Robust Optimization with Recovery: Application to Shortest Paths and Airline Scheduling

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Planning the Recovery

- Developed Recovery Algorithm for disrupted airline schedules

- Want to consider the possibility of recovery during scheduling phase

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- Proactive worst case approach including recovery decisions
- Illustration on the SPPID
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Optimization under Uncertainty

(for a minimization problem)

- Characterization of uncertainty set *U*
  - any knowledge?
  - probabilistic measure?
  - consider extreme cases?

- Proactive vs Reactive approach

Existing Methods' Classification

	Proactive Methods	Reactive Methods
Probabilistic *U*	Stochastic Programming	Stochastic Programming with Recourse
Non Probabilistic *U*	Robust Optimization	On-line Algorithms

Some References

- On-Line Algorithms: Albers, 2003

- Stochastic Optimization: Kall and Wallace, 1994; Wallace and Ziemba, 2005.

- Stochastic with Recourse: Kall and Wallace, 1994; Polychronopoulos and Tsitsiklis , 1996; Provan, 2003.

- Robust Optimization: Soyster, 1973; Ben-Tal and Nemirowski, 2001 (book); Bertsimas and Sim, 2004.

On-Line Optimization

Advantages	Disadvantages
• react in real time • easy computation (decision strategy) • optimality ratio (a posteriori)	• no information on bounds (only ratio) • reaction dictated by nature • difficult to measure performance a priori

Stochastic Programming

Advantages	Disadvantages
• lowest expected cost • low probabilities of high cost • exploits a priori information	• difficult to measure probabilities • estimation of all scenarios • only predictor (depends on scenario) • not efficient for few implementations

Stochastic Programming with Recourse

Advantages	Disadvantages
• includes cost of recourse decision • adapts to information in real time • exploits partial information	• difficult to measure probabilities • estimation of all scenarios • predictor on partially revealed scenario • guided by information revelation

Robust Optimization

Advantages	Disadvantages
• solution always feasible • evaluation on worst scenarios only • get cost bound	• characterize worst scenarios • what if a good scenario occurs?

Definitions

Problem *P*

- set of **feasible solutions** *S*
- uncertainty set *U*
- ***realization*** *or* ***scenario*** $u \in U$
- cost of solution *s* under scenario *u*: $c_u(s)$

Recoverable Approach

- Proactive: want a deterministic solution not dictated by scenario revealing
- Worst + Best: consider both best and worst scenarios (potential)
- Include On-line Decisions (Recovery): consider recovery costs for unfeasible solutions

$$(P') \qquad \min_{s \in S} \left\{ \max_{u \in U} \quad \tilde{c}_u(s) + c_u^{\mathrm{REC}}(s) \quad + \quad \min_{u' \in U} \quad \tilde{c}_{u'}(s) + c_{u'}^{\mathrm{REC}}(s) \right\}$$

Example:

Shortest Path Problem with Interval Data (SPPID):

- Oriented graph $G = (V,A)$
- Unique source s and unique sink t
- Random arc costs $c_{ij} \in [l_{ij}, u_{ij}]$ ($u_{ij} = \infty$ possible)
- Scenario: set of cost realization for every arc cost
- arc cost revealed when source node is reached

On-Line Strategy:

