Discrete choice models with latent classes and variables

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Behavioral framework

Motivation, affect

Attitudes

Information, Knowledge

Perceptions

Preferences

Choice set

Process

Choice
Utility theory

Homo Economicus (source: D. McFadden)

Jeremy Bentham (1789) My notion of man is that ... he aims at **happiness** ... in every thing he does.

Frank Taussig (1912) The fact that [the consumer] is willing to give up something in order to procure an article proves once for all that for him it has **utility**

Herb Simon (1956) The rational man of economics is a **maximizer**, who will settle for nothing less than the best.
Standard discrete choice models

- Attributes
- Utility
- Choice set
- Choice
Motivation

- Standard random utility assumptions are usually violated
- Factors such as attitudes, perceptions, knowledge are not reflected
Example: pain lovers


- Short trial: immerse one hand in water at 14° for 60 sec.
- Long trial: immerse the other hand at 14° for 60 sec, then keep the hand in the water 30 sec. longer as the temperature of the water is gradually raised to 15°.
- Outcome: most people prefer the long trial.
- Explanation:
  - duration plays a small role
  - the peak and the final moments matter
Example: *The Economist*

Example: subscription to *The Economist*

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Web only</td>
<td>@ $59</td>
<td></td>
</tr>
<tr>
<td>Print only</td>
<td>@ $125</td>
<td></td>
</tr>
<tr>
<td>Print and web</td>
<td>@ $125</td>
<td></td>
</tr>
</tbody>
</table>
Example: *The Economist*

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<table>
<thead>
<tr>
<th>Experiment 1</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Web only @ $59</td>
<td>Web only @ $59</td>
<td>68</td>
</tr>
<tr>
<td>0</td>
<td>Print only @ $125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>Print and web @ $125</td>
<td>Print and web @ $125</td>
<td>32</td>
</tr>
</tbody>
</table>

Source: Ariely (2008)

- Dominated alternative
- According to utility maximization, should not affect the choice
- But it affects the perception, which affects the choice.
### Example: good or bad wine?

Choose a bottle of wine...

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>McFadden red at $10</td>
</tr>
<tr>
<td>2</td>
<td>Nappa red at $12</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Most would choose 2</td>
<td>Most would choose 1</td>
</tr>
</tbody>
</table>

- Context plays a role on perceptions
Example: live and let die

Population of 600 is threatened by a disease. Two alternative treatments to combat the disease have been proposed.

<table>
<thead>
<tr>
<th>Experiment 1</th>
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</tr>
</thead>
<tbody>
<tr>
<td># resp. = 152</td>
<td># resp. = 155</td>
</tr>
<tr>
<td>Treatment A:</td>
<td>Treatment C:</td>
</tr>
<tr>
<td>200 people saved</td>
<td>400 people die</td>
</tr>
<tr>
<td>Treatment B:</td>
<td>Treatment D:</td>
</tr>
</tbody>
</table>
| 600 people saved with prob. 1/3
0 people saved with prob. 2/3 |
| 0 people die with prob. 2/3 |
| 600 people die with prob. 2/3 |
Example: live and let die

Population of 600 is threatened by a disease. Two alternative treatments to combat the disease have been proposed.

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| 72% | Treatment A: 200 people saved | Treatment C: 400 people die | 22% |
| 28% | Treatment B: 600 people saved with prob. 1/3 0 people saved with prob. 2/3 | Treatment D: 0 people die with prob. 1/3 600 people die with prob. 2/3 | 78% |

Source: Tversky & Kahneman (1986)
Example: to be free

Choice between a fine and a regular chocolate

<table>
<thead>
<tr>
<th></th>
<th>Experiment 1</th>
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<tbody>
<tr>
<td>Lindt</td>
<td>$0.15</td>
<td>$0.14</td>
</tr>
<tr>
<td>Hershey</td>
<td>$0.01</td>
<td>$0.00</td>
</tr>
<tr>
<td>Lindt chosen</td>
<td>73%</td>
<td>31%</td>
</tr>
<tr>
<td>Hershey chosen</td>
<td>27%</td>
<td>69%</td>
</tr>
</tbody>
</table>

