Modeling advanced disaggregate demand as MILP

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February 2, 2018
I want to open a bar
I want to open a bar

But there is a strong competition...

- La Cour St-Jean
- Le Mad Murphy
- Le Lausanne Express
- La Guimbarde
- ...

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I want to open a bar

But there is a strong competition...

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To be successful...

...I will use Operations Research to optimize my business.
Aggregate demand
Demand analysis

Aggregate demand

- 22000 students in the University of Liège
Demand analysis

Aggregate demand

- 22000 students in the University of Liège
- each student drinks 4.25L of beer per week (source: DH.be)
Demand analysis

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- 45 bars in the “Carré”
Demand analysis

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- each student drinks 4.25L of beer per week (source: DH.be)
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- I should sell about 2000 liters of beer per week
Aggregate demand

- 22000 students in the University of Liège
- each student drinks 4.25L of beer per week (source: DH.be)
- 45 bars in the “Carré”
- I should sell about 2000 liters of beer per week
- Jupiler 25cl at 4€: total revenues = 32000€ per week.
Decisions

Assortment
Decisions

Assortment
Assortment
Decisions

Assortment

- Jupiler
- Leffe
- Orval
Decisions

Assortment and prices

Jupiler: 4 €
Leffe: 6 €
Orval: 8 €
Customers are different
Customers are different

Mathematics
Customers are different

Mathematics

HEC
Disaggregate demand analysis
Customers behavior

- Customers have different tastes
- Customers have different willingness to pay
Disaggregate demand analysis

Customers behavior
- Customers have different tastes
- Customers have different willingness to pay

Customers choice
Outline

1. Choice models
2. MILP
3. Outlook
Variables: $x_{in} = (p_{in}, z_{in}, s_n)$

Attributes of alternative $i$: $z_{in}$
- Price ($p_{in}$)
- Brand
- Color
- Percentage of alcohol
- etc.

Characteristics of customer $n$: $s_n$
- Income
- Age
- Sex
- Type of student
- etc.
Choice models

Behavioral assumptions

Choice set: $C_n$

$y_{in} = 1$ if $i \in C_n$, 0 otherwise
Behavioral assumptions

Choice set: $C_n$

$y_{in} = 1$ if $i \in C_n$, 0 otherwise

Utility function

$U_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$
Behavioral assumptions

Choice set: $C_n$

$y_{in} = 1$ if $i \in C_n$, 0 otherwise

Utility function

$$U_{in} = \sum_{k} \beta_k x_{ink} + \varepsilon_{in}$$

Choice

$$P_n(i|x; C_n) = \Pr(U_{in} \geq U_{jn})$$
Choice models

Logit model

\[
U_{in} = \sum_{k} \beta_k x_{ink} + \varepsilon_{in} = V_{in} + \varepsilon_{in}
\]

\[
P_n(i|x; C_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j \in C} y_{jn}e^{V_{jn}}}
\]
Demand curve

Disaggregate model

\[ P_n(i|p_{in}, z_{in}, s_n) \]

Total demand

\[ D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n) \]

Difficulty

Non linear and non convex in \( p_{in} \) and \( z_{in} \)
Example

**Choice set:** Jupiler
- Lausanne Express $i = 0$
- La Cour St-Jean $i = 1$

**Utility functions**

\[
V_{0n} = -2.2p_0 - 1.3
\]
\[
V_{1n} = -2.2p_1
\]

**Prices**
- Lausanne Express: $[0 - 6 \, \text{€}]$
- La Cour St-Jean: 1.8 €
Demand and revenues

![Graph showing demand and revenues over price]

- **Demand**: The graph plots the choice probability against price. The demand curve peaks around a price of 2 and decreases as price increases, indicating a diminishing demand with higher prices.

- **Revenues**: The revenues are shown as a function of price, likely indicating that revenues increase up to a certain point and then begin to decline as prices rise, due to the drop in demand.