- at node *i* take outgoing arc with lowest cost

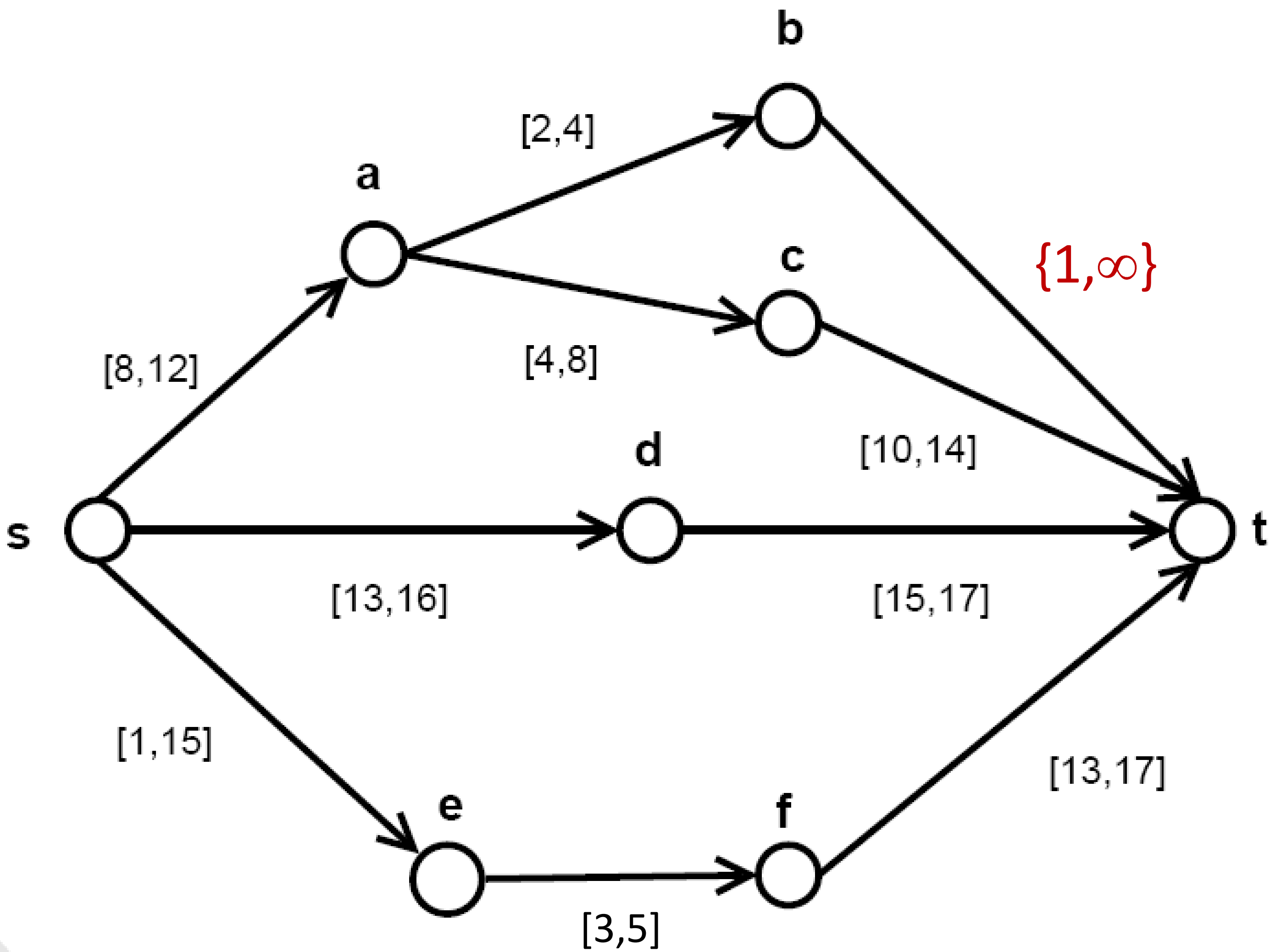
Recovery:

- when reach dead end, take last arc backwards at cost u_{ij}
- remove arc from V

Probabilistic cases:

- symmetric independent distributions

Example: SPPID (3)

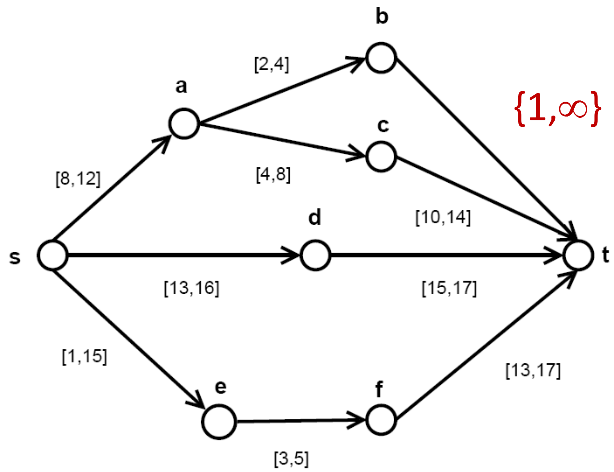


- Stochastic: path {*s,e,f,t*}, mean cost 27

- Robust: path {s,d,t}, worst cost 33

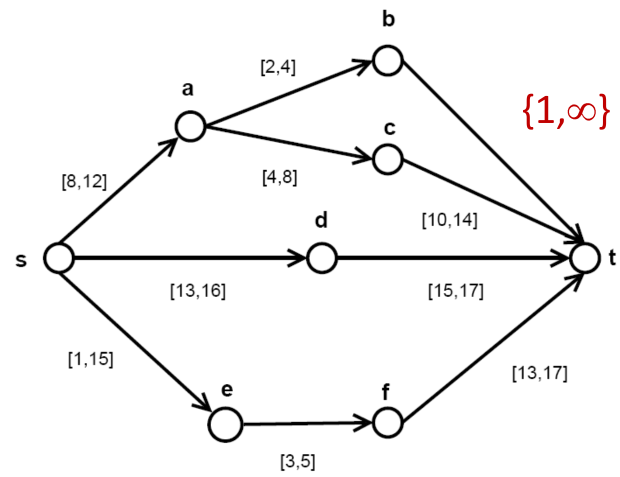
- On-line: either {*s,a,b,t*} or {*s,e,f,t*}

