

Clip-Air, a flexible air transportation system

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Abstract

We introduce a concept of flexible air transportation system called Clip-Air. It is a modular innovative aircraft. The flexibility provided by Clip-Air is due to the decoupling of load and carrying units. In this paper, we introduce the concept, and analyze the impacts from the airlines perspective. An integrated schedule design and fleet assignment model is developed for both standard airline fleets and Clip-Air. The model considers spill and recapture effects to represent the demand in case of capacity shortage. Recapture ratios between available itineraries in each market segment are appropriately calculated through an itinerary choice model. The comparative analysis is carried out under different scenarios which are selected with the purpose of understanding the effects of the network structure, fleet size, fleet configuration and the estimated cost figures for the Clip-Air system. It is observed that Clip-Air is able to carry on the average 5-10 % more passengers by using 20-30% less overall capacity. Moreover, Clip-Air is found to deal better with insufficient transportation capacity. Furthermore, the scheduling decisions are robust to the estimated cost figures of Clip-Air. For the analyzed range of costs Clip-Air is always carrying more passengers with less allocated capacity compared to standard fleet.

1 Introduction

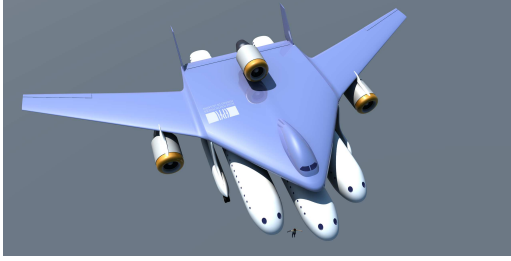
According to the statistics provided by the Association of European Airlines (AEA), air travel traffic has grown at an average rate of 5% per year over the last three decades and in 2012 passenger-km values is expected to be doubled compared to 1997. Consequently, there is an increased number of landings and takeoffs from airports, resulting in frequent congestion and delays that occasionally turn into a major disruption. The steady growth of travel demand during the last decades justifies the need for new concepts and new solutions that can accommodate this demand with a minimal impact on the environment and the economy. We introduce here such a new concept based on a modular aircraft design.

A new family of modular airplane, called Clip-Air, is currently designed at the Ecole Polytechnique Fédérale de Lausanne (EPFL, Leonardi and Bierlaire, 2011). Clip-Air is shown to be feasible from an aircraft design viewpoint and has already been tested in a simulation environment. It is based on two separate structures: a flying wing and capsules. The wing is designed to carry the engines and the flight crew. The capsules are designed to carry the payload that can be passengers and/or freight.

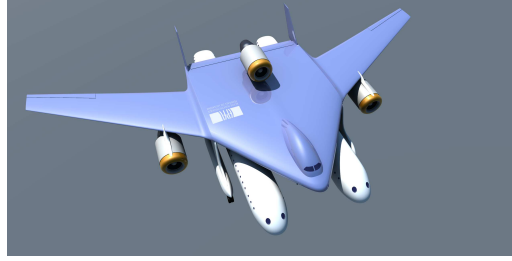
The capsules of Clip-Air are attached (or *clipped*) below the wing as illustrated in Figure 1 with three, two or one capsules. The clipping mechanism facilitates the separate handling of the capsules for airport ground operations such as boarding and unboarding, maintenance operations etc. This modularity is the foundation of the Clip-Air concept for flexible transportation.

The Clip-Air wing has a compact structure and this structure increases the energy efficiency with reduced drag compared to the existing flying wings. Since the wing can be detached from the capsules, it has several advantages. The capsules are easy to produce, transfer and store due to the decoupling from the wing, specifically the engine. Similarly, capsules can be configured to carry freight or even extra fuel due to their simplified structure. Furthermore, the complete separation of pilots and passengers provides security advantages. In case of unexpected events, the capsules can be safely detached from the wing and fatalities are expected to be minimized since no fuel is carried with the passengers.

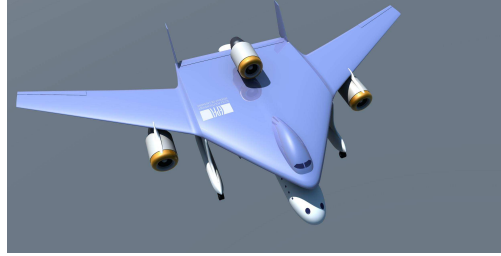
Clip-Air brings the necessary flexibility to address the above mentioned issues. By “flexibility”, we mean “the ability of a system to adapt to external changes, while maintaining satisfactory system performance.” Morlok and Chang, 2004. In the context of air transportation, airlines have dedicated a lot of efforts in increasing the flexibility through demand and revenue management Talluri and van Ryzin, 2004b. Flexibility is obtained namely from differentiated fare products offered to different customer segments with the objective to increase the total revenue. In addition to revenue management, schedule planning of airlines are more and more designed to be robust to unexpected disruptions, such as aircraft breakdowns, airport closures, or bad weather conditions Lan et al., 2006; Gao et al., 2009, and associated recovery strategies are



(a) Three capsules



(b) Two capsules



(c) One capsule

Figure 1: Clip-Air wings and capsules

applied after the occurrence of these disruptions Lettovsky et al., 2000, Eggenberg et al., 2010. Both robust planning and efficient recovery operations in case of disruptions are shown to increase the profitability of airlines.

Flexibility is also being introduced in other modes of transportation and several techniques are studied to increase the flexibility. Railways have flexibility over capacity utilization, which rises from the modularity in fleet. Maritime transportation plays an important role in improving flexibility in the context of multi-modality where the standard unit loads, such as containers, that can be efficiently transferred between trucks and trains.

These various concepts of flexibility (demand management, robustness and recovery, modular capacity, and multi-modality) can be combined in an integrated transportation system. This is the motivation of the Clip-Air system. In a nutshell, the concept of Clip-Air consists in

- bringing the modular capacity of railways to airline operations,
- importing the concept of standard unit loads from freight to passenger transportation, necessary for efficient multimodal operations.

Combined with efficient demand management and robust scheduling methods from the airline operations, the Clip-Air system combines the four types of flexibility mentioned above.

