a builder as a decision maker is faced with the decision of selection of a choice bundle and the associated quantities, while optimizing his expected profits. By doing so, we were able to incorporate not only the relation between various decision dimensions, but also captured the behaviour of the decision maker (builder) and the influence of changing sub-market conditions and regional economy on the decision making. The rest of the paper is organized as follows: we first present the model formulation and then the application of the model to new office space supply model. In the end of this paper we present concluding discussion and future directions.

2. Model of New Built Space Supply

Here we present the theoretical formulation of the model for new built space supply that models the multidimensional decisions of when, where, what type, and how much to build, in a single consistent modelling framework. The decision makers are a set of building construction firms (builders) that are active in the urban area at certain time *t*. They are faced with the decision of choosing the quantity of different types of built space to be built, and the location where to build them. It is assumed that builders take these decisions so as to maximize their expected profits. Profit is determined by the difference of expected revenue and cost. There are various large scale demand models available in the choice modelling literature that model the choice of a discrete bundle of goods (e.g. types of activities in which to engage in) and an associated continuous quantity (e.g. how much time to allocate to each activity) (Bhat, 2005, 2008; Habib *et al.,* 2008; Habib and Miller, 2009, Kim *et al.* 2002). These models predominantly use the well-known random utility modelling (RUM) framework that assumes that the consumer is a utility maximizer. In terms of mathematical model formulation, the assumption of profit maximization by the producer (specifically, builders in the case at hand) is analogous to the utility maximization assumption in these large scale demand models. Moreover, profit from manufacturing a product is a more quantifiable concept than utility. Thus we can pose the problem of expected profit maximization in the same way as RUM does for the utility maximization of consumers faced with choice bundle selection and the associated quantities. This lets us use the same construct of optimization conditions (Kuhn-Tucker conditions) that in recent years has frequently been used in large scale demand models.

2.1 Theoretical Framework

The expected profit (Π) of a building construction firm, from *N* differentiated products that it can decide to build at certain decision point *t* can be represented by:

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ \left(f^r(X_i^r) - f^c(X_i^c)\right)^{\vartheta} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + f^z(z) \qquad [1]$$

Where:

$f^r(X_i^r)$ is a hedonic function that represents the expected unit revenue from selling product *i*

X_i^r is a vector of variables related to product attributes, location features, and built space market conditions that influence the revenue

$f^c(X_i^c)$ is a hedonic function that represents the expected unit cost in building product *i*

X_i^c is a vector of variables related to product attributes, location, state of regional economy, and conditions in various associated markets (labour, material *etc.*) that influence the cost

q_i is the quantity of product *i* that is decided to be built

The formulation here treats the share of profit from individual type of floor space *i* in the same manner as Bhat (2005) and Kim *et al.* (2002) treat the share of individual choices in their utility function for large scale demand systems. The translation parameter γ_i makes sure that there is a possibility of zero production of any given type of built space. Its value is greater or equal to zero ($\gamma_i \geq 0$), so as to make sure that the indifference curves touch the horizontal axis with a finite slope (Bhat, 2008). The parameter α_i is a scale parameter which adjusts the marginal profit with respect to the associated quantity of built space. ϑ here represents the risk behaviour of the builder and the structure of the space market in the region. In the simplest case ϑ could be a constant parameter, but in a more elaborate case it could be parameterized based on a combination of the builder's and the market's characteristics. The value of ϑ greater than one would mean that the builders inflate the expected revenue thus showing a more risk taker attitude, while a lower than one value would represent a risk avoiding attitude due to deflation of the expected revenue. Bhat (2008) performed a comprehensive analysis of the influence of different values of γ_i and α_i on the indifference curves for the utility of the goods consumed. Similar analysis is needed for the formulation suggested in equation [1] for the profit.

2.1.1 Concept of Hicksian/Composite Product

As the builder has other options of investment, besides the set of built space types that we are modelling, we introduce the concept of a Hicksian/composite product in our general formulation, $f^z(z)$. Here, z is in terms of built space units (square foot in the case office space). The value of which comes from the difference of maximum that can be built and the sum of all the quantities (q_*) of different types of built space that were built by the builder in the time step t. If the conditions are extremely favourable, the builder would like to build as much as possible. The only limiting factor will be the technological or zonal constraints, as builders can only build to a certain extent with the current technology in a given time step. On the other hand, if the conditions are not highly favourable, the builder will carefully select the supply levels to an extent so that the profit is maximized and loss is avoided by overbuilding. Profit from the

Hicksian product in equation [1] represents the expected loss that is avoided at a given interest rate at the decision time by not building the built space that could have been built under the technological/zoning constraint. For this, we use a separate profit generation function similar to the one used by von Haefen and Phaneuf (2004), and Habib and Miller (2009) for the composite activity. If we assume that the revenue and cost functions are linear in parameters and the modeller's inability to perfectly observe builder's expected profit is represented by the error term ε_i, then equation [1] can be rewritten as:

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1 - e^{\rho}} z^{(1 - e^{\rho})} \qquad [2]$$

The form of the profit function for composite product here guarantees a positive profit from a nonzero composite product (Habib and Miller, 2009).

