Probabilistic speed-density relationship for pedestrians based on data driven space and time representation

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WORKSHOP ON PEDESTRIAN MODELS 2014

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Introduction

Objective

Mathematical framework providing the detailed characterization of the pedestrian flow

Motivation

- Heterogeneity
- Complex interactions
- Multidirectional flows

Data

Data collection

- Surveys and counting
- Pedestrian tracking

Pedestrian studies

- Field data

  (Fruin, 1971; Navin and Wheeler 1969; Lam et al. 2003; Rastogi et al. 2013)

- Controlled experiments

  (Daamen and Hoogendoorn 2003; Seyfried et al., 2010; Kretz et al., 2006; Wong et al., 2010)

Data

Visiosafe technology

- Spin-off of EPFL
- Gare de Lausanne
- Anonymous sensor based pedestrian tracking

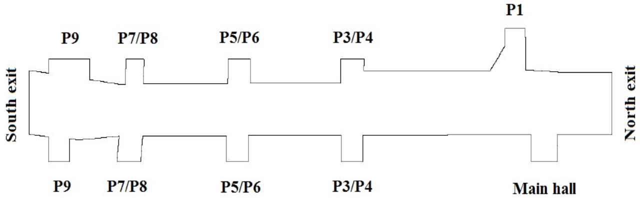
  Thermal sensors

  Range sensors
- Vision processing outcome $(t, x(t), y(t), pedestrian_{id})$

Alahi, A., Jacques, L., Boursier, Y. and Vandergheynst, P. (2011). Sparsity driven people localization with a heterogeneous network of cameras, *Journal of Mathematical Imaging and Vision* 41(1-2): 39-58.

Gare de Lausanne

Pedestrian underpass West

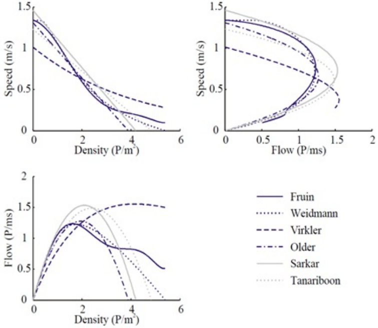


- The busiest walking area in the station
- Area $\approx 685m^2$
- The maximum occupation $\approx$ 250 pedestrians
- Area covered by 32 sensors

Fundamental flow indicators

- Density (k)
- Speed (v)
- Flow (q)
- Fundamental diagram

$$q = v \cdot k$$



source: (Daamen et al., 2005)

Fundamental flow indicators

Issues

- Spatio-temporal discretization is arbitrary
  - Results may be highly sensitive
  - Loss of heterogeneity
- Pedestrian flow is multidirectional
  (Lam et al. 2003;Wong et al., 2010)

Pedestrian-oriented flow characterization

- Detailed pedestrian tracking input
- Data driven space and time discretization

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Density indicator

Pedestrian flow

- Number of pedestrians per unit of space at a given time

Spatial discretization

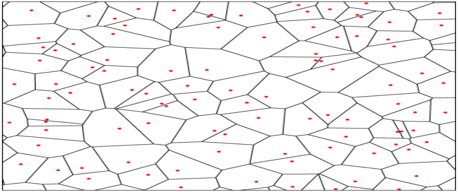
- Discretization units are too small - many remain empty
- Discretization units are too large - loss of information

Spatial discretization

Voronoi tessellations

- $p_1, p_2, ..., p_N$ is a finite set of points
- Voronoi space decomposition assigns a region to each point

$$V(p_i) = \{p | \|p - p_i\| \leq \|p - p_j\|, i \neq j\}$$



Okabe, A., Boots, B., Sugihara, K. and Chiu, S. N. (2009). *Spatial tessellations: concepts and applications of Voronoi diagrams*, Vol. 501, John Wiley & Sons.