Latent concepts

- **latent**: potentially existing but not presently evident or realized (from old French: hidden)
- Here: not directly observed
- Standard models are already based on a latent concept: utility

Drawing convention:

- Latent variable
- Observed variable
- structural relation: 
- measurement: 
- errors: ……
Random utility

\[ V_{in} = \sum_k \beta_{ik} x_{ikn} \]

\[ P_n(i) = \frac{e^{V_{in}}}{\sum_j e^{V_{jn}}} \]
Attitudes

• Psychometric indicators
• Example: attitude towards the environment.
• For each question, response on a scale: strongly agree, agree, neutral, disagree, strongly disagree, no idea.
  • The price of oil should be increased to reduce congestion and pollution
  • More public transportation is necessary, even if it means additional taxes
  • Ecology is a threat to minorities and small companies.
  • People and employment are more important than the environment.
  • I feel concerned by the global warming.
  • Decisions must be taken to reduce the greenhouse gas emission.
Indicators

Indicators cannot be used as explanatory variables. Mainly two reasons:

1. Measurement errors
   - Scale is arbitrary and discrete
   - People may overreact
   - Justification bias may produce exaggerated responses

2. No forecasting possibility
   - No way to predict the indicators in the future
Factor analysis

\[ I_i = \lambda_i + \sum_k L_{ik} X_k^* \]
Measurement equation

Explanatory variables

$\varepsilon_i \rightarrow I_i = \lambda_i + \sum_k L_{ik} X^*_k$

Latent variables $X^*$

$X^*_k = \sum_j \beta_j x_j$

Indicators
Measurement equation

Continuous model: regression

\[ I = f(X^*; \beta) + \varepsilon \]

Discrete model: thresholds

\[ I = \begin{cases} 
1 & \text{if } -\infty < X^* \leq \tau_1 \\
2 & \text{if } \tau_1 < X^* \leq \tau_2 \\
3 & \text{if } \tau_2 < X^* \leq \tau_3 \\
4 & \text{if } \tau_3 < X^* \leq \tau_4 \\
5 & \text{if } \tau_4 < X^* \leq +\infty 
\end{cases} \]
Choice model

- Explanatory variables
- Latent variables
- Utility
- Choice
- Indicators

$\varepsilon_{in}$ $\omega_{in}$
Case study: value of time

- Effect of attitude on value of time
- SP survey, Stockholm, Sweden, 2005
- 2400 households surveyed
- Married couples with both husband and wife working or studying
- Choice between car alternatives
- Data used: 554 respondents, 2216 SP responses
- Attributes:
  - travel time
  - travel cost
  - number of speed cameras
Attitudinal questions

- It feels safe to go by car.
- It is comfortable to go by car to work.
- It is very important that traffic speed limits are not violated.
- Increase the motorway speed limit to 140 km/h.

Likert scale:
- 1: do not agree at all
- 5: do fully agree
Structural models

Attitude model, capturing the positive attitude towards car

\[
\text{Attitude} = \theta_0 \cdot 1 \quad \text{(intercept)} \\
+ \theta_f \cdot \text{female} \\
+ \theta_{\text{inc}} \cdot \text{income} \quad \text{(monthly, in Kronas)} \\
+ \theta_{\text{age}1} \cdot (\text{Age} < 55) \\
+ \theta_{\text{age}2} \cdot (\text{Age} 55–65) \\
+ \theta_{\text{age}3} \cdot (\text{Age} > 65) \\
+ \theta_{\text{edu}1} \cdot (\text{basic/pre high school}) \\
+ \theta_{\text{edu}2} \cdot (\text{university}) \\
+ \theta_{\text{edu}3} \cdot (\text{other}) \\
+ \sigma \cdot \omega \quad \text{(normal error term)}
\]
Structural models

Choice model: 3 alternatives

- Car on route 1
- Car on route 2
- Indifferent (utility = 0)

\[
\text{Utility}_i = \beta_i + \beta_t \cdot \text{travel time}_i + \beta_c \cdot \frac{\text{cost}_i}{\text{Income}} + \gamma \cdot \text{cost}_i \cdot \text{Attitude} / \text{Income} + \beta_{\text{cam}} \cdot \# \text{ cameras}_i + \varepsilon_i
\]