2.2 Estimation Problem

Using equation [2], our optimization problem can be defined as:

Maximize

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1 - e^{\rho}} z^{(1 - e^{\rho})} \qquad [3a]$$

Subject to

$$\sum_{i=1}^{N} q_i + z = K_T \qquad [3b]$$

$$q_i \geq 0 \quad i = 1, 2, \ldots, N \qquad [3c]$$

$$z > 0 \qquad [3d]$$

K_T is the maximum possible space that can be built in the time interval under zoning and technological constraints.

The Lagrangian function for the problem in [3] becomes:

$$\mathcal{L} = \left[\sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1 - e^{\rho}} z^{(1 - e^{\rho})} \right]$$

$$-\lambda[\textstyle\sum_{i=1}^{N} q_i + z - K_T] \qquad [4]$$

λ=Lagrangian multiplier

The Khun-Tucker (KT) first order conditions for optimal allocations here will be:

$$\frac{\partial \Pi}{\partial q_i} - \lambda \leq 0 \qquad \text{for } i = 1,2,\ldots n \qquad [5a]$$

&

$$\frac{\partial \Pi}{\partial z} - \lambda \geq 0 \qquad [5b]$$

Condition in [5a] ensures that, for the selected levels of the product bundle, any further increase in the quantity of product *i* will have no further positive effect on the total profit. Whereas, [5b] ensures that the quantity of the composite product (not investing) is at the level where it has no negative effect on the profit. These conditions help in identifying the solution in the estimation process.

From [5a] and [5b]

$$\frac{\partial \Pi}{\partial q_i} \leq \frac{\partial \Pi}{\partial z} \qquad \text{for } i = 1,2,\ldots n$$

$$(\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\frac{q_i}{\gamma_i} + 1\right)^{(\alpha_i - 1)} \leq z^{-e^{\rho}} \qquad [6]$$

We can show that $\frac{\partial^2 \Pi}{\partial q_i \partial \varepsilon_i}$ is a $i \times i$ non-singular matrix and $\frac{\partial^2 \Pi}{\partial z \partial \varepsilon_i}$ is a zero valued vector. Thus using explicit function theorem (Jittorntrum, 1978), we can express the error term as:

$$\varepsilon_i \leq g_i(\beta_i^r, X_i^r, \beta_i^c, X_i^c, q_i, \vartheta, \gamma_i, \alpha_i, \rho)$$

$$\varepsilon_i \leq \frac{1}{\vartheta}\left[(1-\alpha_i) log\left(\frac{q_i}{\gamma_i} + 1\right) - \vartheta log(\beta_i^r X_i^r - \beta_i^c X_i^c) - e^{\rho} log\left(K_T - \textstyle\sum_{j=1}^{n} q_j\right)\right], \forall i \qquad [7]$$

2.3 Econometric Model Structure

If the joint probability density function, $f(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ of the error terms is known then the probability associated with the quantities of a certain bundle of product that is selected by the building firm for construction is given by:

$$P(Q) = \int_{-\infty}^{g_{m+1}} \dots \int_{-\infty}^{g_n} f(g_1, g_2, \dots g_m, \varepsilon_{m+1}, \dots, \varepsilon_n) |J| d\varepsilon_{m+1} \dots d\varepsilon_n \qquad [8]$$

Where

$Q = [q_1^*, q_2^*, \dots, q_m^*, 0{,}0 \dots 0]$ is the vector of quantities of each type selected by the builder to build.

$|J|$ is the determinant of $m \times m$ Jacobian matrix, whose individual elements are defined by:

$\partial\varepsilon_i / \partial q_i$ (Kim *et al.*, 2002; Bhat, 2005; and Habib and Miller, 2009).

$$|J| = \prod_{i=1}^{N} \frac{1}{\vartheta} \left[\frac{(1-\alpha_i)}{(q_i+\gamma_i)} + \frac{e^{\rho}}{(K_T+q_i)} \right]$$

Most of the discrete-continuous large scale demand models including (Bhat, 2005, 2008; Habib and Miller, 2008, 2009; Habib, 2009; Pinjari and Bhat, 2009), have assumed the error terms to be IID with Type I extreme value distribution. This assumption simplifies [8] and gives a closed form solution for the calculation of choice probabilities. The estimation of model parameters also becomes computationally manageable in cases where the size of choice-set is large. However, we think that this assumption is not valid in the case of new built space. Most of the time, the types of the space that are built by the builder are highly correlated to each other. Builders are localized in terms of their operations (Buzzelli and Harris, 2003). Moreover, builders and their associated contractors/sub-contractors typically specialize in building specific types of space. The builder that builds detached dwellings is more likely to build semi-detached and attached dwellings than high rise apartments. The location case is similar: A zone (business node) that primarily has Type-A office space (BOMA, 2009) will unlikely to get built an inferior, Type-C office space. A more appropriate assumption, therefore is that the error terms are jointly normally distributed with a mean of 0 and covariance matrix Ω. Hence:

$$P(Q) = \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \int_{-\infty}^{g_{m+1}} \dots \int_{-\infty}^{g_n} exp\left(-\frac{1}{2} \mathrm{E}' \Omega^{-1} \mathrm{E}\right) |J| d\varepsilon_1 \dots d\varepsilon_n \qquad [10]$$