In this paper we focus on two dimensions of flexibility: modularity and demand management. We analyze the effect of the modularity of Clip-Air on the schedule planning of airlines and integrate supply-demand interactions through an itinerary choice model.

Before we present our mathematical formulation (section 3) and numerical results (section 4) we provide a literature review on flexibility of transportation systems with a specific focus on airline operations.

2 Related literature

Studies on flexible transportation systems have an increased pace during the last decade. We refer to the work of Morlok and Chang (2004) for the description of the concept of flexibility in transportation and for the techniques to measure the flexibility with a focus on capacity flexibility. Similarly, Chen and Kasikitwiwat (2011) develop network capacity models for the quantitative assessment of capacity flexibility.

Flexibility is studied for different transportation systems including land, rail and air transportation. Brake et al. (2007) provide examples of Flexible Transportation System (FTS) applications that aim to improve the connectivity of public transport networks in the context of land transportation. Crainic et al. (2010) work on the flexibility concept with Demand-Adaptive Systems which combine the features of traditional fixed-line services and purely on-demand systems. Zeghal et al. (2011) studies flexibility for airlines in terms of the active fleet and departure time of flights. An airline can increase or decrease the fleet size renting or renting out planes. Departure times can be adjusted within a given time-window. These flexibilities facilitate the integration of schedule design, fleet assignment, and aircraft routing decisions.

Since we analyze the performance of the Clip-Air system in airline fleet assignment process we refer to integrated fleet assignment models. Yan and Tseng (2002) develop a model that simultaneously decides the flight schedule and the fleet assignment with path-based demand considerations. With a similar idea of itinerary-based demand, Barnhart et al. (2002) build an integrated schedule design and fleet assignment model where they consider spill and recapture effects in case of insufficient capacity. Their model considers fare class segmentation so that passenger demand is represented separately for each fare class. Lohatepanont and Barnhart (2004) build a similar model with the network effects including the demand adjustment in case of flight cancellations.

As mentioned previously, airlines profit from the efficient use of revenue management techniques. We refer to Talluri and van Ryzin (2004b) for a comprehensive presentation of revenue management approaches. Recently, additional attention has been paid to better represent the demand through advanced demand models. Coldren et al. (2003) work on logit models for travel demand, Coldren and Koppelman (2005) extend the models of the previous work using GEV, particularly nested logit model. Koppelman et al. (2008) apply logit models to analyze the effect of schedule delay by modeling

the time of day preferences. Carrier (2008) and Wen and Lai (2010) work on advance demand modeling that enable customer segmentation with the utilization of latent class choice modeling. We refer to the work of Garrow (2010) for a comprehensive presentation of different specifications of choice behavior models.

Advanced demand models are integrated into optimization models in different levels of the airline scheduling process. Talluri and van Ryzin (2004a) integrate discrete choice modeling into the single-leg, multiple-fare-class revenue management model. Authors provide characterization of optimal policies for the problem of deciding which subset of fare products to offer at each point in time under a general choice model of demand. Schön (2006) develops a market-oriented integrated schedule design and fleet assignment model with integrated pricing decisions. It is assumed that customers can be segmented according to their characteristics and different fares can be charged for these segments using pricing models. Different pricing models are considered including simple linear models as well as discrete choice models. The objective is to maximize the revenue by determining the fare products to be included in the schedule and the fleet assignment for the selected flights.

In addition to demand management, the application of robust schedule planning models increases the profitability of airlines introducing flexibility to adapt to unexpected disruptions. In the literature, robustness is introduced for different subproblems of airline scheduling. Rosenberger et al. (2004) study a robust fleet assignment model that reduces the hub connectivity and embeds cancellation cycles in order to decrease the sensitivity to disruptions and they obtain a better performance compared to traditional fleet assignment models. Shebalov and Klabjan (2006) work on robust crew scheduling models where they introduce robustness by maximizing the number of crew pairs that can be swapped in case of unexpected situations. Lan et al. (2006) present two approaches to minimize passenger disruptions: a robust aircraft maintenance routing problem where they aim to reduce the delay propagation and a flight schedule re-timing model where they introduce time windows for the departure times of flight legs. Similarly, Weide (2009) studies an integrated aircraft routing and crew pairing model where the departure time of flights are allowed to vary in a time window. Inclusion of time windows in the schedule is shown to increase the flexibility of the model having improved results.

Flexibility in rail transportation rises from modular carrying units and several operations research techniques are applied to improve this flexibility. We refer to Huisman et al. (2005) for a review on the models and techniques used in passenger railway transportation for different planning phases. Kroon et al. (2009) discuss the construction of a new timetable for Netherlands Railways which improves the robustness of the system decreasing the delays. Similarly, Jespersen-Groth et al. (2009) study the disruption management problems in passenger railway transportation drawing the analogies with airline disruption management.

Multi-modality is widely studied in the context of freight transportation where

standard unit loads are transferred between maritime, land and rail transportation systems. In freight transportation, each movement of a loaded vehicle generates an empty flow and for the efficient use of the transportation system these empty flows need to be paid attention. We refer to Dejax and Crainic (1987) for a review of empty vehicle flow problems and proposed models on the subject. They also point out the potential advantages of an integrated management of loaded and empty vehicle movements. In maritime transportation Crainic et al. (1993) present models for the repositioning of empty containers in the context of a land transportation system. Olivo et al. (2005) study the repositioning problem in a multi-modal network where empty containers are transported by both maritime and land transportation. Di Francesco et al. (2009) consider empty container management problem under uncertainty and present a multi-scenario formulation regarding different realizations of uncertain parameters.

3 Integrated schedule planning

As mentioned in section 1 we focus on the aspects of modular capacity and demand management in the context of airline operations.

Modular capacity is provided by the design of Clip-Air and we analyze the impacts of modularity on fleet assignment process. As illustrated in section 1 capsules can be detached from the wing. This feature generates an additional level of assignment decisions to be made in comparison to the assignment problem of standard planes. Therefore we build an integrated schedule design and fleet assignment model which enables the appropriate assignment of wing and capsules (3.1).

As a demand management dimension, we integrate supply-demand interactions into the fleet assignment problem through spill and recapture effects. In case of insufficient transportation capacity the movement of spilled passengers is driven by an itinerary choice model based on the attributes of the itineraries (3.2).

