Integrated berth allocation and yard assignment problem using column generation

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Agenda

1. Introduction
2. Problem Definition
3. Branch and Price
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1 Introduction
   • Berth Allocation Problem
   • Yard Assignment Problem
   • Motivation

2 Problem Definition

3 Branch and Price

4 Results

5 Conclusion
Berth Allocation Problem

Figure: Lacon ltd.'s plan for extension of the Riga’s port, Latvia
Yard Assignment Problem

Figure: Port of Weipa, Queensland, Australia
Motivation

Figure: Vessels queueing at Newcastle port, Australia (queue hits 60 vessels as max)
1. Introduction

2. Problem Definition
   - Input
     - Vessel
     - Port
     - General Data
   - Output

3. Branch and Price

4. Results

5. Conclusion
Input – Vessel

Information
- Number of Vessels
- Arrival Time
- Length
- Draft (omitted)
- Cargo
  - Quantity
  - Cargo Type
Input – Draft Omitted
Input – Port

Information

- Number of Sections
  - Length
  - Draft (omitted)
  - Coordinates
  - Resources
- Number of Cargo Locations
  - Coordinates
  - Neighbouring Locations
Input – General Data

- Time Horizon
- Number of Cargo Types
- Incompatible Cargo Types
- Distances
- Transfer Rate
- Crane Handling Rate
- Bulk Ports (No Containers)
Output

Minimize

- Handling Time + Delay = Service Time
- Parallel Handling
Introduction

Problem Definition

Branch and Price
- Framework
- Initial Solution
- Master Problem
- Sub-Problem
- Branch and Bound

Results

Conclusion
Framework

- **Initial Solution**
- **Column Generation – Lower Bound**
- **Branch and Bound – Optimal Integer Solution**
Initial Solution

**Branch and Price**

Start

Initial Columns

Solve Master Problem

Dual Variables

Pricing Solver

New Columns with Negative Reduced Cost

Integer Solution?

Output Solution

End

Add Current Column And Delete it From Initial List

Is Pool Empty?

Is Current Column Compatible With Pool & column.vessel = i?

i = 0

i < number of vessels

i++

j = 0

j < columns.size

j++

no

no

done

done

yes

yes

End

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Master Problem – Parameters

Parameters

- \( \Omega \) – set of all feasible assignments
- \( \Omega_1 \subset \Omega \) – current pool of columns
- \( c_a \) – cost of assignment \( a \in \Omega (\Omega_1) \)
- \( N = \{1..n\} \) – set of vessels
- \( K = \{1..m\} \) – set of sections
- \( W = \{1..w\} \) – set of cargo types
- \( T = \{1..h\} \) – set of time steps
- \( L = \{1..q\} \) – set of locations
- \( ct_w \) – number of vessels carrying cargo type \( w \)
Master Problem – Parameters

Decision Variables

- $\lambda_a \in (0, 1)$ – 1 if assignment $a$ is selected, 0 otherwise
- $\mu^l_w \in (0, 1)$ – 1 if location $l$ is storing cargo of type $w$, 0 otherwise
- relaxed

Idea

- obtain berth and yard schedule
  - section
    - used by vessel at time
  - location
    - stored cargo
    - used by vessel at time
Master Problem – Parameters

Parameters

\[ A_{ia}^a = \begin{cases} 
1 & \text{if vessel } i \text{ is assigned in assignment } a, \\
0 & \text{otherwise.} 
\end{cases} \]

\[ B_{kat}^a = \begin{cases} 
1 & \text{if section } k \text{ is occupied at time } t \text{ in assignment } a, \\
0 & \text{otherwise.} 
\end{cases} \]

\[ C_{lw}^a = \begin{cases} 
1 & \text{if cargo } w \text{ is stored at location } l \text{ in assignment } a, \\
0 & \text{otherwise.} 
\end{cases} \]

\[ D_{lt}^a = \begin{cases} 
1 & \text{if cargo location } l \text{ is handling assignment } a \text{ at time } t, \\
0 & \text{otherwise.} 
\end{cases} \]
Master Problem – Objective Function

\[ \text{minimize } \sum_{a \in \Omega} c_a \cdot \lambda_a \] (1)
Master Problem – Constraints
All Vessels Served

\[ \sum_{a \in \Omega_1} A^i_a \cdot \lambda_a = 1, \quad \forall i \in N, \]  

(2)
Master Problem

Master Problem – Constraints

Section Occupation

\[ \sum_{a \in \Omega_1} B_{a}^{kt} \cdot \lambda_a \leq 1, \quad \forall k \in K, \forall t \in T, \]  

(3)

Figure: Illustration of Philadelphia Experiment
Master Problem – Constraints

Location Occupation

\[
\sum_{a \in \Omega_1} D_a^{lt} \cdot \lambda_a \leq 1, \quad \forall l \in L, \forall t \in T, \tag{4}
\]
Master Problem – Constraints