Where $\mathrm{E} = [\varepsilon_1, \dots, \varepsilon_m, \varepsilon_{m+1} \dots \varepsilon_n]$

Equation [10] involves computing an (n–m) dimensional integral of the function that will have a high computational cost associated for large choice sets. In the case of built space however, the builder is faced with very few choices (e.g. 3 in case of office space and 4 to 5 in the case of housing). Thus the evaluation of [10] remains computationally viable.

The resulting likelihood function from [10] for all the builders thus becomes:

$$L(Q|\beta_i^r, \beta_i^c, \vartheta, \gamma_i, \alpha_i, \rho, \Omega) =$$

$$\prod_{b=1}^{B} \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \int_{-\infty}^{g_{k+1}} \dots \int_{-\infty}^{g_n} exp\left(-\frac{1}{2}\mathrm{E}^{'}\Omega^{-1}\mathrm{E}\right) |J| d\varepsilon_1 \dots d\varepsilon_n \quad [11]$$

2.4 Parameter Estimation

The likelihood maximization based parameter estimation process involves two basic steps, the generation/evaluation of the candidate parameter values, and the evaluation of the likelihood function. In the logit-based conventional discrete choice models, the likelihood function has a closed form, so the evaluation of likelihood, gradient, and hessian of the function is trivial. Gradient based search methods like Newton-Raphson (NS), Broyden–Fletcher–Goldfarb–Shanno (BFGS), Berndt–Hall–Hall–Hausman (BHHH), David–Fletcher–Powell, Polak–Ribiere conjugate gradient, and simulated annealing (Ben-Akiva and Lerman, 1985; Washington *et al.*, 2003; and Train, 2009) are used to estimate the parameters and their statistical properties.

In case of parameter estimation for probit models, the evaluation of the likelihood function becomes non-trivial, because of the involvement of the multi-dimensional integral. In case of the classic probit model the multidimensional integral involved in the likelihood function is approximated using one of several methods: numerical integration, tabulation, numerical approximation, and Monte Carlo simulation (Sheffi *et al.* 1982). MC Simulation is most widely used in which the likelihood function is evaluated using various simulation techniques like Accept–Reject (AR), smoothed AR, and Geweke–Hajivassiliou–Keane (GHK). The resulting approximate log-likelihood function is called a Simulated Log-Likelihood (SLL). The gradient of the function required for optimization problem can be approximated by dividing the change in SLL by the change in the parameter values (Train, 2009). Bolduc (1999) suggests a simulation based procedure for the analytical solution of the gradient in the GHK-simulator. Another approach for the probit model parameter estimation is the Bayesian based Markov Chain Monte Carlo (MCMC) simulation technique that avoids the direct evaluation of the likelihood function. Instead it derives the posterior distributions from the prior belief and the data. The moments and other statistical properties are derived by sampling from the posterior distribution using simulation techniques like Metropolis-Hasting (M-H), Adaptive M-H, and Gibb's sampler (Kim *et al.,* 2002; Kim *et al.*, 2003; and Train, 2009). Kim *et al.* (2003) used Markov chain Gibb's sampler to draw directly from posterior distribution and performed finite sample likelihood inference.

Bhat (2001) used a quasi-random Monte Carlo simulation technique to estimate parameters for a mixed logit model. A Halton sequence for each dimension of the integral in the likelihood function was drawn by pairing k-sequences. The sequence ensured that the whole region under the integral is uniformly covered. The cyclic nature of the Halton sequence results in correlation issues. To avoid this problem, a scrambling technique was used, but this adds an exponential overhead with each dimension, so as to produce a "good" permutation (Hess and Polak, 2003). It is however not very clear what maximization criteria were used and how the approximate gradient/scores and hessian were calculated. It is also not very clear how the local maxima were avoided in the estimation process.