Spatial discretization

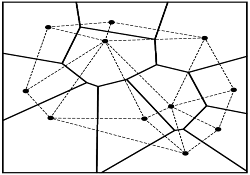
Numerical instability

- Small polygons allocated to pedestrians in very dense areas

Delaunay triangulation

- Clustering of critical cells
- ξ, threshold distance

  $d(p_i, p_j) < \xi, \forall i, j$



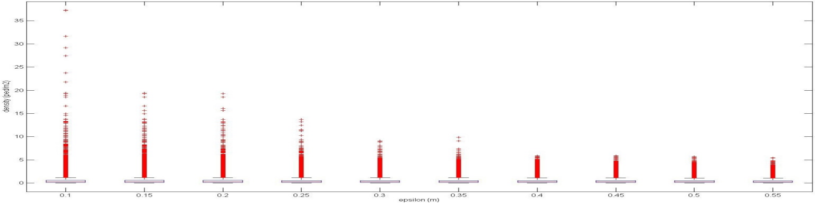
Spatial discretization

Numerical instability

- Small polygons allocated to pedestrians in very dense areas

Sensitivity analyses

- $\xi = 0.4m$
- ω_i, weight associated to the corresponding space

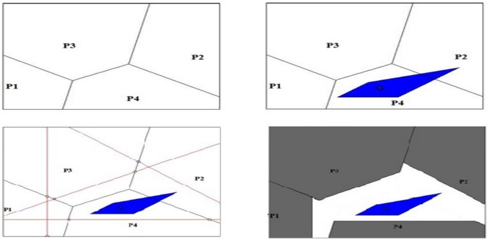


Spatial discretization

Presence of obstacles

- Assumption: two points can be connected by a straight line
- Voronoi diagram for points and Voronoi diagram areas

$$d\left(p_i, O\right) = min_{o_j}\left\{\|p_i - o_j\| \,|o_j \in O\right\}$$



Density indicator

Definition

- Set of points: pedestrians

$$p_i = (x_i, y_i, t_i)$$

- Pedestrian-oriented density indicator

$$k_i = \frac{\omega_i}{|V(p_i)|}$$

Voronoi density map

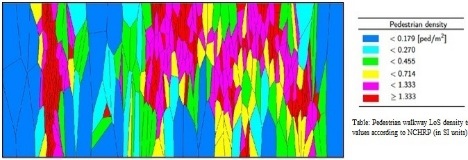


Table: Pedestrian walkway LoS density treshold values according to NCHRP (in SI units)

Speed indicator

Pedestrian flow

- Instantaneous speed - rate of change of position of a pedestrian with respect to time and at a particular point.

Time discretization

- Discretization interval is too small - noisy observations
- Discretization interval is too large - lower precision

Time discretization

Moment	$v_{\Delta t=0.1s}$	$v_{\Delta t=0.2}$	$v_{\Delta t=0.3s}$	$v_{\Delta t=0.4s}$	$v_{\Delta t=0.5s}$	$v_{\Delta t=0.6s}$	$v_{\Delta t=0.7s}$	$v_{\Delta t=0.8s}$	$v_{\Delta t=0.9s}$	$v_{\Delta t=1s}$
1	1.1161	1.1158	1.1156	1.1155	1.1153	1.1152	1.1150	1.1149	1.1148	1.1147
2	0.4175	0.3296	0.2956	0.2747	0.2591	0.2465	0.2358	0.2263	0.2179	0.2104
3	5.7853	2.5957	1.7703	1.4310	1.2544	1.1476	1.0740	1.0188	0.9744	0.9363
4	134.4926	31.2621	15.5319	10.9042	9.0167	8.0657	7.4917	7.0994	6.8045	6.5660

- Kruskal-Wallis test (H=4.61, df=9, p=0.87)

  The moments represent the same population at 95% confidence level

Speed indicator

Definition

- Space-time representation

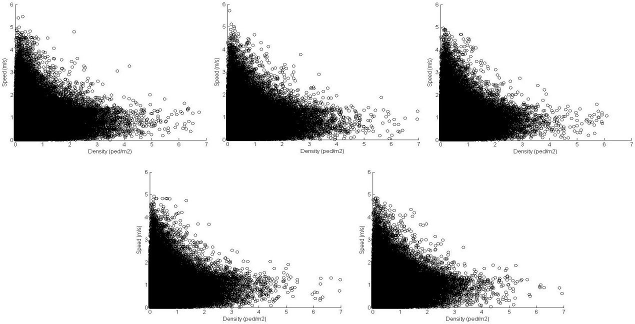
$$p_i = (x_i, y_i, t_i)$$

- Pedestrian-oriented speed indicator

$$v_i = \frac{\|p_i(t+\Delta t) - p_i(t-\Delta t)\|}{2\Delta t}, \ \Delta t = 1\text{s}$$

Empirical speed-density relationship

Speed-density profiles



February 11.-15., 2013.: morning peak hour

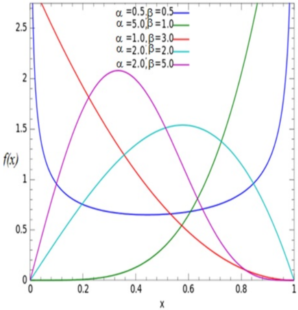
Probabilistic approach

Kumaraswamy distribution

- Defined on the bounded region $[l, u]$
- Two non-negative shape parameters α and β
- The simple closed form of pdf *f(x)* and cdf *F(x)*

$$f(x) = \frac{\alpha \cdot \beta \cdot (x-l)^{\alpha-1} \cdot ((u-l)^{\alpha} - (x-l)^{\alpha})^{(\beta-1)}}{(u-l)^{\alpha \cdot \beta}}$$

$$F(x) = 1 - (1 - (\frac{x-l}{u-l})^{\alpha})^{\beta}$$



Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes, *Journal of Hydrology* 46(1): 79-88.