3.1 Integrated schedule design and fleet assignment model

We present an integrated schedule design and fleet assignment model which is an extension of the models of Barnhart et al. (2002) and Lohatepanont and Barnhart (2004). Since we want to come up with a comparative analysis between standard planes and Clip-Air, the model is developed for both cases.

The most important difference of Clip-Air from standard planes is that the fleet assignment includes both the assignment of wing and capsules. A flight can not be realized if there is no wing assigned to that flight. When a wing is assigned there is another decision about the number of capsules to be attached to the wing. Secondly, the operating cost allocation is different such that the costs are decoupled between wing and capsules. Flight crew cost is related only to the wing and cabin crew cost is related to the capsules. As will be explained in section 4.1, some other cost figures are

also decoupled according to the weights of wing and capsules.

The model for a fleet composed of Clip-Air wings and capsules, which considers a single airline, is presented in Figure 2. Schedule design is modeled with two sets of mandatory and optional flights such that schedule design decision is to operate the optional flights or to cancel them. Let F be the set of flights, mandatory flights and optional flights are represented by the sets of F^M and F^O . S is the set of market segments, which is taken as distinct origin and destination pairs in this study. I represents the set of itineraries, subset I_s being the itineraries in segment s . We include a set of no-revenue itineraries I'_s for each segment s which stands for the itineraries offered by other airlines. A represents the set of airports and K represents the set of aircraft types which can be a Clip-Air wing with one, two or three capsules. The schedule is represented by time-space network such that $N(a, t)$ is the set of nodes in the time-line network, a and t being the index for airports and time respectively.

The objective (1) is to minimize the operating cost and loss of revenue due to unsatisfied demand. Operating cost for each flight f , has two components that correspond to operating cost for wings and capsules which are represented by C_f^w and $C_{k,f}$ respectively. These are associated with binary decision variables of x_f^w and $x_{k,f}$. x_f^w equals one if there is a wing assigned to flight f . $x_{k,f}$ represents the number of capsules assigned to flight f in such a way that it is one if there are k capsules assigned to flight f . $t_{i,j}$ is the decision variable for the number of passengers redirected from itinerary i to itinerary j typically when there is insufficient capacity. $b_{i,j}$ is the proportion of passengers who accept to be redirected from itinerary i to j .

Constraints (2) ensure that every mandatory flight should be assigned at least one capsule. Constraints (3) maintain the wing capsule relation such that if there is no wing assigned to a flight, there can be no capsule assigned to that flight. On the other hand if there is a wing assigned there can be up to three capsules flying. Constraints (4) and (7) are for the flow conservation of wings and capsules. $y_{a,t-}^w$ and $y_{a,t-}^k$ represent the number of wings and capsules at airport a just before time t . Similarly $y_{a,t+}^w$ and $y_{a,t+}^k$ stand for the number of wings and capsules just after time t . Constraints (5) and (8) limit the usage of fleet by the available amount which is represented by R_w and R_k for wings and capsules respectively. In this study it is assumed that the number of wings and capsules at each airport at the beginning of the period, which is one day, is the same as the end of the period. Constraints (6) and (9) ensure this cyclic schedule property.

Constraints (10) maintain the capacity availability, Q being the capacity of one capsule. The assigned number of seats for a flight should be consistent with the demand for the corresponding itineraries considering the spill effects, that will be explained in detail in section 3.2. Similarly when a flight is canceled, all the related itineraries should not realize any demand. Constraints (11) are for demand conservation for each itinerary saying that total redirected passengers from itinerary i to all other itineraries should not exceed its expected demand D_i .

$$\begin{aligned}
\text{Min } & \sum_{f \in F} (C_f^w x_f^w + \sum_{k \in K} C_{k,f} x_{k,f}) \\
& + \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} (\sum_{j \in I_s} t_{i,j} - \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i \tag{1} \\
\text{s.t. } & \sum_{k \in K} x_{k,f} = 1 \quad \forall f \in F^M \tag{2} \\
& \sum_{k \in K} x_{k,f} \leq x_f^w \quad \forall f \in F \tag{3} \\
& y_{a,t^-}^w + \sum_{f \in \text{In}(a,t)} x_f^w = y_{a,t^+}^w + \sum_{f \in \text{Out}(a,t)} x_f^w \quad \forall [a, t] \in N \tag{4} \\
& \sum_{a \in A} y_{a,t_n}^w + \sum_{f \in \text{CT}} x_f^w \leq R_w \quad \forall k \in K \tag{5} \\
& y_{a, \min E_a^-}^w = y_{a, \max E_a^+}^w \quad \forall a \in A \tag{6} \\
& y_{a,t^-}^k + \sum_{k \in K} k x_{k,f} = y_{a,t^+}^k + \sum_{k \in K} k x_{k,f} \quad \forall [a, t] \in N \tag{7} \\
& \sum_{a \in A} y_{a,t_n}^k + \sum_{\substack{f \in \text{CT} \\ k \in K}} k x_{k,f} \leq R_k \tag{8} \\
& y_{a, \min E_a^-}^k = y_{a, \max E_a^+}^k \quad \forall a \in A \tag{9} \\
& \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} \delta_f^i D_i - \sum_{j \in I_s} \delta_f^i t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} \delta_f^i t_{j,i} b_{j,i} \\
& \leq \sum_{k \in K} Q k x_{k,f} \quad \forall f \in F \tag{10} \\
& \sum_{j \in I_s} t_{i,j} \leq D_i \quad \forall s \in S, i \in (I_s \setminus I'_s) \tag{11} \\
& x_f^w \in \{0, 1\} \quad \forall f \in F \tag{12} \\
& x_{k,f} \in \{0, 1\} \quad \forall k \in K, f \in F \tag{13} \\
& y_{a,t}^w \geq 0 \quad \forall [a, t] \in N \tag{14} \\
& y_{a,t}^k \geq 0 \quad \forall [a, t] \in N \tag{15} \\
& t_{i,j} \geq 0 \quad \forall s \in S, i \in (I_s \setminus I'_s), j \in I_s \tag{16}
\end{aligned}$$

Figure 2: Integrated schedule planning model for Clip-Air

3.2 Spill effects

Although the purpose of the fleet assignment is to optimize the assignment of aircraft to the flight legs, capacity restrictions and the uncertainties in demand may result with lost passengers or under utilized capacity. In case of capacity shortage some passengers, who can not fly on their desired itineraries, may accept to fly on other available itineraries in the same market segment offered by the company. This effect is referred as spill and recapture effect. In this paper we model explicitly the spill and recapture in order to better represent the demand.