Train (2009) outlined a Bayesian based MCMC method for the parameter estimation in mixed logit models. Bayesian methods relax the constraint of maximizing the simulated-likelihood function, which could be complicated in cases where there might be various local maxima and thus might result in identification problems. In Bayesian methods, the prior distribution plays an important role and is assumed to be near the values that globally maximizes the likelihood function. Bayesian methods are also superior to standard simulated-likelihood maximization methods in terms of consistency and efficiency. In the case of large-scale demand model estimation, Monte Carlo simulation, quasi-Monte Carlo simulation, and Markov Chain based Monte Carlo simulation methods are commonly used (Bhat, 2001, Kim *et al.,* 2002, Habib, 2009). Bhat (2005) and Habib and Miller (2009) used the quasi-random Monte Carlo simulation procedure outlined in Bhat (2001) for the likelihood function that had extreme valued error terms and normally distributed parameters. Kim *et al.* (2002), von Haefen and Phaneuf (2004), and Habib (2009), used Markov Chain Monte Carlo (MCMC) based on Metropolis-Hasting method to estimate their parameters from the likelihood function involving the normal distribution. The likelihood functions in the cases of von Haefen and Phaneuf (2004), and Habib (2009) had extreme valued error terms and normally distributed correlated parameters. Kim *et al.* on the other hand had a normally distributed correlated error terms as well. They used GHK simulator to evaluate the multidimensional integral involved within the log likelihood function. The statistical properties of the estimated parameters were computed using Gibb's sampling. The likelihood function in equation [11] also involves correlated error terms that are normally distributed. In the estimation of parameters from this function, we also decided to use Bayesian MCMC with Gibb's sampling approach. For the evaluation of the multidimensional normal probability function involved in equation [11], we used a technique based on randomized lattice rules that seeks to fill the hyper integration space evenly using a deterministic process. In principle, these lattice rules construct regular patterns, such that the projections of the integration points onto each axis produce an equidistant subdivision of the axis (Genz, & Bretz 2002, 2009). Robust integration error bounds are obtained by introducing additional shifts of the entire set of integration nodes in random directions. Since this additional randomization step is only performed to introduce a robust Monte Carlo error bound, 10 simulation runs are usually sufficient. We preferred this method from the more widely used Halton sequence based simulation procedure, because it has been numerically proven to outperform Halton or Sobel sequences in terms of efficiency and doesn't suffer from correlation issues (Lai, 2009).

2.5 Estimation Procedure

The procedure that we used to estimate parameters in equation [11] is as follows:

Let the parameters in the likelihood function be paired as $\zeta_b = (\beta_b^r, \beta_b^c, \gamma, \rho)$, α, and ϑ. The parameters in $\zeta_b$have a mean of $\bar{\zeta}_b$ are correlated to each other by matrix Ω_ζ.

1. Initialize $\zeta_b, \alpha, \vartheta, \bar{\zeta}_b, \Omega_\zeta$

2. Generate $\{\zeta_b, b = 1, \ldots, B\}$ from

$$\psi(\zeta_b|\{q_{bt}, t = 1, \ldots, T\}, \alpha, \bar{\zeta}_b, \Omega_\zeta\} \propto det\left|\Omega_\zeta\right|^{\frac{1}{2}} exp[-\frac{1}{2}(\zeta_b - \bar{\zeta}_b)'\Omega_\zeta{}^{-1}(\zeta_b - \bar{\zeta}_b)] \prod_t^T L_{bt}$$

Where

ψ is a $N \times 1$ vector representing all the alternatives

t represents the decision occasion

Generate a random number $\tau_\psi -> N(0,0.0025)$, then the candidate value of $\zeta_b$for iteration *k* will be:

$$\zeta_b^k = \zeta_b^{(k-1)} + \tau_\psi$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\zeta_b^{(k)} - \bar{\zeta}^{(k)})'\Omega_\zeta(\zeta_b^{(k)} - \bar{\zeta}^{(k)})] \prod_t^T L_{bt}^k}{exp[-\frac{1}{2}(\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})'\Omega_\zeta(\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})] \prod_t^T L_{bt}^{k-1}}, 1\right]$$

3. Generate $\bar{\zeta}_b$from

$$\psi\left(\bar{\zeta}_b \middle| \{\zeta_b, b = 1, \ldots, B\}, \Omega_\zeta\right\} = N\left(\frac{\sum_1^B \zeta_b}{B}, \frac{\Omega_\zeta}{B}\right)$$

4. Generate $\Omega_\zeta$from

$$\psi\left(\Omega_\zeta \middle| \{\zeta_b, b = 1, \ldots, B\}, \bar{\zeta}_b,\right\} \propto Inverted\ Wishart\left(d_0 + B.D_0 + \sum_1^B (\zeta_b - \bar{\zeta}_b)'(\zeta_b - \bar{\zeta}_b)\right)$$

Where d_0 is the prior degrees of freedom and D_0 is the sum of squares of Ω_ζ

5. Generate α from

$$\psi(\alpha|\{q_{bt}, b = 1 \ldots B\ and\ t = 1, \ldots, T\}, \{\zeta_b, b = 1 \ldots B\}, \bar{\alpha}_0, \Sigma_0\} \propto det|\Omega_0|^{\frac{1}{2}} \exp[-\frac{1}{2}(\alpha - \bar{\alpha}_0)'\Omega_0{}^{-1}(\alpha - \bar{\alpha}_0)] \prod_b^B \prod_t^T L_{bt}$$

