Integrating advanced discrete choice models in mixed integer linear optimization

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Outline

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3. Optimization model
4. Demand-based benefit maximization
5. Case study
6. Conclusions and ongoing work
1 Introduction

2 Choice model

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6 Conclusions and ongoing work
Discrete choice models

- Demand modeling
- Disaggregate level
- Heterogeneity of the population
- Predict the choice
Why are they not in OR?

- Tractability
- Linearity and/or convexity
- MILP models
Why are they not in OR?

- Tractability
- Linearity and/or convexity
- MILP models

- Behavioral realism
- Unrealistic assumptions
- Complex formulations
Bridging the gap

- General framework integrating
  - choice model \((demand)\)
  - MILP model \((supply)\)
- Simulation to linearize the choice model
**General framework**

- **Exogenous variables explaining the choice**: $x^d \in \mathbb{R}^D$
**Introduction**

**General framework**

- **Exogenous** variables explaining the choice: $x^d \in \mathbb{R}^D$
- **Exogenous** variables involved in the optimization model: $x^s \in \mathbb{R}^S$
General framework

- **Exogenous** variables explaining the choice: $x^d \in \mathbb{R}^D$
- **Exogenous** variables involved in the optimization model: $x^s \in \mathbb{R}^S$
- **Endogenous** variables appearing in both models: $x^e \in \mathbb{R}^E$
  - Characterize the interactions (e.g.: price)
  - $\ell^e \leq x^e \leq m^e$
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Choice set and population

- Set of alternatives $C \ (i)$
- Capacity: $c_i \geq 1$
- $N$ individuals ($n$)
- Individual choice set $C_n \subseteq C$
Choice set and population

- Set of alternatives $C (i)$
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$$y_{in} = \begin{cases} 1 & \text{if alternative } i \text{ is offered to } n \\ 0 & \text{otherwise} \end{cases}$$
Utility function and behavioral assumption

\[
U_{in}(x^d, x^e; \varepsilon_{in}) = V_{in}(x^d, x^e) + \varepsilon_{in}
\]

- **Utility function**
  - Deterministic part: \( V_{in}(x^d, x^e) = \sum_k \beta_k x_{ink}^e + g^d(x^d) \)
  - Random term: \( \varepsilon_{in} \)
Utility function and behavioral assumption

\[ U_{in}(x^d, x^e; \varepsilon_{in}) = V_{in}(x^d, x^e) + \varepsilon_{in} \]

- **Utility function**
  - Deterministic part: \( V_{in}(x^d, x^e) = \sum_k \beta_k x^e_{ink} + g^d(x^d) \)
  - Random term: \( \varepsilon_{in} \)

- **Behavioral assumption**
  - \( i \) chosen by \( n \) if \( U_{in} \geq U_{jn}, \forall j \in C_n \)
  - \( P_n(i|x^d, x^e) = \Pr(U_{in} \geq U_{jn}, \forall j \in C_n) \)
Simulation

- $R$ draws from the distribution of $\varepsilon_{in}$
- $\xi_{in1}, \ldots, \xi_{inR}$
- Behavioral scenario

\[ U_{inr} = \sum_k \beta_k x_{ink}^e + g^d(x^d) + \xi_{inr} \]
Availability

- Not considered
- \( y_{in} = 0 \forall i \notin C_n \)
Availability

- Not considered
- \( y_{in} = 0 \ \forall \ i \notin C_n \)

- Not offered
- Model’s decision
Availability

- Not considered
- \( y_{in} = 0 \quad \forall i \notin C_n \)

- Not offered
- Model’s decision

- Reached capacity
- \( y_{inr} \leq y_{in} \)
Discounted utility

$z_{inr} = \begin{cases} 
U_{inr} & \text{if } y_{inr} = 1 \\
\ell_{nr} & \text{if } y_{inr} = 0 
\end{cases}$

\[
\ell_{nr} \leq z_{inr} \\
z_{inr} \leq \ell_{nr} + M_{inr} y_{inr} \\
U_{inr} - M_{inr} (1 - y_{inr}) \leq z_{inr} \\
z_{inr} \leq U_{inr}
\]
Choice model

\[ U_{nr} = \max_{i \in C} z_{inr} \]

