A Lagrangian relaxation method for solving choice-based mixed linear optimization models that integrate supply and demand interactions

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Outline

Motivation

Demand

- Choices of customers
- Discrete choice models
- Nonlinear and nonconvex formulations

Supply

- Design and configuration of the system
- Mixed Integer Linear Problems (MILP)

Demand model

- Population of N customers (n)
- Choice set $\mathcal{C}$ (i)
- $\mathcal{C}_n \subseteq \mathcal{C}$: alternatives considered by customer n ($\mathcal{N}_i = \{n \geq 1 | i \in \mathcal{C}_n\}$)

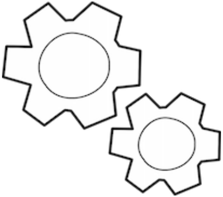
Behavioral assumption

- $U_{in} = V_{in} + \varepsilon_{in}$
- $V_{in} = \sum_k \beta_{ink} x^e_{ink} + q^d(x^d)$
- $P_n(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$

Simulation

- Distribution ε_{in}
- R draws $\xi_{in1}, \ldots, \xi_{inR}$
- $U_{inr} = V_{in} + \xi_{inr}$

Supply model



- Operator selling services to a market
  - Price p_{in} (to be decided)
  - Capacity c_i
- Benefit (revenue − cost) to be maximized
- Opt-out option ($i = 0$)

Price characterization

- Lower and upper bound
- Discretization: price levels
- Binary representation ($\lambda_{in\ell}$)

Capacity allocation

- Exogenous priority list of customers
- Here it is assumed as given
- Capacity as decision variable

MILP model (in words)

MILP

max	benefit
subject to	availability
	utility definition
	discounted utility
	choice
	capacity allocation
	price selection

MILP model

MILP

max	benefit
subject to	**availability**
	utility definition
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$y_i \in \{0,1\}$	operator decision
$y^d_{in} \in \{0,1\}$	customer decision (data)
$y_{in} \in \{0,1\}$	product of decisions
$y_{inr} \in \{0,1\}$	capacity restrictions

Relations between availabilities

$$y_{in} = y^d_{in} y_i \quad \forall i, n \tag{1}$$

$$y_{inr} \leq y_{in} \quad \forall i, n, r \tag{2}$$

MILP model

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Utility

$$U_{inr} = \overbrace{\beta_{in} p_{in} + q_d(x_d)}^{V_{in}} + \xi_{inr} \quad \forall i, n, r \qquad (3)$$

MILP model

MILP

max benefit
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$$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases} \quad \text{discounted utility}$$

Discounted utility

$$\ell_{nr} \leq z_{inr} \quad \forall i, n, r \quad (4)$$

$$z_{inr} \leq \ell_{nr} + M_{inr} y_{inr} \quad \forall i, n, r \quad (5)$$

$$U_{inr} - M_{inr}(1 - y_{inr}) \leq z_{inr} \quad \forall i, n, r \quad (6)$$

$$z_{inr} \leq U_{inr} \quad \forall i, n, r \quad (7)$$

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$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr}$$

$$w_{inr} = \begin{cases} 1 & \text{if } i = \arg\max\{U_{nr}\} \\ 0 & \text{otherwise} \end{cases} \qquad \text{choice}$$

Choice

$$z_{inr} \leq U_{nr} \qquad \forall i, n, r \qquad (8)$$

$$U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \qquad \forall i, n, r \qquad (9)$$

$$\sum_i w_{inr} = 1 \qquad \forall n, r \qquad (10)$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r \qquad (11)$$

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Priority list

$$y_{in^- r} \geq y_{inr} \qquad \forall i > 0, n < N, r \qquad (12)$$

Capacity cannot be exceeded $\Rightarrow y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1) y_{inr} + (n-1)(1 - y_{inr}) \quad \forall i > 0, n > c_i, r \quad (13)$$

Capacity has been reached $\Rightarrow y_{inr} = 0$

$$c_i (y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} \qquad \forall i > 0, n, r \qquad (14)$$

MILP model

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$$p_{in} = \frac{1}{10^k}\left(\ell_{in} + \sum_{\ell=0}^{L_{in}-1} 2^\ell \lambda_{in\ell}\right)$$

- When calculating the benefit: $\lambda_{in\ell} w_{inr}$
- $\alpha_{inr\ell} = \lambda_{in\ell} w_{inr}$

Linearization of $\alpha_{inr\ell}$ + Price bounded from above

$$\lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell} \quad \forall i > 0, n, r, \ell \quad (15)$$

$$\alpha_{inr\ell} \leq \lambda_{in\ell} \quad \forall i > 0, n, r, \ell \quad (16)$$

$$\alpha_{inr\ell} \leq w_{inr} \quad \forall i > 0, n, r, \ell \quad (17)$$

$$\ell_{in} + \sum_{\ell=0}^{L_{in}-1} 2^\ell \lambda_{in\ell} \leq m_{in} \quad \forall i > 0, n \quad (18)$$

MILP model

MILP

max	**benefit**
subject to	availability
	utility definition
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$$\max \sum_{i>0} (R_i - C_i)$$

Revenue

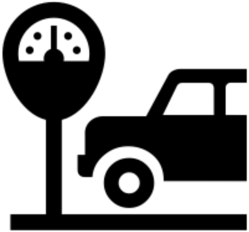
$$R_i = \frac{1}{R} \frac{1}{10^k} \left[\sum_n \sum_r \left(\ell_{in} w_{inr} + \sum_\ell 2^\ell \alpha_{inr\ell} \right) \right]$$

Cost

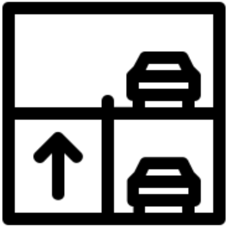
$$C_i = (f_i + v_i c_i) y_i$$

Parking choices[1]

PSP



PUP



FSP (opt-out)

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$

- $p_{in} = p_i \quad \forall n$
- Mixtures of a logit model

[1] A. Ibeas, L. dellOlio, M. Bordagaray, et al., "Modelling parking choices considering user heterogeneity," Transportation Research Part A: Policy and Practice, vol. 70, pp. 41 –49, 2014.