$\bar{\alpha}_0$and Ω_0 are the prior parameters

Generate a random number $\tau_\alpha -> N(0,0.01)$, then the candidate value of $\alpha$for iteration k will be:

$$\alpha^k = \alpha^{(k-1)} + \tau_\alpha$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\alpha^k - \bar{\alpha}_0)'\Omega_0{}^{-1}(\alpha^k - \bar{\alpha}_0)] \prod_b^B \prod_t^T L_{bt}^k}{exp[-\frac{1}{2}(\alpha^{(k-1)} - \bar{\alpha}_0)'\Omega_0{}^{-1}(\alpha^{(k-1)} - \bar{\alpha}_0)] \prod_b^B \prod_t^T L_{bt}^{(k-1)}}, 1\right]$$

6. Generate ϑ from

$$\psi(\vartheta|\{q_{bt}, b = 1 \ldots B\ and\ t = 1, \ldots, T\}, \{\zeta_b, b = 1 \ldots B\}, \bar{\vartheta}_0, \Sigma_0\} \propto det|\Omega_{0'}|^{\frac{1}{2}} exp[-\frac{1}{2}(\vartheta - \bar{\vartheta}_0)'\Omega_0{}^{-1}(\vartheta - \bar{\vartheta}_0)] \prod_b^B \prod_t^T L_{bt}$$

$\bar{\vartheta}_0$and $\Omega_{0'}$ are the prior parameters

Generate a random number $\tau_\vartheta -> N(0,0.01)$, then the candidate value of ϑ for iteration k will be:

$$\vartheta^k = \vartheta^{(k-1)} + \tau_\vartheta$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\vartheta^{k}-\bar{\vartheta}_0)'{\Omega_0}^{-1}(\vartheta^{k}-\bar{\vartheta}_0)]\prod_b^B\prod_t^T L_{bt}^{k}}{exp[-\frac{1}{2}(\vartheta^{(k-1)}-\bar{\vartheta}_0)'{\Omega_0}^{-1}(\vartheta^{(k-1)}-\bar{\vartheta}_0)]\prod_b^B\prod_t^T L_{bt}^{(k-1)}},1\right]$$

7. Iterate back to step 1

The simulation has to be run for a sufficient numbers of iterations before drawing inferences. It is suggested that around 25,000 iterations should be enough for the burn-in (Kim *et al.,* 2002; von Haefen and Phaneuf, 2004; Train, 2009; and Habib, 2009). Gibb's sampling is then done to construct the distributional summary statistics for $\zeta_b, \bar{\zeta}_b, \Omega_\zeta, \alpha, \vartheta$. Gibb's sampling induces a serial correlation in the parameters. To avoid this correlation, it is also suggested that every 10th iteration be used in the simulation after warm up (von Haefen and Phaneuf, 2004; and Train, 2009).

2.6 Solution Identification

Parameter estimation from the data based on the underlying model structure is fundamentally an optimization problem that may have a non-unique solution set. The identification problem is the problem of determining what conclusions drawn from the data about a model parameters are feasible (Manski, 1995; Train, 2009). Walker *et al.* (2007) defined the identification problem as the problem of determining the set of restrictions to impose in order to obtain a unique vector of consistent parameter estimates. These restrictions can be on the range in which the parameter values should exist, acceptable goodness-of-fit values for the estimated model, and definition of the regions in the search space in which to search. In this context, it seems that the Bayesian method based on Markov Chain Monte Carlo simulation with Gibb's sampling does a better job to properly identify the solution. Compared to the quasi-MC methods, the approach proposed in this paper, gives more flexibility in defining the search space and guiding the search in the proper direction. This is because the search is based on a prior distribution. The prior distribution corresponds to the knowledge that we already have about the parameters and their correlations. It is rarely the case that we do not have any idea about the sign and/or scale of the parameter values. We can thus control the starting direction of search based on our prior beliefs about the solution. Secondly, the Metropolis-Hasting based search process itself is more controlled and directed. The MH search updates the value of the parameter distributions based on the increase in the likelihood from the new values. The statistical properties of the parameters in the solution are drawn from the posterior distribution of the parameters that are not just based on the likelihood values from the data, but also on the prior distribution and the search process. Lastly, by introduction of the Hicksian good, it is ensured that the solution space for the problem is reduced to finding only those parameters that represent the set of quantities within the technological constraints. The parameters estimation also takes into account the fact that a builder will only invest in built space to an extent where it can maximize profit, but also have the option of investing elsewhere (or for that matter not investing at all).