We assume that the spilled passengers are recaptured by the other itineraries with a recapture ratio based on a logit choice model. Choice of an itinerary is modeled by defining the utilities of the alternatives. To explain the utilities, we have used *fare*, *time of day*, and *level of service* as found to be important in the context of itinerary choice in the studies of Coldren et al. (2003), Coldren and Koppelman (2005) and Garrow (2010). Therefore the utility for itinerary i is given by:

$$V_i = -0.050 p_i + 0.139 \text{ morning}_i + 0.900 \text{ nonstop}_i,$$

where p_i is the fare price of itinerary i , morning_i is a dummy variable for the time of day which is 1 if departure time is between 07:00-11:00 and 0 otherwise. Lastly nonstop_i is a dummy variable for the number of stops which is 1 if it is a non-stop itinerary and 0 otherwise. The parameters have been estimated by maximum likelihood estimation using a dataset from a major European airline company.

The logit model allows us to calculate the recapture ratios $b_{i,j}$ which represent the proportion of recaptured passengers by itinerary j among $t_{i,j}$ spilled passengers from itinerary i . The recapture ratio is calculated for the itineraries that are in the same market segment as given in equation (17) where the desired itinerary i is excluded from the choice set. Therefore lost passengers may be recaptured by the remaining alternatives of the company or by the no-revenue options which represent the alternatives provided by competitors. Since no-revenue itineraries are out of the network we assume that no spill exist from them.

$$b_{i,j} = \frac{\exp(V_j)}{\sum_{k \in I_s \setminus i} \exp(V_k)} \quad \forall s \in S, i \in (I_s \setminus I'_s), j \in I_s, \quad (17)$$

We illustrate the concept with the itineraries in market segment A-B including the no-revenue itinerary A-B'. The attributes for the itineraries can be seen in Table 1. Using the logit formulation, recapture ratios are calculated as given in Table 2. These ratios are given as an input to the integrated schedule planning model.

For example, in case of capacity shortage for itinerary 2, at most 5.6% and 71% of spilled passengers will be recaptured by itineraries 1 and 3 respectively. 23.4% will be lost to the itineraries offered by competitive airlines. Recapture ratio from itinerary 2 to itinerary 1 is the lowest since it is the most expensive itinerary and it is not a

Table 1: A-B itineraries

OD	fare	nonstop	time of day
A-B ₁	262	0	0
A-B ₂	162	1	1
A-B ₃	162	1	0
A-B'	185	1	1

Table 2: Recapture ratios for A-B

	A-B ₁	A-B ₂	A-B ₃	A-B'
A-B ₁	0	0.464	0.403	0.133
A-B ₂	0.056	0	0.71	0.234
A-B ₃	0.051	0.738	0	0.211

nonstop itinerary. The ratio from itinerary 2 to itinerary 3 is the highest having the same fare price and being a nonstop itinerary.

4 Results on the potential performance of Clip-Air

For carrying out the comparative analysis between standard planes and Clip-Air fleet we work with a dataset from a major European airline company. Data provides information for the sets of airports, aircraft, flights and itineraries. Apart from these we need the estimated cost figures for Clip-Air wings and capsules which will be explained in section 4.1.

As Clip-Air exists only in a simulated environment we make the following assumptions to obtain results:

- Model with standard fleet has different available plane types at hand and is free to use the optimal fleet composition. On the other hand Clip-Air capsules are of the same size. This is a clear advantage for standard fleet since it is able to adjust the fleet composition according to the characteristics of the network.
- Total available transportation capacity in number of seats is sufficient to serve all the demand in the network for all the analyzed instances. It will be explained in section 4.5 that this is in favor of standard fleet and whenever the capacity is restricted Clip-Air performs significantly better than standard fleet.
- The schedule is assumed to be cyclic so that the number of aircraft/wings/capsules at each airport is the same at the beginning and at the end of the period, which is one day. This a limiting factor for Clip-Air since the modularity of the capsules is not efficiently used in such a case. By the design of Clip-Air, capsules are easy to transfer and store which could be utilized better with the repositioning of the capsules.

The assumptions above lead to a conservative comparison between Clip-Air and standard fleet which makes us more confident about our results.

We have implemented our model in AMPL and results are obtained with GUROBI solver. In this section we first present a small example to illustrate the advantages of the enhanced flexibility of Clip-Air system. Then we present the results for different scenarios regarding the network configuration, fleet size, fleet type and the costs of Clip-Air fleet. For each test case we present the data instances with the following variables:

- Number of airports in the network.
- Number of flights in the network.
- Average number of flights per route which is used as a measure of the *flight density* of the network.
- Capsule capacity of Clip-Air in number of seats.
- Total number of expected passengers.
- Number of itineraries.
- Available plane types for standard fleet.

The results are described with the following attributes:

- Operating cost.
- Spill cost due to the lost passengers.
- Revenue.
- Total number of transported passengers.
- Flight count which is the total number of realized flights.
- Total flight duration which is the total time traveled in minutes for the flights.
- Used fleet which is the fleet composition for standard fleet and the number of wings and capsules for Clip-Air.
- Used aircraft which correspond to the total number of planes/wings assigned to the flights.
- Used seats which correspond to the total number of seats allocated to the flights.

- Available seat kilometers (ASK): The number of seats available multiplied by the number of kilometers flown. This is a widely used measure for the passenger carrying capacity. Since our data does not provide information on the kilometers flown for the flights, we convert the total flight duration to kilometers with a speed of 850 kilometers per hour.
- Transported passengers per available seat kilometers (TPASK): A productivity measure which we adapt to compare the standard fleet and Clip-Air. It is the total number of transported passengers divided by the available seat kilometers and measures the productivity of the allocated capacity.

