Waste collection inventory routing with non-stationary stochastic demands

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Outline

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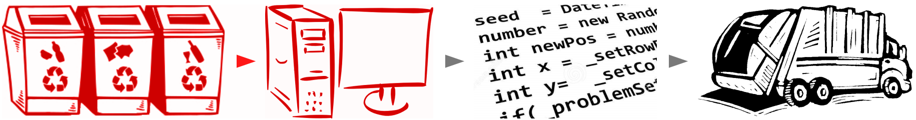
- Sensorized containers for recyclables periodically send waste level data to a central database.



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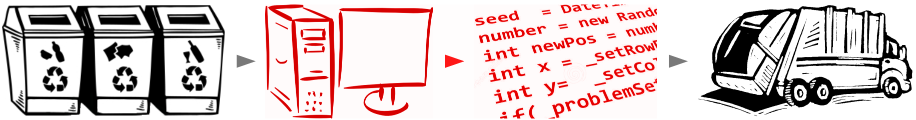

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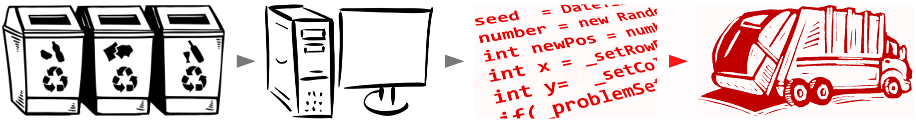
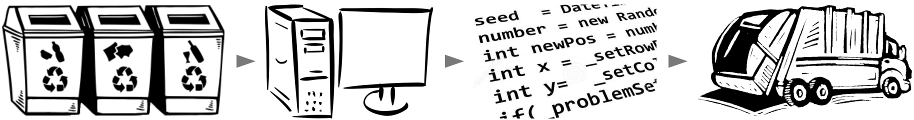


Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and route planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection thus depends on the ability to:
  - forecast container levels,
  - select the containers to collect each day,
  - and route the vehicles in an (near-)optimal way.



Problem Definition

- The setup falls within the framework of the Stochastic Inventory Routing Problem (SIRP) with:
  - stochastic demands,
  - Order-Up-to level (OU) policy,
  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).

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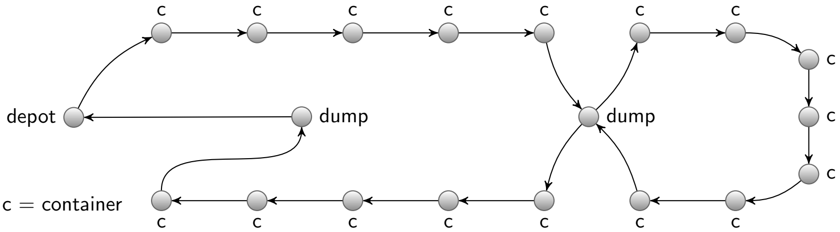
  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).
- The routing component includes:
  - intermediate facility visits (recycling plants),
  - heterogeneous capacitated vehicles,
  - site dependencies,
  - vehicle-to-period availabilities,
  - time windows,
  - maximum tour duration.

Routing Component

Figure 1: Example of a Collection Tour



Outline

SIRP Research Directions

- Early research on optimal replenishment policies in a stochastic setting:
  - Trudeau and Dror (1992), Jaillet et al. (2002), Bard et al. (1998).
- Robust optimization:
  - Solyalı et al. (2012).
- Chance constraints:
  - Soysal et al. (2015), Abdollahi et al. (2014), Yu et al. (2012).
- Scenario based:
  - roll-out/branch-and-cut: Bertazzi et al. (2013), Bertazzi et al. (2015).
  - stochastic optimization: Hemmelmayr et al. (2010), Nolz et al. (2014), Adulyasak et al. (2015).

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.

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  - we have a monetary value associated with the realization of undesirable events.
- Rich routing features rarely considered in the IRP literature.
- Probabilistic approach superior wrt alternative practical policies.

Outline

Selected Notation

Sets

o	origin	d	destination
$\mathcal{D}$	set of dumps	$\mathcal{P}$	set of containers
$\mathcal{N}$	$= \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$	$\mathcal{K}$	set of vehicles
$\mathcal{T}$	$= \{0, ..., u\}$	$\mathcal{T}^+$	$= \{1, ..., u+1\}$

Parameters

π_{ij}	travel distance of arc (i, j)
ρ_{it}	demand of container i on day t (random variable)
σ_{it}	$= 1$ if container i is in a state of full and overflowing on day t, 0 otherwise
ς	forecasting model error (st. dev. of the fit's residuals)
ω_i	capacity of container i
χ	container overflow cost (monetary)
ζ	emergency collection cost (monetary)

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Parameters

φ_k	daily deployment cost of vehicle k (monetary)
β_k	unit-distance running cost of vehicle k (monetary)
θ_k	unit-time running cost of vehicle k (monetary)
Ω_k	capacity of vehicle k

Selected Notation

Decision variables: binary

$$x_{ijkt} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses arc } (i,j) \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ikt} = \begin{cases} 1 & \text{if vehicle } k \text{ visits point } i \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{kt} = \begin{cases} 1 & \text{if vehicle } k \text{ is used on day } t \\ 0 & \text{otherwise} \end{cases}$$

Decision variables: continuous

Q_{ikt}	expected cumulative quantity on vehicle k at point i on day t
I_{it}	expected inventory in container i at the start of day t
S_{ikt}	start-of-service time of vehicle k at point i on day t

Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.