3. Application to Office Space Supply

Using the model formulation proposed in section 2, here we estimate a model for the new office space supply evolution in the Greater Toronto Area (GTA). The office space market in the GTA is a vibrant and growing market. Greater Toronto Area is the third largest financial centre of North America, only after New York and Chicago. It is the centre of activity in Canada for various office-based employment sectors, including finance, information technology, banking, insurance, and legal consulting. The consistent demand of high quality office space with good accessibility and location arising from the office based employment sectors has driven the growth in office space market in the GTA. In addition to its relevance in understanding travel behaviour, modelling and understanding the office space market in general and office space supply in particular has high economic benefits. The large capital requirements and long development periods make office investment riskier than other types of built space (Tse and Webb, 2003). Using office space models for forecasting and understanding of the working of building industry could decrease these investment risks. Few aggregate (country, municipality or CBD-suburb level) office space supply models can be found in the real estate and integrated land-use and transportation modelling literature. There are very few examples, however, of serious modelling efforts at the more disaggregate submarket level. This is in spite of the fact that there is strong evidence to suggest that sub-markets within a metropolitan area are temporally asynchronous from each other in terms of growth and are characterised by a high level of agglomeration by industry type. The availability of certain types of office floor space has an effect on firm location and relocation decisions as well. Another important dimension in the modelling of office space market is that the location, quality, and quantity are interconnected decisions. At anytime, a location may have excess stock of one type, but is under-stocked in other type. Similarly, some locations are suitable for only a few specific types of built space while not suited for others. For instance, downtown Toronto has a high concentration of Type A and B office space, but rarely Type C space (BOMA, 2009). The quantity that could be built at certain location is also influenced by the neighbourhood characteristics (zoning by-laws, technological constraints).

In the real estate literature, quantity is mostly modelled at a very high level of aggregation (Lentz and Tse, 1977; Rosen, 1984; Viezer, 1999; Hendershott *et al.*, 1999; McDonald, 2000; Nanthakumaran *et al.*, 2000; Tse and Webb, 2003; Ho, 2005; and Fürst, 2006). Operational integrated urban modelling frameworks model these decisions at a lower level of aggregation (census tracts, small grids), but do not treat them as related decisions within a single framework (Martínez, 1996; Martínez and Hurtubia, 2006; Martínez and Henríquez, 2007; Waddell *et al.*, 2003, 2008; Waddell and Ulfarsson, 2003, 2004; Hunt and Abraham, 2003; and Miller *et al.* 2011). Instead the individual dimensions are modelled separately, and then some kind of simulation or rule based allocation is used to simulate the built space evolution. In ILUTE for instance, Miller *et al.* (2011) used a separate model for the location choice probabilities for each type of dwelling and another model for the quantities to produce in the study area. These two models are then used in a Monte Carlo simulation to allocate the new stock to individual locations.

The building industry generally, and in the case of the GTA in particular, is an oligopoly with very few firms and these firms move in and out of the market very frequently with the boom and bust cycles of the built space market (Buzzelli and Harris, 2003). It is important to bring in the builders' behaviour within the decision modelling framework. Their attitude towards risk taking and the expectation of profit might vary among individual builders. A modelling framework that can incorporate these issues is currently missing in the literature. Farooq *et al.* (2010) reported a high spatial variation in the rent of office space. It was also reported that there were not only inter-cluster variation, but also intra-cluster variations. Under the profit maximizing assumption for the builder of space, this variation in rent will influence the decision to select the best location for the new office space. Considering the mentioned shortcomings in the existing literature, the estimation of a multidimensional decision model for office space that models the when, where, what, and how much decisions in a single framework, is a significant contribution towards the better understanding of the office space supply evolution.

We used the office space data collected by Coldwell Banker Richard Ellis (CBRE) consultants for the Greater Toronto Area from 1986 to 2005 to estimate the model. Office space is distributed across the study area in various identifiable clusters. Based on their geographic concentration in various regions, CBRE divided the study area into 36 visually identifiable business nodes (Figure 3) in the survey. The CBRE dataset that was used in this study is a quarterly account of the total inventory and market conditions, including new supply, vacancy rates, gross and net rent levels, and absorption rates in these business nodes for different types of office space. For the estimation of the model here, we converted the data to yearly values and used the data that were available from first quarter of 1986 to third quarter of 2005. The dataset classified office space into four standard types (A, B, C, and G), as defined by the Building Owners and Managers Association (BOMA). This is a subjective classification that uses a combination of factors including rent, building finishes, system standards and efficiency, building amenities, location/accessibility, and market perception (BOMA, 2009). Type A floor space has a high quality standard finish, state of the art systems in the building, exceptional accessibility and a definite market presence. Downtown Toronto and regional centres are dominated by Type A office space. Type A space has higher than average rents for the area. Type B office space has fair to good facilities and infrastructure, while Type C office space buildings are only providing a functional space at a lower rent level compared to the area average. Type G office space are government owned buildings. Type G buildings don't follow the general demand for office space in the market and are not included in this analysis. Table 1 provides the summary statistics and description of the dependent and explanatory variables used in the estimated model.