- Stochastic with recourse: either {*s,e,f,t*} or {*s,a,b,t*}



Example: SPPID (5)

- Recoverable: path $\{s,a,b,t\}$



Path	Best	Worst	Potential	Mean
$\{s,a,b,t\}$	11	42	53	∞
$\{s,a,c,t\}$	22	34	56	28
$\{s,d,t\}$	28	33	61	30.5
$\{s,e,f,t\}$	17	37	54	27

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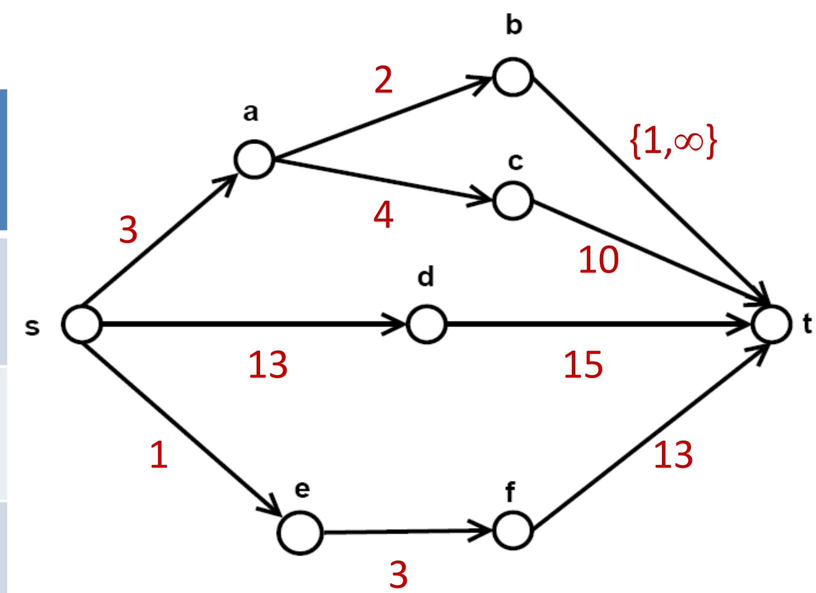
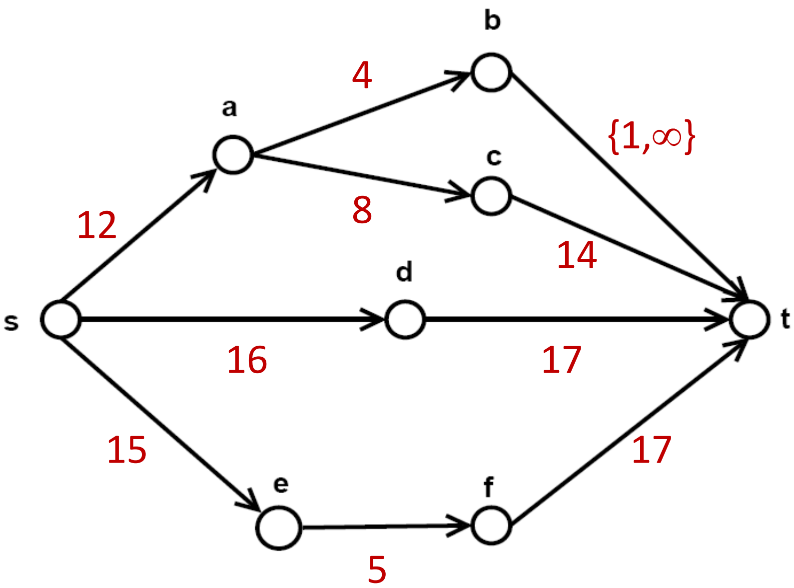
Example: SPPID (6)

Method	Path	Cost
On-Line	{*s,e,f,t*}	17
Stochastic with Recourse	{*s,e,f,t*}	17
Stochastic	{*s,e,f,t*}	17
Robust	{*s,d,t*}	28
Recoverable	{*s,a,b,t*} Or {s,a,b,a,c,t}	11 or 28



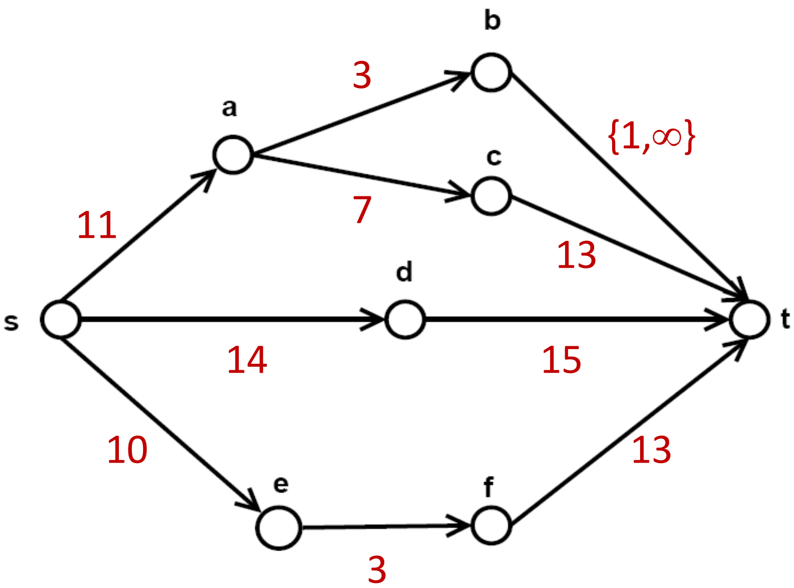
Example: SPPID (7)