General experiments

Uncapacitated vs Capacitated case

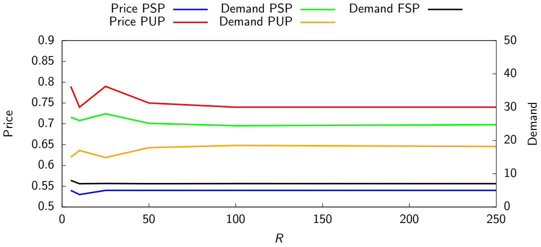
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation

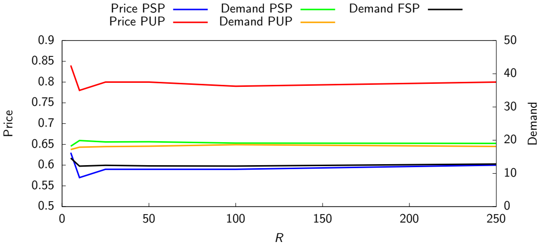
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is obliged to offer a reduced price

Uncapacitated vs Capacitated case

Uncapacitated

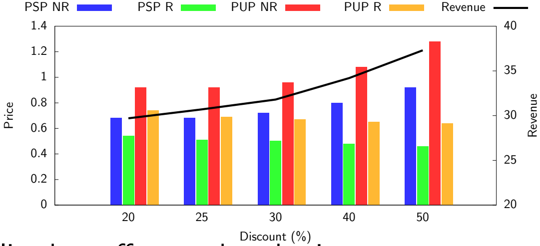


Capacitated

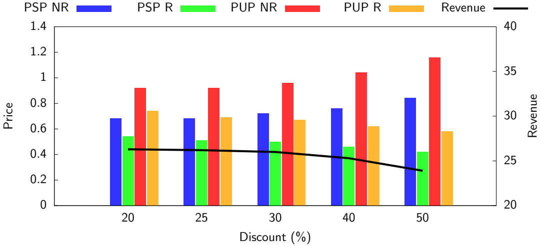


Price differentiation by population segmentation

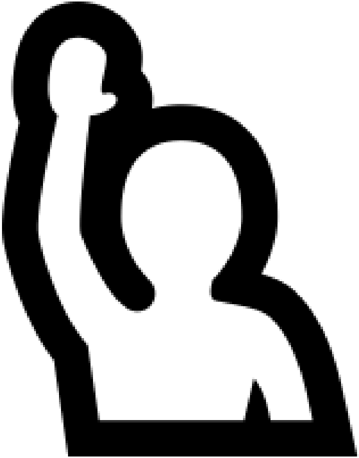
Subsidy offered by the municipality



Operator is obliged to offer a reduced price



But...



Computational time

	Uncapacitated case				Capacitated case			
R	Sol time	PSP	PUP	Rev	Sol time	PSP	PUP	Rev
5	2.58 s	0.54	0.79	26.43	12.0 s	0.63	0.84	25.91
10	3.98 s	0.53	0.74	26.36	54.5 s	0.57	0.78	25.31
25	29.2 s	0.54	0.79	26.90	13.8 min	0.59	0.80	25.96
50	4.08 min	0.54	0.75	26.97	50.2 min	0.59	0.80	26.10
100	20.7 min	0.54	0.74	26.90	6.60 h	0.59	0.79	26.03
250	2.51 h	0.54	0.74	26.85	1.74 days	0.60	0.80	25.93

General idea

1. Relax complicating constraints $\Rightarrow$ Lagrangian subproblem
2. Define 2 separable subproblems:
   - Identify common variable
   - Create a copy
   - Relax associated constraints
3. Solve the subproblems independently
4. Solve the Lagrangian dual to provide an upper bound (subgradient method)

First attempt: subproblems

Relaxed constraints

- Common variable: w_{inr} (copy: v_{inr})
- Transferred to the objective function:
  - Copy constraints (γ)
  - Utility definition: involved in the choice + contains price variables (θ)

Choice subproblem

- **Variables:** U_{inr} and w_{inr}
- Decomposes by n and r
- **Choice subproblem:** $Z^c_{nr}(\theta, \gamma)$

Price subproblem

- **Variables:** $\lambda_{in\ell}$, $\alpha_{inr\ell}$ and v_{inr}
- Decomposes by n
- **Price subproblem:** $Z^p_n(\theta, \gamma)$

First attempt: drawbacks

- Variables reaching the bounds:
  - Utility
  - Price
- $\Rightarrow$ poor upper bound
- High importance placed on the subgradient method

Current approach: sketch

Relaxed constraints

- Relation between availability at operator and customer level
- Copy constraints for the choice variables also introduced
- 2 subproblems:
  - Operator subproblem
  - Customer subproblem

Current approach: subproblems

Operator subproblem

- Capacitated Facility Location Problem

Customer supbroblem

- Assumption: utility decreases as a function of the price
- Iterate over customers (priority list) and over scenarios
- Highest price such that the customer does not change the choice

Ongoing research and future work

Ongoing research

- Implementation of the 2 subproblems
- Subgradient method to solve the Lagrangian dual

Future work

- Provide a lower bound on the original problem
- If the gap between bounds is significant $\Rightarrow$ column generation

Questions?

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