Figure 3
Study area and approximate location of the business nodes

Table 1 Summary Statistics

Variable	Description	Mean/Proportion	Std. Dev.	Min.	Max.
Dependent Variable					
Supply_A	Supply of Type A (1000 sq. ft.)	76.57	226.78	0	2601.88
Supply_B	Supply of Type B (1000 sq. ft.)	11.92	44.99	0	525.00
Supply_C	Supply of Type C (1000 sq. ft.)	2.14	12.58	0	150
Independent Variable					
Built_A	Already Built Type A in the Node (million sq. ft.)	1.833	3.815	0	24.667
Built_B	Already Built Type B in the Node (million sq. ft.)	1.307	1.448	0	5.987
Built_C	Already Built Type C in the Node (million sq. ft.)	0.368	0.438	0	1.640
Gr_Rtl_Rt_A	Average Rent of Type A in the Node (CAN $)	25.46	6.888	2.30	50.30
Gr_Rtl_Rt_B	Average Rent of Type B in the Node (CAN $)	20.82	5.525	1.77	40.89
Gr_Rtl_Rt_C	Average Rent of Type C in the Node (CAN $)	18.73	5.000	7.40	37.62
Vac_Rt_A	Vacancy Rate of Type A in the Node	0.139	0.077	0	0.442
Vac_Rt_B	Vacancy Rate of Type B in the Node	0.160	0.096	0	0.56
Vac_Rt_C	Vacancy Rate of Type C in the Node	0.158	0.175	0	1
Con_Wrks	Number of Construction Workers in the GTA (×1000)	41.93	9.87	2.73	61.30
Wage_Rt	Hourly Wage Rate for Construction Workers	18.27	2.69	12.83	22.91
Con_Cost	Construction Cost per sq. ft.	85.15	13.75	70.5	117.55

Total observations: 720

3.1 Model Estimates

Using the dataset described in the previous section, we estimated the model of new office space supply for the Greater Toronto Area. The estimation process was implemented in R statistical language. This language was selected due to the open source nature of the language and availability of various support packages. The code is written in a very generic form and could be readily used for estimation of the models that are based on the framework proposed in section 2. The execution time for the estimation code is around 6 hours. The large computation time is due to the fact that we need to run the MCMC process for a long time to warm it up and then to avoid correlation among iterations, we only use 1 in 10 iterations to generate the distributions for the parameters. In future we intend to work on a faster implementation of the algorithm.

The dependent variable here is the probability of selection of a vector (3×1) of quantities for each type of office space to be built in a business node n at certain year t from 1986 to 2005. The explanatory variables used in the model represent the market conditions and land use characteristics of the business node and the state of regional economy at the time of decision to build. Parameter estimates and associated statistics are reported in Table 2. Table 3 reports the correlations between different types of office space. The constant term for Type A space is the highest, followed by Type B. For Type C the constant term is negative. This suggests that builders in general prefer to build higher quality space. The office employment sector in the GTA is dominated by financial, accounting, law, and technology firms that generate the demand for high quality office space. Higher constants for Type A and B seems to be the response of builders to this demand and higher profit margins. Haider and Miller (2004) reported the phenomena of spatial inertia in the new housing supply of the GTA. We observe the same phenomena in the office space supply. The attractiveness which is captured by the amount of office space that is already available (Buit_*), is the highest in case of Type A, while it is lowest in the case of Type C. The rent per sq. ft. of the type office space at the time of decision was used as the indicator of market and the growth of office based employment. In general there is a positive effect of the supply decisions with the higher rents and this effect is highest in the case of Type C buildings. This result is unexpected as one would expect that the higher quality space will be more sensitive to the increase in the rent. One reason for this might be the fact that in general there is a higher temporal variation found in the rent of Type C office space. While in case of Type A and B, the variation is both in terms of time and space. We used average vacancy rate in the node at time of decision as another indicator of the demand for office space. The model reports negative sensitivity of the supply decisions to the increase in vacancy rates. This effect is highest in case of Type A space. A higher project cost is associated with Type A space and at the same time the revenue (indicated by rents) from it is the highest as well. This explains the higher sensitivity to vacant space in case of Type A supply decisions. The number of construction workers in the labour force at the time of decision is used as the indicator of building inertia and state of the regional economy. A positive effect is found on the supply decisions due to the increase in number of construction workers. With the increase in the wage rates the cost of the project increases and thus effects the new office space supply decision in the negative fashion. Similar behaviour is evident in the case of increase in the construction cost. Unfortunately, due to the unavailability of the data, we were not able to model the probability of selection due to difference in cost by type.

Table 2 Model Parameter Estimates

Parameters	Estimates	Std. Error	t-statistics
Explanatory Variable			
Const_A	29.71	0.182	163.29
Const_B	19.99	0.112	178.74
Const_C	-10.72	0.119	-89.75
Built_A	1.76	0.202	8.68
Built_B	0.68	0.057	11.94
Built_C	0.57	0.082	6.99
Gr_Rtl_Rt_A	0.69	0.219	3.13
Gr_Rtl_Rt_B	0.80	0.166	4.81
Gr_Rtl_Rt_C	1.11	0.108	10.31
Vac_Rt_A	-2.44	0.140	-17.44
Vac_Rt_B	-1.15	0.167	-6.91
Vac_Rt_C	-0.37	0.236	-1.58
Con_Wrks	0.84	0.072	11.73
Wage_Rt	-1.32	0.116	-11.37
Con_Cost	-0.64	0.135	-4.73
Model structure parameters			
Gamma_A	100.10	0.080	1246
Gamma_B	100.06	0.088	1135
Gamma_C	99.88	0.138	723.28
Rho	4.49	0.108	41.64
Alpha	0.66	0.173	1.107
Theta	1.59	0.226	15.44