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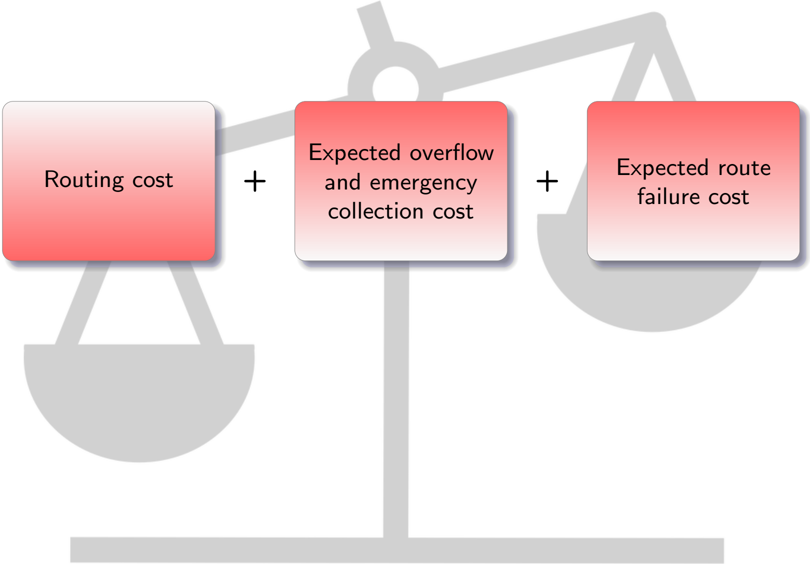
- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error ς.
- The forecasting error is the standard deviation of the residuals based on a historical fit.
- Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures.

Objective Function



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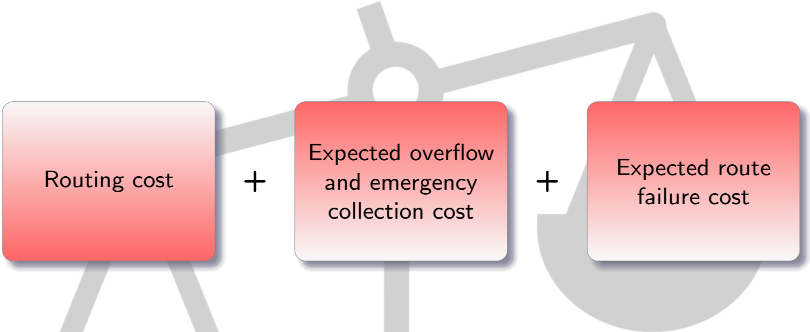

- Lower routing cost is counterbalanced by more overflows and route failures, and vice versa.
- Our goal is to minimize the expected monetary value of all components.

Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
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- Two types of container collection:
  - regular collection of container i on day t: $\exists k \in \mathcal{K} : y_{ikt} = 1$,
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.

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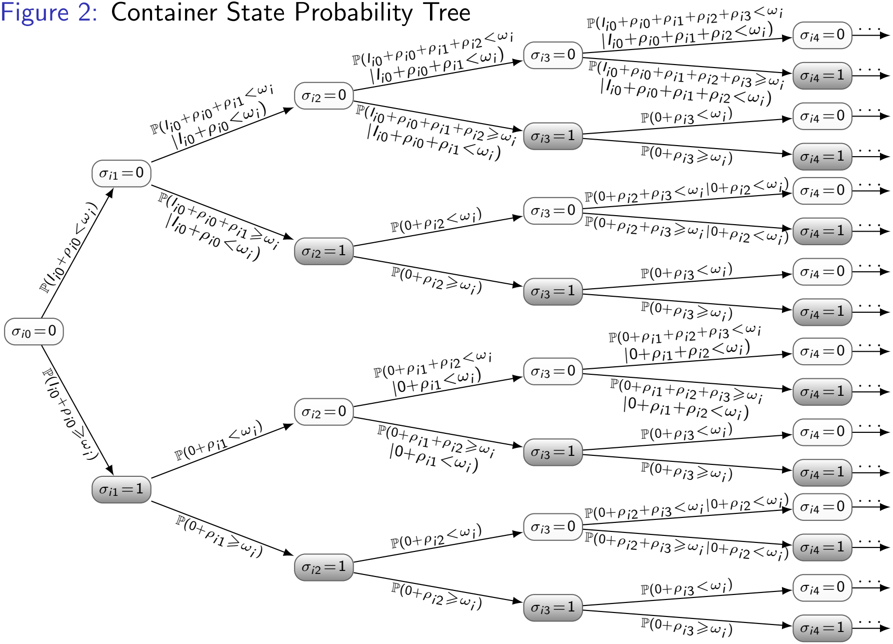
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.
- Related costs:
  - overflow cost χ: paid in state $\sigma_{it} = 1$,
  - emergency collection cost ζ: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in \mathcal{K}$.