4.1 Cost figures for Clip-Air

As mentioned previously Clip-Air exists only in a simulated environment. Therefore estimated values are used for the operating cost of Clip-Air using analogies with A320. In Table 3 we present the weight values for Clip-Air flying with one, two and three capsules in comparison to one, two and three aircraft of type A320. As seen from the Table Clip-Air is 63% heavier than one A320 plane when it is flying with one capsule. However when flying with two capsules Clip-Air becomes advantageous being 1% lighter than two A320 planes. This advantage is more obvious when flying with three capsules. We use these weight differences to proportionally decrease/increase the fuel cost and air navigation charges.

The adjustment of the cost figures resulting from the weight differences is explained in the work of de Tenorio (2009).

Table 3: Clip-Air configuration

		Clip-Air	A320
Maximum Capacity		3x145 (435 seats)	150 seats
Engines		3 engines	2 engines
Maximum	1 (plane/capsule)	126t (+63%)	77.5t
Aircraft Weight	2 (planes/capsules)	153t (-1%)	2x77.5t (155t)
	3 (planes/capsules)	180t (-23%)	3x77.5t (232t)

Furthermore we make adjustment on the crew cost due to the decoupling of wing and capsules. Flight crew cost is associated with the wing and the cabin crew cost is associated with the capsules. Clip-Air flies with one set of flight crews regardless of the number of capsules used for the flight which is the source of crew cost reduction. It is given by the study of Aigrain and Dethier (2011) that flight crew constitutes 60% of the total crew cost for A320. Therefore Clip-Air decreases the total crew cost by 30% and 40% when flying with two and three capsules respectively. Remaining operating cost values are assumed to be the same as A320 for the utilization of each capsule.

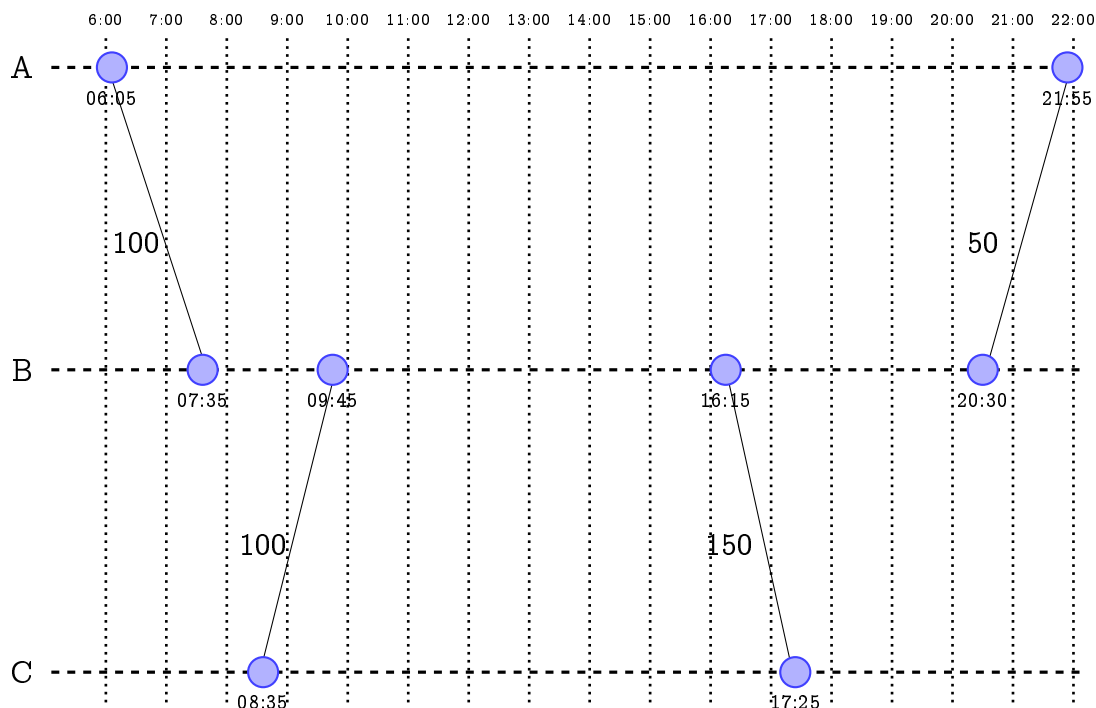


Figure 3: Time-line network for the illustrative example

4.2 An illustrative example

We present results for a small data instance to illustrate the flexibility provided by Clip-Air system. The network consists of four flights with the given demand and departure-arrival times as in Figure 3. There is an expected demand of 250 passengers which is generated by 4 itineraries between airports A-C, B-C, C-A and C-B. The available fleet capacity is not limited and the circular property of the schedule is ignored for this example. It is assumed that there are three types of standard planes which have 50, 100 and 150 seats. On the other hand Clip-Air capsules have a capacity of 50 seats.

Results are provided in Table 4. It is seen that model for standard fleet decides to use 4 aircraft with a total of 400 seats to cover the demand in the network. On the other hand Clip-Air starts with two capsules both from airport A and C in the morning. For the evening flights Clip-Air flies with three capsules to airport C and one capsule to airport A by clipping one capsule to the flight from B to C. Therefore Clip-Air is able to serve all the demand by allocating 50% less capacity and having 3% more profit compared to the standard fleet thanks to its enhanced flexibility. Since the same number of passengers are carried with less capacity compared to standard fleet, Clip-Air uses the allocated capacity more productively as seen from the TPASK measure.

Table 4: Results for the illustrative example

	Standard Fleet	Clip-Air
Operating cost	46,436	44,120
Revenue	118,900	118,900
Profit	72,464	74,780
Transported pax.	250	250
Flight count	4	4
Total flight duration	315 min	315 min
Used fleet	1 A50, 2 A100, 1 A150	2 wings, 4 capsules
Used aircraft	4	2
Used seats	400	200
ASK	1,785,000	892,500
TPASK ($\times 10^{-5}$)	14	28

4.3 Network effect

The type of the network is an important factor that needs to be analyzed for quantifying the performance of Clip-Air. For this matter, we present results for three different network structures: airport pair, hub-and-spoke network with single hub and peer-to-peer well connected network. Flight densities of these networks are different from each other which affects the performance of Clip-Air.