(ASC) (EV error term)

Note: standard model obtained with $\gamma = 0$. 
Value of time

- Model without attitude variable ($\gamma = 0$)

$$VOT = \frac{\beta_t}{\beta_c} \cdot Income$$

- Model with attitude variable

$$VOT = \frac{\beta_t}{\beta_c + \gamma \cdot Attitude} \cdot Income$$

Note: distributed
Measurement equations

- **Choice:**
  \[
  y_i = \begin{cases} 
  1 & \text{if } U_i \geq U_j, j \neq i \\
  0 & \text{otherwise}
  \end{cases}
  \]

- **Attitude questions:** \( k = 1, \ldots, 4 \)
  \[
  I_k = \alpha_k + \lambda_k \text{Attitude} + \mu_k
  \]
  where \( I_k \) is the response to question \( k \).
Model estimation

- Simultaneous estimation of all parameters
- with Biogeme 2.0
- Important: both the choice and the indicators reveal something about the attitude.
Measurement equations

- It feels safe to go by car.

\[ I_1 = \text{Attitude} + 0.5666 \, \nu_1 \]

- It is comfortable to go by car to work.

\[ I_2 = 1.13 + 0.764 \, \text{Attitude} + 0.909 \, \nu_2 \]

- It is very important that traffic speed limits are not violated.

\[ I_3 = 3.53 - 0.0716 \, \text{Attitude} + 1.25 \, \nu_3 \]

- Increase the motorway speed limit to 140 km/h.

\[ I_4 = 1.94 + 0.481 \, \text{Attitude} + 1.37 \, \nu_4 \]
### Structural model

**Attitude towards car:**

<table>
<thead>
<tr>
<th>Param.</th>
<th>Estim.</th>
<th>$t$-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>5.25</td>
<td>8.99</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>-0.0185</td>
<td>-0.34</td>
</tr>
<tr>
<td>$\theta_{inc}$</td>
<td>0.0347</td>
<td>1.99</td>
</tr>
<tr>
<td>$\theta_{age1}$</td>
<td>-0.0217</td>
<td>-1.85</td>
</tr>
<tr>
<td>$\theta_{age2}$</td>
<td>0.00797</td>
<td>0.88</td>
</tr>
<tr>
<td>$\theta_{age3}$</td>
<td>0.0231</td>
<td>2.35</td>
</tr>
<tr>
<td>$\theta_{edu1}$</td>
<td>-0.147</td>
<td>-0.94</td>
</tr>
<tr>
<td>$\theta_{edu2}$</td>
<td>-0.252</td>
<td>-5.22</td>
</tr>
<tr>
<td>$\theta_{edu3}$</td>
<td>-0.157</td>
<td>-0.85</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.934</td>
<td>16.18</td>
</tr>
</tbody>
</table>
# Structural model

Utility:

<table>
<thead>
<tr>
<th>Param.</th>
<th>Estim.</th>
<th>$t$-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>4.01</td>
<td>15.58</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>2.84</td>
<td>10.57</td>
</tr>
<tr>
<td>Time</td>
<td>-0.0388</td>
<td>-8.10</td>
</tr>
<tr>
<td>Cost/Income</td>
<td>-2.02</td>
<td>-3.63</td>
</tr>
<tr>
<td>Cost $\cdot$ Attitude/Income</td>
<td>0.265</td>
<td>2.11</td>
</tr>
<tr>
<td>Speed camera</td>
<td>-0.109</td>
<td>-2.75</td>
</tr>
</tbody>
</table>
Value of time

CDF of VOT (full sample)  

CDF of VOT (income group 1)  

CDF of VOT (income group 2)  

CDF of VOT (income group 3)  

CDF of VOT (income group 4)  

CDF of VOT (income group 5)  

CDF of VOT (income group 6)  

CDF of VOT (income group 7)  

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Base model  
Model with latent attitudes
Conclusions

- Attitudes and perceptions can be accounted for in the discrete choice framework
- Latent variables
- Case study in Stockholm
  - Median value of time increases with income
  - Variability of value of time increases with income
References and Acknowledgment