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Disaggregate demand as MILP
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Heterogeneous population

Two groups in the population

\[ V_{0n} = -\beta_n p_0 + c_0 \]

Mathematics: 25%
\[ \beta_1 = -4.5, \]
\[ c_1 = -1.3 \]

Business: 75%
\[ \beta_2 = -0.25, \]
\[ c_2 = -1.3 \]
Demand per market segment

```
<table>
<thead>
<tr>
<th>Price</th>
<th>Demand math</th>
<th>Demand business</th>
<th>Total demand</th>
</tr>
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<td>0.0</td>
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Choice models

Demand and revenues

![Graph showing choice probability and revenues over price]

Disaggregate demand as MILP

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Optimization

Pricing

- Non linear optimization problem.
- Non convex objective function.
- Multimodal function.
- May feature many local optima.
- In practice, the groups are many, and interdependent.
- Optimizing each group separately is not feasible.
Choice models

Optimization

Pricing
- Non linear optimization problem.
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Assortment
What about assortment?
Heterogeneous population, two products

Utility functions: math

\[ V_{LE, Jupiler, m} = -4.5 p_{LE, Jupiler} - 1.3 \]
\[ V_{LE, Orval, m} = -4.5 p_{LE, Orval} - 1.3 + 3 \]
\[ V_{CSJ, Jupiler, m} = -4.5 p_{CSJ, Jupiler} \]
\[ V_{CSJ, Orval, m} = -4.5 p_{CSJ, Orval} + 3 \]

Utility functions: HEC

\[ V_{LE, Jupiler, b} = -0.25 p_{LE, Jupiler} - 1.3 \]
\[ V_{LE, Orval, b} = -0.25 p_{LE, Orval} - 1.3 + 1 \]
\[ V_{CSJ, Jupiler, b} = -0.25 p_{CSJ, Jupiler} \]
\[ V_{CSJ, Orval, b} = -0.25 p_{CSJ, Orval} + 1 \]
**Total revenues**

![Graph showing total revenues for Price Jupiler, Revenues Jupiler, Revenues Orval, and Total revenues.](image-url)
Orval only

Revenues Orval

Price Jupiler

Revenues
Optimization

Assortment and pricing

- Combinatorial problem
- For each potential assortment, solve a pricing problem
- Select the assortment corresponding to the highest revenues
- MINLP
- Non convex relaxation
### Disaggregate demand models

#### Advantages
- Theoretical foundations
- Market segmentation
- Taste heterogeneity
- Many variables
- Estimated from data

#### Disadvantages
- Complex mathematical formulation
- Not suited for optimization
- Literature: heuristics
Research objectives

Observations
- Revenues is not the only indicator to optimize,
- e.g. customer satisfaction.
- Many OR applications need a demand representation

Goal
- Generic mathematical representation of choice models,
- designed to be included in MILP,
- linear in the decision variables.
Outline

1. Choice models
2. MILP
3. Outlook
The main idea
The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.
The main idea

Linearization

• Hopeless to linearize the logit formula (we tried...)
• Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem
The main idea

Linearization
- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles
Each customer solves an optimization problem

Solution
Use the utility and not the probability
A linear formulation

Utility function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}. \]

Simulation

- Assume a distribution for \( \varepsilon_{in} \)
- E.g. logit: i.i.d. extreme value
- Draw \( R \) realizations \( \xi_{inr} \), \( r = 1, \ldots, R \)
- The choice problem becomes deterministic
Scenarios

Draws

- Draw $R$ realizations $\xi_{inr}$, $r = 1, \ldots, R$
- We obtain $R$ scenarios

\[ U_{inr} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \xi_{inr}. \]

- For each scenario $r$, we can identify the largest utility.
- It corresponds to the chosen alternative.
Demand may exceed supply
Each alternative $i$ can be chosen by maximum $c_i$ individuals.
An exogenous priority list is available.
Can be randomly generated, or according to some rules.
The numbering of individuals is consistent with their priority.
Choice set

Variables

\[ y_i \in \{0, 1\} \quad \text{operator decision} \]
\[ y_{in}^d \in \{0, 1\} \quad \text{customer decision (data)} \]