Table 3 Correlation Matrix between the error terms (significant with 95% confidence)

	Type of Office Space		
	A	*B*	*C*
A	1	0.25	-0.25
B		1	-0.10
C			1

The greater than one theta variable that represents the behaviour of the builders towards risk shows that they are risk takers. Builders overbuild in the boom of construction cycles anticipating future revenues. This fact is evident from the discussion in the data description section. We would like to introduce more detailed behaviour of the builders and possible heterogeneity in the model by further parameterizing this variable using data on builders' characteristics. The translation parameters that make the corner solutions possible, are almost the same for all three types. The standard errors are also very small. We also tried various starting values; these resulted in approximately the same values. It will be interesting to observe the effect of fixing the value of translation parameter to 100 and running the estimation again. This way, we will be able to control the bias that variable translation parameters may have introduce in the estimation process. The scale parameter seems to be in the acceptable range. The rho parameter that is associated with the parameterization for the Hicksian good is also in the right range. A limitation that arises due to the use of Bayesian-based MCMC estimation process is the inability to generate model level goodness-of-fit statistics. The goodness-of-fit test for these types of method is an evolving research topic. An alternate approach to test the goodness of fit for this model could be to use simulation forecasting and compare the results with the observed data. Once new data for the years after 2005 are available, we plan on performing the simulation tests.

4. Discussion and Concluding Remarks

We presented a model for new office space supply for the Greater Toronto Area. The model is based on the novel multidimensional decision modelling framework for the supply of new built space. The modelling framework assumes that builders attempt to maximize expected profit. To our knowledge, this work is the first that models the where, when, how much, and what type of office space to supply in a single framework at a fairly disaggregate spatial zoning system. The estimated model is dynamic in the sense that it captures the lagged effects of market conditions on the new supply. We observed the phenomena of spatial inertia in terms of location choices for different types of office space. The behaviour of the builders in terms of risk is explicitly incorporated and estimated in the model. Builders are risk takers and tend to overbuild in the boom cycles. In future we intend to bring in more detail in the model in terms of builder's behaviour and heterogeneity among builders. That will require specialized survey of builders and their characteristics and observations about their decision making behaviour in the context of new built space supply. The changes in the construction project's expected cost on the builder's decisions are also modelled. Depending on the data availability we intend to introduce more detailed representation of costs and its variation by type and location in the model.

The use of Markov Chain Monte Carlo simulation has a short-coming in the sense that the model fitness statistics cannot be estimated. On the estimation side, we intend to assess the goodness of fit for the model using simulation and comparison of simulation results with the observed data. Unfortunately the data for the new built space supply after 2005 is not currently available. Another option for assessing the performance of the model would be to randomly pick some observations from the current dataset and not include them in the estimation process. We could then test the prediction power of the model on the excluded observations. This would provide some indication concerning how good the model is, however the size of the dataset would be reduced further (currently 720 observations), which might affect the quality of the estimation process. In the future we also intend to develop an estimation procedure for the model using a maximum likelihood method. Such a method will require at least developing procedures to evaluate the first and second order derivative of the likelihood function (equation [11]), discussed in section 2. The advantage of using maximum likelihood based estimation is that we will be able to compute goodness-of-fit values from the estimation process. On the other hand, we will have to exercise extra caution so as to avoid getting stuck in the local maxima during the estimation process. Due to the higher degree from of the likelihood function and the problem's dimensionality, it is not very clear if the global maximum exists. In any case, we want to make sure that the solution that we find does make sense and is usable in planning and forecasting. For instance, if we set the parameters value to infinity (with appropriate signs) that will give us the maximum value for the likelihood function, but the resulting model estimates will be of no use to us in terms of giving insight and its predictive power.

This model is part of our ongoing efforts towards operationalization of the office space market within Integrated Land Use Transportation and Environment (ILUTE) modelling framework, currently under development at the University of Toronto. In the general market-clearing framework of ILUTE, the asking rent model captures the role of accessibility, neighbourhood characteristics, quality of space, and market conditions to determine the asking price at each simulation year. The model of asking rent of office space at building level has already been developed by Farooq *et al.* (2010). The models for demand of office space in the Greater Toronto Area by small- and medium-sized firms have also been developed by Elgar *et al.* (2009). With the available demand and supply, the asking rents are then to be used in the market clearing module to match the space to the demander at a transaction rent that is endogenously determined. In next simulation year, the lagged transaction rents then influence the builders' decisions of where, how much, and what type of office space to supply.

Note that the model estimated in section 3 is based on 36 business nodes that are spatially quasi-independent sub-markets. While in the context of modelling the office space supply, it makes sense to use this spatial system, for the implementation of the model in ILUTE, we will need another level of model that distribute the built space within a business node to the census tracts or dissemination areas that are marked as commercial by the zoning by-laws. This model can be a similar location choice model as the one estimated for the new housing supply by

Haider (2003). This model will then we used in the similar fashion as we used Haider's model for operationalizing housing supply in ILUTE v1.0 (Farooq, 2010).

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