Airport-pair network

We present a network with 2 airports and 35 flights which are balanced for the two routes. The description of the data set is given in Table 5 and the results are provided in Table 6. It is observed that Clip-Air carries 4% more passengers with allocating 32% less seats which results with a clear increase in TPASK measure. The increase in the number of transported passengers is also reflected by the spill cost which is higher for standard fleet. Therefore the profit is 6% higher when flying with Clip-Air. Considering the number of aircraft used, Clip-Air uses 6 wings, on the other hand model with standard fleet uses 10 planes. This is important in terms of the needed flight crews. With standard fleet, the minimum number of needed flight crew pairs is 10. However this value is 6 for Clip-Air. Furthermore airport operations will also be simplified with Clip-Air having less aircraft.

Hub and spoke network with a single hub

The behavior of the Clip-Air system is analyzed for a hub-and-spoke network with a single hub where all the flights need to connect through the hub. Details for the data instance are given in Table 7. With Clip-Air, there is a 7% increase in profit and 6% increase in total transported passengers allocating 7% less capacity. Since the flight density is low with 3.38 flights per route the advantage of Clip-Air is less evident

Table 5: Data instance for the airport-pair network

Airports	2
Flights	35
Flights/route	17.5
Capsule capacity	35
Passengers	2,321
Itineraries	67
Standard fleet types	A35 (35), A70 (70), A105 (105)

Table 6: Results for the airport-pair network

	Standard fleet	Clip-Air
Operating cost	302,695	306,916
Spill cost	61,062	44,550
Revenue	496,537	513,049
Profit	193,842	206,133
Transported pax.	2,023	2,103
Flight count	34	34
Total flight duration	2,810 min	2,810 min
Used fleet	3 A35 5 A70 2 A105	6 wings 13 capsules
Used aircraft	10	6
Used seats	665	455
ASK	26,472,542	18,112,792
TPASK ($\times 10^{-5}$)	7.64	11.61

compared to the airport-pair network which has 17.5 flights per route. However we are still using one less aircraft with Clip-Air which will reduce the needed number of flight crews and simplify the ground operations for airports. We need to mention that in this particular instance the incoming and outgoing flights from the hub are balanced in terms of the demand for each spoke airport. Therefore standard fleet can also perform well in this situation.

Well connected peer-to-peer network

In this section we present results for a peer-to-peer network where the airports are well connected with 44 flights and 3,314 expected passengers as seen in Table 9. Model with standard fleet and Clip-Air result with a similar number of transported passengers. However Clip-Air uses the capacity more efficiently so that 20% less capacity is allocated to carry these passengers. This is also supported by the increased TPASK measure. When we look at the used number of aircraft we see that there is a clear difference between standard fleet and Clip-Air. Therefore the minimum number of needed flight crews is 23% less for Clip-Air which is important for the crew scheduling decisions.

Table 7: Data instance for the hub-and-spoke network

Airports	5
Flights	27
Flights/route	3.38
Capsule capacity	33
Passengers	1,644
Itineraries	42
Standard fleet types	A33 (33), A66 (66), A99 (99)

Table 8: Results for the hub-and-spoke network

	Standard fleet	Clip-Air
Operating cost	204,299	209,720
Spill cost	41,567	26,074
Revenue	355,072	370,565
Profit	150,773	160,845
Transported pax.	1,427	1,509
Flight count	26	26
Total flight duration	2,020 min	2,020 min
Used fleet	3 A33 3 A66 2 A99	7 wings 14 capsules
Used aircraft	8	7
Used seats	495	462
ASK	14,165,250	13,220,900
TPASK ($\times 10^{-5}$)	10.07	11.41

The density of the network is higher compared to the hub-and-spoke instance which helps to reveal the advantages of the flexibility of Clip-Air.

4.4 Effect of the standard fleet configuration

Clip-Air is composed of modular capsules, on the other hand standard fleet can be composed of any aircraft type and the model has the opportunity to select the best fleet composition. Therefore it is important to see the effect of the fleet configuration when comparing with the performance of Clip-Air. This analysis enables us to figure out which type of airlines may profit better from the Clip-Air system.

We use a data instance given in Table 11. We change the available standard fleet configuration by gradually decreasing the fleet heterogeneity. The total transportation capacity is kept high enough to serve the whole demand for all the tested instances. The results for Clip-Air and standard fleet with different fleet configurations are provided in Table 12. It is observed that the richer the fleet configuration, the better the performance of standard fleet. The profit and the transported passengers dramatically decrease when the fleet configuration is highly restricted. The change of profit and

Table 9: Data instance for the peer-to-peer network

Airports	4
Flights	44
Flights/route	3.67
Capsule capacity	39
Passengers	3,314
Itineraries	64
Standard fleet types	A39 (39), A78 (78), A117 (117)

Table 10: Results for the peer-to-peer network

	Standard Fleet	Clip-Air
Operating cost	375,078	367,621
Spill cost	75,356	64,884
Revenue	589,334	599,806
Profit	214,256	232,185
Transported pax.	2,936	2,988
Flight count	40	40
Total flight duration	2,955 min	2,955 min
Used fleet	5 A39 4 A78 4 A117	10 wings 20 capsules
Used aircraft	13	10
Used seats	975	780
ASK	40,815,938	32,652,750
TPASK ($\times 10^{-5}$)	7.19	9.15

total number of transported passengers with the fleet configuration can be seen more clearly in Figure 4. When we look at the results with 1 plane type, which has the same capacity as 1 capsule, the decrease in profit is 70% and 46% less passengers are carried. Similarly, measure of TPASK gets worse except the last case where the utilization of the capacity is very high due insufficient capacity allocation.

4.5 Effect of the available transportation capacity

All the previous results are obtained without any limit on the total capacity so that it is enough to cover the total expected demand. However in reality there may be capacity shortage in case of unexpected events, weather conditions or in high season. Therefore it is important to test the performance of Clip-Air compared to standard fleet when there is limited capacity. The data instance seen in Table 13, that consists of 108 flights, is used for the tests. Available capacity is decreased gradually and the results corresponding to each level of capacity is presented in Table 14.