Probabilistic approach

Speed-density relationship

$$V \sim f\left(\alpha\left(k\right), \beta\left(k\right), l\left(k\right), u\left(k\right)\right)$$

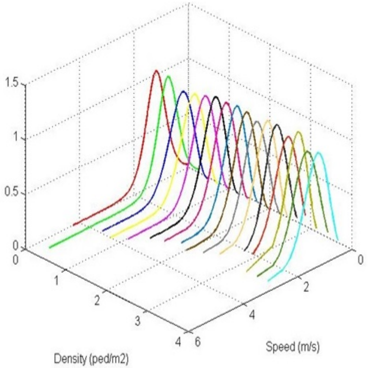
f - Kumaraswamy pdf

V - speed

k - density level

α, β - shape parameters

u,l - boundary parameters



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Probabilistic approach

Specification of speed-density relationship

$$V \sim f\left(\alpha\left(k\right), \beta\left(k\right), l\left(k\right), u\left(k\right)\right)$$

Parameter	Specification#1	Specification#2
$\alpha(k)$	$a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha$	$a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha$
$\beta(k)$	$a_\beta exp(b_\beta k)$	$a_\beta exp(b_\beta k)$
u(k)	$a_u exp(b_u k)$	$a_u k^3 + b_u k^2 + c_u k + d_u$
l(k)	0	0

Probabilistic approach

Maximum likelihood estimation

$$log\mathcal{L} = \sum_{i=1}^{n} log(\alpha(k_i)) + \sum_{i=1}^{n} log(\beta(k_i)) + \sum_{i=1}^{n}(\alpha(k_i) - 1)log(v_i - l(k_i)) + \sum_{i=1}^{n}(\beta(k_i) - 1)log((u(k_i) - l(k_i))^{\alpha(k_i)} - (v_i - l(k_i))^{\alpha(k_i)}) - \sum_{i=1}^{n} \alpha(k_i)\beta(k_i)log(u(k_i) - l(k_i))$$

Parameter	Specification#1	Specification#2
a_α	-0.0076	0.0498
b_α	0.0961	-0.2823
c_α	-0.3781	-0.0207
d_α	2.2185	2.0089
a_β	44.8191	45.362
b_β	-0.1057	-0.5945
a_u	7	0.0002
b_u	0	-0.0002
c_u		-0.0010
d_u		8.0017
$log\mathcal{L}$	-891880	-932990

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Probabilistic approach

Speed-density relationship

$$V \sim f\left(\alpha\left(k\right), \beta\left(k\right), l\left(k\right), u\left(k\right)\right)$$

$$\alpha(k) = a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha$$
$$\beta(k) = a_\beta exp(b_\beta k)$$
$$u(k) = 7$$
$$l(k) = 0$$

$$a_\alpha = -0.0076, b_\alpha = 0.0961, c_\alpha = -0.3781, d_\alpha = 2.2185$$
$$a_\beta = 44.8191, b_\beta = -0.1057$$

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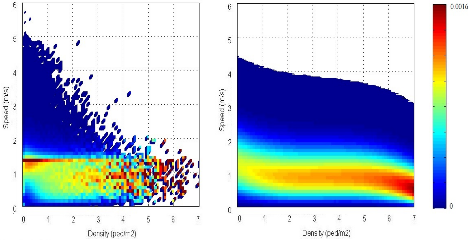
Probabilistic approach

Validation

- Moments of empirical and predicted discrete joint distributions
- Kruskal-Wallis test (H=0.33, df=1, p=0.5637)

  The model and data represent the same population at 95% confidence level



Moments	Data	Model prediction
1	0.9333	0.9856
2	0.1845	0.2376
3	0.0426	0.0648
4	0.1521	0.1769

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Conclusion

- Pedestrian-oriented flow characterization
- Data-driven space and time discretization
- Probabilistic methodology to describe observed heterogenaity
- Model estimation and validation based on pedestrian tracking input
- Case study: Gare de Lausanne

Future directions

- The framework is insufficient to explain the multidirectional nature of pedestrian flows
- Solution investigated: a stream-based approach
- Final objective: integration of the stream-based concept with the developed probabilistic framework

Thank you

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