One Cargo per Location

\[
\sum_{a \in \Omega_1} C_{a}^{lw} \cdot \lambda_a - c_{tw} \cdot \mu_{w}^{l} \leq 0, \quad \forall l \in L, \forall w \in W, \\
\sum_{w \in W} \mu_{w}^{l} \leq 1, \quad \forall l \in L, 
\]
Master Problem – Constraints
Compatible Neighbours

\[ \mu'_w + \mu'_{\overline{w}} \leq 1, \quad \forall l \in L, \forall \overline{l} \in \overline{L}, \]
\[ \forall w \in W, \forall \overline{w} \in \overline{W}, \quad (7) \]
**Sub-Problem – Parameters**

**Idea**
- run for each vessel separately
- get n columns (one per vessel)

**Sets**
- $K = \{1..m\}$ – set of sections
- $W = \{1..w\}$ – set of cargo types
- $T = \{1..h\}$ – set of time steps
- $L = \{1..q\}$ – set of locations

**Dual Variables**
- $\alpha, \beta_{kt}, \gamma_{lt}, \delta_{lw}$
Sub-Problem – Objective Function

\[\begin{align*}
    \text{minimize} & \quad (c + s - a) - (\alpha + \sum_{k \in K} \sum_{t \in T} \beta_{kt} \cdot \beta_{kt} + \\
    & \sum_{l \in L} \sum_{t \in T} \gamma_{lt} \cdot \gamma_{lt} + \sum_{l \in L} \sum_{w \in W} \delta_{lw} \cdot \delta_{lw}) \\
\end{align*}\]  

(8)

Parameters

- \( a \) – arrival time

Decision Variables

- \( c \geq 0 \) – handling time
- \( s \geq 0 \) – start time of service
- related to duals:
  - \( \beta_{kt} \in (0, 1) \) – 1 if vessel occupies section \( k \) at time \( t \), 0 otherwise
  - \( \gamma_{lt} \in (0, 1) \) – 1 if vessel uses location \( l \) at time \( t \), 0 otherwise
  - \( \delta_{lw} \in (0, 1) \) – 1 if cargo type \( w \) is stored at location \( l \), 0 otherwise
Sub-Problem – Constraints

\[ s - a \geq 0, \quad (9) \]
\[ c \geq h_t_k \cdot fraction_{jk} - M \cdot (1 - s_{sj}), \quad \forall k, j \in K, \quad (10) \]

**Parameters**

- \( fraction_{jk} \) – fraction of cargo handled at section \( k \), if the starting section of the vessel is section \( j \)
- \( M \) – large enough number (set to 1,000,000, could be the largest quantity multiplied by the longest service time)

**Decision Variables**

- \( h_t_k \geq 0 \) – handling time of section \( k \)
- \( s_{sj} \in (0, 1) \) – 1 if section \( j \) is the starting section of the vessel
Sub-Problem – Constraints

\[
\sum_{j \in K} ss_j = 1, \tag{11}
\]

\[
\sum_{j \in K} ss_j \cdot sc_j + \text{length} \leq ql, \tag{12}
\]

Parameters

- \( sc_j \) – starting coordinate of section \( j \)
- \( \text{length} \) – length of the vessel
- \( ql \) – quay length
Sub-Problem – Constraints

\[ \sum_{l \in L} \text{split}_l \leq Z, \]  

(13)

**Parameters**
- \( Z \) – maximum number of locations used by vessel

**Decision Variables**
- \( \text{split}_l \in (0, 1) \) – 1 if vessel uses location \( l \)
Sub-Problem – Constraints

\[ \text{split}_l \leq \text{delta}_{lw}, \quad \forall l \in L, \quad (14) \]
\[ \sum_{l \in L} \text{cs}_l = \text{quantity}, \quad (15) \]
\[ \text{cs}_l \leq \text{split}_l \cdot \text{quantity}, \quad \forall l \in L, \quad (16) \]
\[ \text{split}_l \leq \text{cs}_l, \quad \forall l \in L, \quad (17) \]

Parameters
- \( w \) – cargo type carried on the vessel

Decision Variables
- \( cs_l \geq 0 \) – quantity of cargo stored at location \( l \)
Sub-Problem – Constraints

\[ td_k = \left( \sum_{l \in L} d_{kl} \cdot cs_l \right) / \text{quantity}, \quad \forall k \in K, \quad (18) \]

\[ ht_k = F / \text{cranes}_k + V_w \cdot td_k, \quad \forall k \in K, \quad (19) \]

Parameters

- \( d_{kl} \) – distance between section \( k \) and location \( l \)
- \( \text{cranes}_k \) – number of cranes in section \( k \)
- \( F \) – crane handling rate, \( V_w \) – cargo transfer rate

Decision Variables

- \( td_k \geq 0 \) – total average distance for section \( k \)
Sub-Problem – Constraints

\[
\sum_{t \in T} time_t = c, \quad (20)
\]

\[
t + M \cdot (1 - time_t) \geq s + 1, \quad \forall t \in T, \quad (21)
\]

\[
t \leq s + c + M \cdot (1 - time_t), \quad \forall t \in T, \quad (22)
\]

Parameters

- \(M\) in 21 – minimum value is \(s + 1\)
- \(M\) in 22 – minimum value is \(T - s + c\)