Method	Path	Cost
On-Line	*{s,a,b,t}* or {s,a,b,a,c,t}	17 42
Stochastic with Recourse	*{s,a,b,t}* or {s,a,b,a,c,t}	17 42
Stochastic	*{s,e,f,t}*	37
Robust	*{s,d,t}*	33
Recoverable	*{s,a,b,t}* or {s,a,b,a,c,t}	17 42



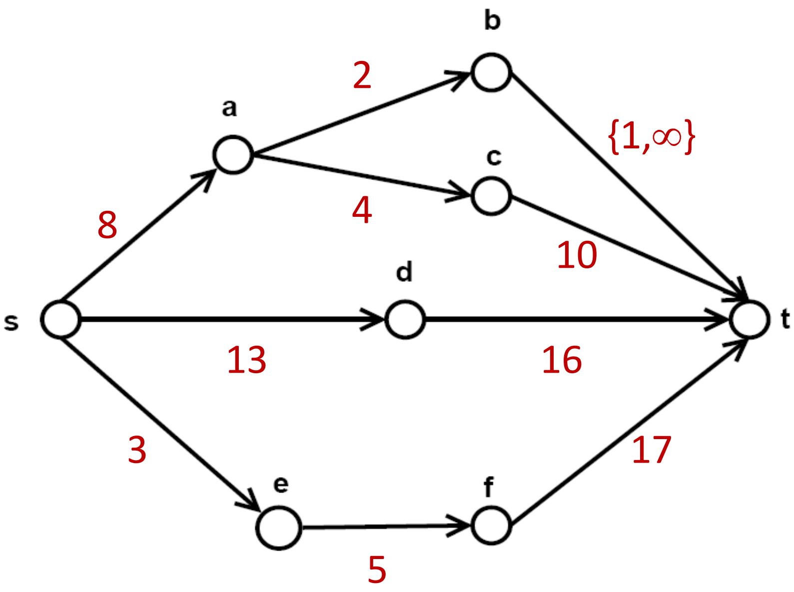
Example: SPPID (8)

Method	Path	Cost
On-Line	*{s,e,f,t}*	26
Stochastic with Recourse	*{s,a,b,t}* or {s,a,b,a,c,t}	15 38
Stochastic	*{s,e,f,t}*	26
Robust	*{s,d,t}*	29
Recoverable	*{s,a,b,t}* or {s,a,b,a,c,t}	15 38



Example: SPPID (9)

Method	Path	Cost
On-Line	{*s,e,f,t*}	25
Stochastic with Recourse	{*s,e,f,t*}	25
Stochastic	{*s,e,f,t*}	25
Robust	{*s,d,t*}	29
Recoverable	{*s,a,b,t*} or {s,a,b,a,c,t}	11 28



Average costs over 8 different scenarios

Method	Path	Cost
On-Line	Scenario dependent	26.875
Stochastic with Recourse	Scenario dependent	26.375
Stochastic	{*s,e,f,t*}	26.250
Robust	{*s,d,t*}	29.875
Recoverable	{*s,a,b,t*} or {s,a,b,a,c,t}	24.250

- Recoverable solution both highest and lowest cost
- Robust and Stochastic reject possibly unfeasible paths
- Reactive dependent on scenario

REMARK

Path {*s*,*a*,*b*,*t*} has least expected cost (24.5) if consider stochastic with recourse in a proactive way

Recoverable Schedule for Airlines

Difficulties:

- evaluation of feasibility
- recovery problem *NP*-hard
- characterization of worst scenario impossible
- evaluation on whole *U* impossible

Research Directions:

- Scenario Sampling
- Uncertainty set structure
- Recovery costs estimations
- Implicit measures and multi-objective optimization
- Explicit measures in recovery algorithm

Future Work:

- Develop a specific formalism
- Explore more deeply possible directions
- Develop an efficient algorithm
- Benchmark with real problems

Conclusions:

- Proactive best-worst case framework including information on unfeasible solutions
- Few scenario estimations, no probabilistic measures
- Provides recovery plan for each scenario
- Trade-off between proactive and reactive (planning the recovery)

THANKS for your attention!

Any Questions?