For the unlimited capacity case, Clip-Air is able to carry 1% more passengers with 40% less transportation capacity. In case of capacity restrictions, the advantage of

Table 11: Data instance for the tests with different fleet configurations

Airports	3
Flights	48
Flights/route	9.6
Capsule capacity	50
Passengers	3,520
Itineraries	50
Standard fleet types	Varying

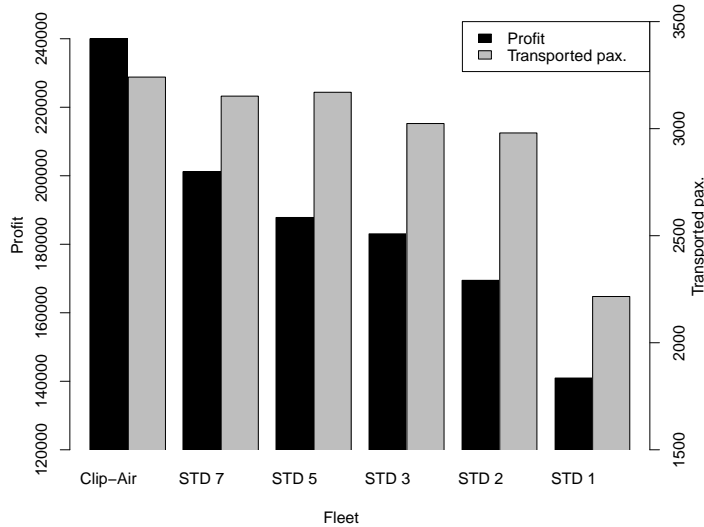


Figure 4: Profit and transported passengers for different fleet configurations

Clip-Air over standard fleet becomes more evident as the restriction becomes harder to overcome. For example for the case with a capacity of 1260 seats, Clip-Air is able to carry 3.6% more passengers with 4 less flights. When the results regarding the measure of TPASK are analyzed it is seen that Clip-Air’s productivity is always higher for the allocated capacity compared to standard fleet.

As mentioned previously, in the set of flights there are mandatory flights which need to be served. Our dataset does not include information about the mandatory flights and to be able to represent the schedule design decision we randomly select a percentage of the flights to be mandatory. In this instance 43.75% of the flights are mandatory. The model with standard fleet becomes unfeasible when capacity is decreased further since it can not cover these mandatory flights.

4.6 Sensitivity analysis on the costs of Clip-Air

Since Clip-Air system does not exist yet, sensitivity analysis needs to be carried out for the operating cost of Clip-Air. As mentioned in section 4.1, we adjusted the crew cost, fuel cost, airport and air navigation charges for Clip-Air. Therefore we present

Table 12: Results with varying standard fleet configuration

	Clip-Air	Standard fleet				
		7 plane types	5 plane types	3 plane types	2 plane types	1 plane type
Operating cost	382,483	404,763	421,892	398,832	398,424	298,658
Spill cost	50,264	66,781	63,018	90,856	104,836	233,126
Revenue	622,466	605,949	609,712	581,874	567,894	439,604
Profit	239,983	201,186 (-19%)	187,820 (-28%)	183,042 (-31%)	169,470 (-42%)	140,946 (-70%)
Transported pax.	3,241	3,152 (-3%)	3,170 (-2%)	3,024 (-7%)	2,980 (-9%)	2,216 (-46%)
Flight count	47	47	47	47	47	47
Total flight duration	3,660 min	3,660 min	3,660 min	3,660 min	3,660 min	3,660 min
Used fleet	9 wings 14 capsules	1 A318(123) 1 A319(79) 1 BAE300(100) 2 CRJ100(50) 2 CRJ700(72) 4 ERJ135(37) 1 ERJ145(50)	1 A318(123) 1 A319(79) 1 BAE300(100) 2 CRJ700(72) 6 ERJ145(50)	1 A318(123) 4 A319(79) 6 ERJ145(50)	3 A318(123) 7 ERJ145(50)	9 ERJ145 (50)
Used aircraft	9	12	11	11	10	9
Used seats	700	744	746	739	719	450
ASK	36,295,000	38,576,400	38,680,100	38,317,150	37,280,150	23,332,500
TPASK ($\times 10^{-5}$)	8.93	8.17	8.20	7.89	7.99	9.50

Table 13: Data instance for the tests with different available capacity

Airports	5
Flights	108
Flights/route	6.75
Capsule capacity	42
Passengers	8,370
Itineraries	200
Standard fleet types	A318(123), A319(79), BAE200(94), BAE300(100), CRJ100(50), CRJ700(72), ERJ135(37), ERJ145(50), F100(100)

an analysis regarding these cost figures. Fuel cost, airport and air navigation charges are analyzed with the cases of 10% lower and higher values compared to the reference values we have initially used. Regarding the crew cost, we analyze the sensitivity of the results to the percentage of the flight crew cost. We consider the cases where flight crew constitutes the 50%, 60% and 70% of the total crew cost.

The analysis is carried for the same data instance used for the analysis of the effect of transportation capacity in section 4.5. The results in Table 15 are presented in comparison to the results for standard fleet given in Table 14 for the case of unlimited capacity.

It is observed that scheduling decision is the same for almost all of the cases having 19 assigned aircraft and allocating 33.73%-40.70% less seats compared to the standard fleet. This is a good indicator which says that our model is robust in the analyzed range and the general conclusions will remain similar in case we are provided with better estimates of the cost figures of Clip-Air.

The number of transported passengers is higher for Clip-Air for all the analyzed cases and the range of this increase is between 0.62%-2.24%. The highest increase in profit is 7.48% and for 89% of the cases Clip-Air is making more profit than the standard fleet. A decrease in profit is only observed when all the cost figures are in favor of standard fleet such that the fuel cost, airport and air navigation charges are high and the flight crew percentage is low. These cases are highlighted in the table.

It is observed that both the increase in the fuel cost and the increase in airport and air navigation charges decrease the profit as expected. However the total number of transported passengers is not considerably affected by the change of the costs. When the percentage of the flight crew cost increases, Clip-Air uses the advantage of the decoupling of wing and capsules and reduces the crew cost considerably. Although the number of carried passengers is not highly affected, it is increased when the flight crew percentage is high.