\[ y_{in} \in \{0, 1\} \quad \text{product of decisions} \]
\[ y_{inr} \in \{0, 1\} \quad \text{capacity restrictions} \]

Constraints

\[ y_{in} = y_{in}^d y_i \quad \forall i, n \]
\[ y_{inr} \leq y_{in} \quad \forall i, n, r \]
Utility

Variables

\[ U_{inr} \]

\[ z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases} \]

(\( \ell_{nr} \) smallest lower bound)

Constraint: utility

\[ U_{inr} = \beta_{in} p_{in} + q_d(x_d) + \xi_{inr} \quad \forall i, n, r \]
Utility (ctd)

Constraints: discounted utility

\[
\begin{align*}
l_{nr} & \leq z_{inr} & \forall i, n, r \\
z_{inr} & \leq l_{nr} + M_{inr} y_{inr} & \forall i, n, r \\
U_{inr} - M_{inr}(1 - y_{inr}) & \leq z_{inr} & \forall i, n, r \\
z_{inr} & \leq U_{inr} & \forall i, n, r
\end{align*}
\]
Choice

Variables

\[ U_{nr} = \max_{i \in C} z_{inr} \]

\[ w_{inr} = \begin{cases} 
1 & \text{if } z_{inr} = U_{nr} \\
0 & \text{otherwise}
\end{cases} \]

Constraints

\[ z_{inr} \leq U_{nr} \quad \forall i, n, r \]

\[ U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \quad \forall i, n, r \]

\[ \sum_{i} w_{inr} = 1 \quad \forall n, r \]

\[ w_{inr} \leq y_{inr} \quad \forall i, n, r \]
Capacity cannot be exceeded ⇒ $y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr}) \quad \forall i > 0, n > c_i, r$$

Capacity has been reached ⇒ $y_{inr} = 0$

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}, \quad \forall i > 0, n, r$$
A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.
A case study

Challenge
- Take a choice model from the literature.
- It cannot be logit.
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Parking choice
- [Ibeas et al., 2014]
Parking choices [Ibeas et al., 2014]

Alternatives
- Paid on-street parking
- Paid underground parking
- Free street parking

Model
- $N = 50$ customers
- $C = \{\text{PSP, PUP, FSP}\}$
- $C_n = C \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models
General experiments

Uncapacitated vs Capacitated case
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price
Uncapacitated vs Capacitated case

Uncapacitated

![Graph showing Price, Demand, PSP, and PUP for Uncapacitated case]

Capacitated

![Graph showing Price, Demand, PSP, and PUP for Capacitated case]
## Computational time

<table>
<thead>
<tr>
<th>$R$</th>
<th><strong>Uncapacitated case</strong></th>
<th></th>
<th><strong>Capacitated case</strong></th>
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<td>PUP</td>
<td>Rev</td>
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<td>2.58 s</td>
<td>0.54</td>
<td>0.79</td>
<td>26.43</td>
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<tr>
<td>10</td>
<td>3.98 s</td>
<td>0.53</td>
<td>0.74</td>
<td>26.36</td>
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<tr>
<td>25</td>
<td>29.2 s</td>
<td>0.54</td>
<td>0.79</td>
<td>26.90</td>
</tr>
<tr>
<td>50</td>
<td>4.08 min</td>
<td>0.54</td>
<td>0.75</td>
<td>26.97</td>
</tr>
<tr>
<td>100</td>
<td>20.7 min</td>
<td>0.54</td>
<td>0.74</td>
<td>26.90</td>
</tr>
<tr>
<td>250</td>
<td>2.51 h</td>
<td>0.54</td>
<td>0.74</td>
<td>26.85</td>
</tr>
</tbody>
</table>
Outline

1. Choice models
2. MILP
3. Outlook
Linear formulation of choice models

Generic framework

- Not only logit: any choice model.
- Choice models from the literature can be used as such.
- Disaggregate: the choice of every individual for every draw is available.
- Many indicators can be derived.

Challenges

- Large scale
- Simulation noise
- Additional linearization may be necessary (e.g. revenue = \( p \cdot w \))
Linear formulation of choice models

Opportunities: decomposition methods
- Lagrangian relaxation
- Decomposable by individual
- Decomposable by draw

Future work
- Game theory
- Parameter estimation (discrete maximum likelihood)
- Link with machine learning (SVM, random forests, etc.)
Thank you!
Thank you!

Key contributors

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Introduction to discrete choice models
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Merci
Dank u wel
Danke schön