Incorporating behavioral model into transport optimization

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Demand and supply

1. Demand and supply
2. Disaggregate demand models
3. A simple example
4. A generic framework
5. MILP
6. Conclusion
Mobility as a service

Demand orientation [Jittrapirom et al., 2017]

- User-centric paradigm
- Best from customer’s perspective
- Demand responsive

Personalization

- Every user has different needs
- Tailor-made solutions
- Social network
Mobility as a service

Key challenges [Jittrapirom et al., 2017]
- Demand-side modeling
- Supply-side modeling
- Governance and business model to match supply and demand
Outline

1. Demand and supply
2. Disaggregate demand models
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5. MILP
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Choice models

Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models
Choice models

Theoretical foundations

- Random utility theory
- Choice set: $C_n$
- $y_{in} = 1$ if $i \in C_n$, 0 if not
- Logit model:

$$P(i|C_n) = \frac{y_{in}e^{v_{in}}}{\sum_{j \in C} y_{jn}e^{v_{jn}}}$$
Logit model

Utility

\[ U_{in} = V_{in} + \varepsilon_{in} \]

- Decision-maker \( n \)
- Alternative \( i \in C_n \)

Choice probability

\[ P_n(i|C_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j \in C} y_{jn}e^{V_{jn}}} . \]
Variables: \( x_{in} = (p_{in}, z_{in}, s_n) \)

Attributes of alternative \( i \): \( z_{in} \)
- Cost / price \( (p_{in}) \)
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker \( n \): \( s_n \)
- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.
Demand curve

Disaggregate model

$$P_n(i|p_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in $p_{in}$ and $z_{in}$
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Example

Choice set: Jupiler
- ’t Klooster $i = 0$
- Belvédère $i = 1$

Utility functions

\[
V_{0n} = -2.2p_0 - 1.3 \\
V_{1n} = -2.2p_1
\]

Prices

- ’t Klooster: [0 – 6 €]
- Belvédère: 1.8 €
Demand and revenues

![Graph showing demand and revenues](image)

- **Demand**
- **Revenues**

Choice probability vs. Price

- **Price** range: 0 to 6
- **Demand** curve:
  - Peak at around 1.5
- **Revenues** curve:
  - Higher values at lower prices

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Behavioral models and optimization

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Heterogeneous population

Two groups in the population

$$V_{0n} = -\beta_n p_0 + c_0$$

Mathematics: 25%

$$\beta_1 = -4.5, \quad c_1 = -1.3$$

Business: 75%

$$\beta_2 = -0.25, \quad c_2 = -1.3$$
Demand per market segment

A simple example

Choice probability vs Price

Demand math
Demand business
Total demand

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Demand and revenues

A simple example

Revenues
Demand

Price

Choice probability

Revenues

Demand

0 1 2 3 4 5 6

0 0.2 0.4 0.6 0.8 1

0 0.2 0.4 0.6 0.8 1
Optimization

Pricing

- Non linear optimization problem.
- Non convex objective function.
- Multimodal function.
- May feature many local optima.
- In practice, the groups are many, and interdependent.
- Optimizing each group separately is not feasible.
Optimization

Pricing

- Non linear optimization problem.
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Assortment

What about assortment?
Heterogeneous population, two products

Utility functions: math

\[ V_{K, \text{Jupiler}, m} = -4.5p_{K, \text{Jupiler}} - 1.3 \]
\[ V_{K, \text{Orval}, m} = -4.5p_{K, \text{Orval}} - 1.3 + 3 \]
\[ V_{B, \text{Jupiler}, m} = -4.5p_{B, \text{Jupiler}} \]
\[ V_{B, \text{Orval}, m} = -4.5p_{B, \text{Orval}} + 3 \]

Utility functions: HEC

\[ V_{K, \text{Jupiler}, b} = -0.25p_{K, \text{Jupiler}} - 1.3 \]
\[ V_{K, \text{Orval}, b} = -0.25p_{K, \text{Orval}} - 1.3 + 1 \]
\[ V_{B, \text{Jupiler}, b} = -0.25p_{B, \text{Jupiler}} \]
\[ V_{B, \text{Orval}, b} = -0.25p_{B, \text{Orval}} + 1 \]
A simple example

Total revenues

Revenues Jupiler
Revenues Orval
Total revenues

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In transportation

Assortment and pricing

- Airlines
- Deregulated railways
- Mobility as a service
Assortment and pricing

- Combinatorial problem
- For each potential assortment, solve a pricing problem
- Select the assortment corresponding to the highest revenues
- MINLP
- Non convex relaxation
Disaggregate demand models

Advantages
- Theoretical foundations
- Market segmentation
- Taste heterogeneity
- Many variables
- Estimated from data

Disadvantages
- Complex mathematical formulation
- Not suited for optimization
- Literature: heuristics
Research objectives

Observations
- Revenues is not the only indicator to optimize,
  e.g. customer satisfaction.
- Many transportation applications need a demand representation.

Goal
- Generic mathematical representation of choice models,
  designed to be included in MILP,
  linear in the decision variables.
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The main idea
The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.
The main idea

Linearization
- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles
Each customer solves an optimization problem
The main idea

Linearization
- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles
Each customer solves an optimization problem

Solution
Use the utility and not the probability
A linear formulation

Utility function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}. \]

Simulation

- Assume a distribution for \( \varepsilon_{in} \)
- E.g. logit: i.i.d. extreme value
- Draw \( R \) realizations \( \xi_{inr}, r = 1, \ldots, R \)
- The choice problem becomes deterministic
Draws

- Draw $R$ realizations $\xi_{inr}$, $r = 1, \ldots, R$
- We obtain $R$ scenarios

$$U_{inr} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$ 

- For each scenario $r$, we can identify the largest utility.
- It corresponds to the chosen alternative.
MILP (in words)

\[
\begin{align*}
\text{MILP} & \\
\text{max} \quad & \text{benefit} \\
\text{subject to} \quad & \text{utility definition} \\
& \text{availability} \\
& \text{discounted utility} \\
& \text{choice} \\
& \text{capacity allocation} \\
& \text{price selection}
\end{align*}
\]
A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.
A case study

Challenge

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Parking choice

• [Ibeas et al., 2014]
Parking choices [Ibeas et al., 2014]

Alternatives
- Paid on-street parking
- Paid underground parking
- Free street parking

Model
- $N = 50$ customers
- $C = \{\text{PSP, PUP, FSP}\}$
- $C_n = C \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models
General experiments

Uncapacitated vs Capacitated case
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price
Uncapacitated vs Capacitated case

Uncapacitated

Capacitated
# Computational time

<table>
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<th>$R$</th>
<th><strong>Uncapacitated case</strong></th>
<th></th>
<th><strong>Capacitated case</strong></th>
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<td>PSP</td>
<td>PUP</td>
<td>Rev</td>
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<tr>
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<tr>
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<td>20.7 min</td>
<td>0.54</td>
<td>0.74</td>
<td>26.90</td>
</tr>
<tr>
<td>250</td>
<td>2.51 h</td>
<td>0.54</td>
<td>0.74</td>
<td>26.85</td>
</tr>
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</table>
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Summary

Demand and supply
- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models
- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models
Optimization

Discrete choice models
- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation
- Linear in the decision variables
- Large scale
- Fairly general
Ongoing research

- Decomposition methods.
- Competitive markets: several suppliers.