- \( z_{inr} \leq U_{nr} \)
- \( U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \)
- \( w_{inr} \leq y_{inr} \)
- \( \sum_{i \in C} w_{inr} = 1 \)
Expected demand

\[ D_i = \frac{1}{R} \sum_{r=1}^{R} \sum_{n=1}^{N} W_{inr} \]
Capacity allocation: priority list

- Exogenous to the model
- Relationship with individuals
- Numbering of individuals
- Example: random arrival
Choice model

Capacity allocation: capacity cannot be exceeded

\[
\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr})
\]
Choice model

Capacity allocation: capacity has been reached

\[ c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} \]
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General optimization model (MILP)

\[ g^s(x^s, x^e) \]

- Objective function (linear)
- Relates decisions at an aggregate level

\[ h^s(x^s, x^e) = 0 \]

- Set of constraints (linear)
- Feasible configuration of the variables

\[ \ell^e \leq x^e \leq m^e \]

\[ x^e_z \in \mathbb{Z} \]
Applications

- Design of a train timetable
- Objective: maximize profit
- Constraints: passenger satisfaction

- Schedule, what movie...
- Objective: total benefit
- Constraints: one movie per theater, capacity
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General setting

- Set of services $C(i)$
- Capacity: $c_i \geq 1$
- Opt-out option ($i = 0$)
- Maximize benefit

- $N$ customers ($n$)
- Individual choice set $C_n \subseteq C$
- $0 \in C_n \forall n$
- Price to pay: $p_{in}$
Demand-based benefit maximization

Benefit maximization

\[
\max \sum_{i>0} (G_i - C_i)
\]

\[
G_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} p_{in} w_{inr}
\]

\[
C_i = (f_i + v_i c_i) y_i
\]

- Product of variables
- Binary representation of price
- \( \frac{1}{10^k} a_{in} \leq p_{in} \leq \frac{1}{10^k} b_{in} \)
- \( L_{in} = \lceil \log_2(b_{in} - a_{in} + 1) \rceil \) variables
- \( f_i \) fixed cost
- \( v_i \) variable cost
- \( y_i = 1 \) if \( i \) is offered \( \forall n \)
- \( y_{in} = y_i \ \forall n \)
Price characterization

\[ p_{in} = \frac{1}{10^k} \left( a_{in} + \sum_{\ell=0}^{L_{in}-1} 2^\ell \lambda_{in\ell} \right) \]

\[ G_i = \frac{1}{R} \frac{1}{10^k} \sum_{n=1}^{N} \sum_{r=1}^{R} \left( a_{in} + \sum_{\ell=0}^{L_{in}-1} 2^\ell \lambda_{in\ell} \right) w_{inr} \]

\[ = \frac{1}{R} \frac{1}{10^k} \left[ \sum_{n=1}^{N} \sum_{r=1}^{R} \left( a_{in} w_{inr} + \sum_{\ell=0}^{L_{in}-1} 2^\ell \alpha_{inr\ell} \right) \right] \]

where \( \alpha_{inr\ell} = \lambda_{in\ell} w_{inr} \)
Resulting MILP

\[
\max \text{ benefit} \\
\text{subject to } \text{availability} \\
\text{utility definition} \\
\text{discounted utility} \\
\text{choice} \\
\text{capacity allocation} \\
\text{price selection}
\]
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Parking choices

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP, PUP, FSP}\}$
- $\mathcal{C}_n = \mathcal{C}$ $\forall n$

Mixtures of a logit model

- $y_{in} = y_i$ $\forall n$
- $p_{in} = p_i$ $\forall n$
General experiments

- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is obliged to offer a reduced price
Uncapacitated vs Capacitated case (1)

Uncapacitated

[Graph showing Log Solution time (s) vs Revenue for different values of R for the Uncapacitated case]

Capacitated

[Graph showing Log Solution time (s) vs Revenue for different values of R for the Capacitated case]
Uncapacitated vs Capacitated case (2)

Uncapacitated

Capacitated
Case study

Price differentiation by population segmentation

Subsidy offered by the municipality

Operator is obliged to offer a reduced price
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Conclusions and ongoing work

- Powerful tool to configure systems based on heterogeneous behavior
- Computationally expensive
- Decomposition techniques $\Rightarrow$ Lagrangian relaxation
  - Operator subproblem: FLP
  - Customer subproblem: iterative method (customers)
Questions?

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