It can be concluded that the fleet assignment decisions are kept the same for the given range of the analyzed parameters. Furthermore, crew cost and fuel cost are more critical compared to airport and air navigation charges in terms of the profit and the

Table 14: Results with varying available capacity
Clip-Air

	Unlimited	1470 seats	1260 seats	1050 seats	840 seats
Operating cost	1,086,607	1,053,590	980,511	892,351	781,960
Spill cost	126,994	167,461	268,087	414,079	614,595
Revenue	1,893,918	1,853,451	1,752,825	1,606,833	1,406,317
Profit	807,311	799,861	772,314	714,482	624,357
Transported pax.	7,677	7,508	7,018	6,294	5,336
Flight count	104	105	99	95	90
Total flight duration	7,965	8,015	7,545	7,245	6,885
Used fleet	19 wings 40 capsules	18 wings 35 capsules	17 wings 30 capsules	18 wings 25 capsules	18 wings 20 capsules
Used wings	19	18	17	18	18
Used seats	1,680	1,470	1,260	1,050	840
ASK	189,567,000	166,912,375	134,678,250	107,769,375	81,931,500
Pax. per ASK ($\times 10^{-5}$)	4.05	4.50	5.21	5.84	6.51
Standard Fleet					
	Unlimited	1470 seats	1260 seats	1050 seats	840 seats
Operating cost	1,090,695	1,041,703	972,790		
Spill cost	141,268	214,417	310,584		
Revenue	1,879,644	1,806,495	1,710,328		
Profit	788,949 (-2.3%)	764,792 (-4.6%)	737,538 (-4.7%)		
Transported pax.	7,589(-1.2%)	7,254 (-3.5%)	6,773 (-3.6%)		
Flight count	106	105	103	<i>Unfeasible</i>	<i>Unfeasible</i>
Total flight duration	8,105	8,010	7,875		
Used aircraft	33	19	17		
Used seats	2,833	1,466	1,256		
ASK	325,287,421	166,354,350	140,122,500		
TPASK ($\times 10^{-5}$)	2.33	4.36	4.83		

Table 15: Sensitivity analysis for the cost figures of Clip-Air

		Fuel cost	-10%			-			+10%		
		Flight crew %	50%	60%	70%	50%	60%	70%	50%	60%	70%
airport & air navi- gation charges	-10%	Profit	+5.24%	+5.99%	+7.48%	+3.00%	+3.73%	+5.22%	+0.76%	+1.49%	+2.97%
		Transported pax.	+1.98%	+1.88%	+2.24%	+0.62%	+1.16%	+1.16%	+0.62%	+0.62%	+1.16%
	-	Profit	+3.83%	+4.58%	+6.06%	+1.60%	+2.33%	+3.81%	-0.64%	+0.09%	+1.56%
		Transported pax.	+1.88%	+1.16%	+1.16%	+0.62%	+1.16%	+1.16%	+0.62%	+0.62%	+1.16%
	+10%	Profit	+2.44%	+3.17%	+4.66%	+0.20%	+0.93%	+2.41%	-2.04%	-1.31%	+0.16%
		Transported pax.	+0.62%	+1.16%	+1.16%	+0.62%	+0.62%	+1.16%	+0.62%	+0.62%	+1.16%

number of transported passengers, although there is not a significant effect on the scheduling decisions.

5 Conclusions and future directions

In this paper we present a comparative analysis of airline operations between a new flexible transportation system called Clip-Air, and an existing standard configuration. For this purpose an integrated schedule design and fleet assignment model is developed for both Clip-Air and a fleet with standard planes. The model considers spill and recapture effects to better represent the reality. The recapture ratios between itineraries are defined based on an itinerary choice model explained by fare price, number of stops and departure time of day.

Since the Clip-Air system does not exist yet, the estimation of the cost is based on reasonable assumptions. In order to perform a conservative comparison, our scenarios include some advantages for the standard fleet compared to Clip-Air. For instance, we do not allow Clip-Air to use different types of capsules, while the standard fleet can rely on different plane types.

Different scenarios are analyzed to quantify the performance of Clip-Air. The scenarios are designed to test the effects of the network type, fleet size, fleet configuration and the estimated cost of the Clip-Air system. In all analyzed cases, Clip-Air is found to carry more passengers allocating less capacity compared to the standard fleet. This is supported by the high TPASK measures which means that Clip-Air uses the available capacity more efficiently than the standard fleet.

As mentioned previously cost estimation for Clip-Air system is based on various assumptions. Therefore a sensitivity analysis is presented for crew cost, fuel cost and airport and air navigation charges. It is seen that scheduling decisions are not sensitive to the cost in the range of our analysis. Clip-Air is found to always perform better in terms of the number of carried passengers and generates a higher profit in 89% of the instances.

The overall results show that Clip-Air has a significant potential for an efficient use of the capacity, as well as an increase of the airline profits. The conservative nature of the scenarios and the sensitivity analysis suggest that these reported improvements will be outperformed by a real implementation of the system.

The Clip-Air concept opens the door to a wide range of new research opportunities. For instance, a standardization of the Clip-Air capsule would give a multimodal dimension to the system. The capsules could be carried on railways and on trucks, allowing passengers to board outside of the airport. Since the capsules are of simple structure, storage and transfer of them is relatively easy. We believe that the repositioning possibility will increase the flexibility of Clip-Air and help to show more clearly how it can adapt to different situations of the capacity and demand. Moreover, the modularity of Clip-Air allows to have freight and passenger loaded capsules on the same flight which

opens up new frontiers to mixed passenger and cargo transportation. Furthermore, it is more realistic for an airline company to have part of the fleet composed of Clip-Air wings and capsules in the initial phase of the modification of the fleet. Therefore, a model with mixed fleet is crucial to see what types of aircraft should be replaced by Clip-Air. A dynamic business plan for companies can be obtained with the inclusion of the fixed cost for the purchase of the Clip-Air wings and capsules. Furthermore, a business model where the companies operating the wings are different from the companies operating the capsules should be analyzed.

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