Decision Variables

- \(time_t \in (0, 1)\) – 1 if the vessel is at time \(t\) served, 0 otherwise
Sub-Problem – Constraints

\[
\beta_{kt} \geq x_k + \text{time}_t - 1, \quad \forall k \in K, \forall t \in T, \quad (23)
\]

\[
\beta_{kt} \leq x_k, \quad \forall k \in K, \forall t \in T, \quad (24)
\]

\[
\beta_{kt} \leq \text{time}_t, \quad \forall k \in K, \forall t \in T, \quad (25)
\]

\[
\gamma_{lt} \geq \text{split}_l + \text{time}_t - 1, \quad \forall l \in L, \forall t \in T, \quad (26)
\]

\[
\gamma_{lt} \leq \text{split}_l, \quad \forall l \in L, \forall t \in T, \quad (27)
\]

\[
\gamma_{lt} \leq \text{time}_t, \quad \forall l \in L, \forall t \in T, \quad (28)
\]
Branch and Bound

\[ \lambda = 0.734 \]
\[ X = 0.333 \]
1 Introduction

2 Problem Definition

3 Branch and Price

4 Results
   ■ Status
   ■ Tests

5 Conclusion
Status

- **Master problem:**
  - able to solve small instances
  - finished

- **Sub-problem:**
  - validated with Opl and the minimum handling time of generated assignments
  - running time < 3 sec.

- **Column Generation:**
  - without sub-problem, just reduced cost, able to solve small instances
  - strategies:
    - one column per turn – not able to solve in 3 hours (and still working)
    - all negative columns per turn – solved below 20 iterations
  - sub-problem

- **Branch and Bound:**
  - to be implemented
  - probably existing function in CPLEX
Test Instance
Initial Berth Plan
Debugging

To catch a bug, you've got to learn to think like a bug
1 Introduction

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Thank you for your attention.
\begin{align}
\text{min} & \quad z \\
\text{s.t.} & \quad m_i - A_i \geq 0 \\
& \quad \sum_{k \in M} s_i^k = 1 \\
& \quad \sum_{k \in M} (s_i^k b_k) + L_i \leq L \\
& \quad \sum_{p \in M} (\delta_{ik} s_i^k) = x_{ik} \quad \forall k \in M \\
& \quad \sigma_{ik}^j \geq x_{ik} + \theta_{it} - 1 \quad \forall k \in M, \forall t \in H \\
& \quad \sigma_{ik}^j \leq x_{ik} \quad \forall k \in M, \forall t \in H \\
& \quad \sigma_{ik}^j \leq \theta_{it} \quad \forall k \in M, \forall t \in H \\
& \quad (d_i - D_i) x_{ik} \geq 0 \quad \forall k \in M \\
& \quad c_i \geq h_{ik}^{\text{w}_{\text{w}_{\text{w}}}} Q_i - B (1 - s_i^j) \quad \forall l \in L, \forall k \in M \quad \forall w \in W_i \\
& \quad h_{ik} = \alpha_{ik}^{w} + \beta_{ik}^{w} \quad \forall w \in W_i, \forall k \in M \\
& \quad \alpha_{ik}^{w} = T / n_i^{w} \quad \forall w \in W_i, \forall k \in M \\
& \quad \beta_{ik}^{w} = V_{ik}^{w} \quad \forall w \in W_i, \forall k \in M \\
& \quad \sum_{l \in L} \phi_{il} \leq 2 \\
& \quad r_{ik} = \sum_{l \in L} (r_{ik}^{l} \lambda_{il}) / Q_i \quad \forall k \in M \\
& \quad \sum_{w \in W_i} (r_{ik}^{l} \lambda_{il}) \leq 1 \quad \forall l \in L \\
& \quad \phi_{il} \leq \pi_{il} \quad \forall w \in W_i, \forall l \in L \\
& \quad \omega_{il} \geq \phi_{il} + \theta_{it} - 1 \quad \forall w \in W_i, \forall l \in L, \forall t \in H \\
& \quad \omega_{il} \leq \phi_{il} \quad \forall w \in W_i, \forall l \in L, \forall t \in H \\
& \quad \omega_{il} \leq \theta_{it} \quad \forall w \in W_i, \forall l \in L, \forall t \in H \\
& \quad \sum_{t \in H} \theta_{it} = c_i \\
& t + B (1 - \theta_{it}) \geq m_i + 1 \quad \forall t \in H \\
& t \leq m_i + c_i + + B (1 - \theta_{it}) \quad \forall t \in H \\
& \quad Q_i = \sum_{l \in L} \lambda_{il} \\
& \quad \lambda_{il} \leq \phi_{il} Q_i \quad \forall w \in W_i, \forall l \in L \\
& \quad \phi_{il} \leq B \lambda_{il} \quad \forall l \in L \\
& \quad \lambda_{il} \leq \sum_{w \in W_i} \sum_{t \in H} (R_{w} \omega_{il}^{t} + B (1 - \pi_{it}^{w})) \quad \forall l \in L
\end{align}