Figure 2: Container State Probability Tree



Objective Function: Formulation

- Routing Cost (RC):

$$\sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} \left(\varphi_k z_{kt} + \beta_k \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij} x_{ijkt} + \theta_k \left(S_{dkt} - S_{okt} \right) \right) \tag{1}$$

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- Expected Overflow and Emergency Collection Cost (EOECC):

$$\sum_{t\in\mathcal{T}\cup\mathcal{T}^+}\sum_{i\in\mathcal{P}}\left(\mathbb{P}\left(\sigma_{it} = 1 \mid \max\left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1\right)\right)\left(\chi + \zeta - \zeta\sum_{k\in\mathcal{K}} y_{ikt}\right)\right) \quad (2)$$

Objective Function: Formulation

- Expected Route Failure Cost (ERFC):

$$\sum_{t \in \mathcal{T} \setminus 0} \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathscr{S}_{kt}} \left(\psi C_{\mathcal{S}} \, \mathbb{P} \left(\sum_{s \in \mathcal{S}} (I_{st} > \Omega_k \mid \max(0, g < t \colon y_{skg} = 1)) \right) \right), \quad (3)$$

where

- $\mathscr{S}_{kt}$ is the set of depot-to-dump or dump-to-dump trips for vehicle k on day t,
- $\mathcal{S}$ is the set of containers in a particular trip,
- $C_{\mathcal{S}}$ is the average routing cost of going from $\mathcal{S}$ to the nearest dump and back to $\mathcal{S}$,
- ψ is the Route Failure Cost Multiplier (RFCM), controlling the degree of conservatism wrt this component.

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- The objective function becomes:

$$\min z = \text{RC} + \text{EOECC} + \text{ERFC} \quad (4)$$

and is non-linear, thus resulting in an MINLP.

Constraints

- Basic routing constraints.
- Inventory constraints:
  - container inventory tracking,
  - no expected overflows for $\mathcal{T}^+$,
  - single-day backorder limit,
  - OU policy.
- Capacity tracking and renewal constraints.
- Time tracking and time window constraints.
- Domain definition.

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- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

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- 10 runs for each instance.
- Simulation-based validation to assess probability information captured by the objective function.

Probabilistic Policies

- We consider two types of objective function:
  - **complete**: minimizes the full probabilistic objective defined by expression (4).
  - **routing-only**: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.

Probabilistic Policies

- We consider two types of objective function:
  - **complete**: minimizes the full probabilistic objective defined by expression (4).
  - **routing-only**: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.
- Probability-related costs:
  - overflow cost χ: 100 CHF (fixed by municipality),
  - emergency collection cost ζ: 100 CHF, 50 CHF, 25 CHF (does not apply to routing-only $=$ 0 CHF),
  - Route Failure Cost Multiplier (RFCM) ψ: 1.00, 0.50, 0.25 (does not apply to routing-only $=$ 0 CHF).

Probabilistic Policies

Table 1: Cost and KPI

Objective	ECC	RFCM	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)	Avg Collected Volume (L)	Liters Per Unit Cost	Liters Per Unit Routing Cost
Complete	100.00	1.00	579.78	99.73	0.03	47,234.59	69.51	81.47
Complete	100.00	0.50	579.46	99.33	0.05	47,225.62	69.57	81.50
Complete	100.00	0.25	577.84	99.93	0.04	47,455.19	70.01	82.13
Complete	100.00	0.00	578.83	98.28	0.00	47,662.90	70.39	82.34
Complete	50.00	1.00	559.44	102.72	0.02	45,646.48	68.93	81.59
Complete	50.00	0.50	558.37	103.82	0.09	45,852.89	69.24	82.12
Complete	50.00	0.25	558.47	103.35	0.07	45,949.94	69.42	82.28
Complete	50.00	0.00	557.16	104.77	0.00	45,788.15	69.17	82.18
Complete	25.00	1.00	547.74	103.46	0.04	44,682.00	68.61	81.57
Complete	25.00	0.50	548.10	103.83	0.11	44,653.66	68.48	81.47
Complete	25.00	0.25	547.75	104.05	0.06	44,678.38	68.54	81.57
Complete	25.00	0.00	546.34	105.37	0.00	44,773.34	68.70	81.95
Routing-only	0.00	0.00	425.08	0.00	0.00	25,286.94	59.49	59.49

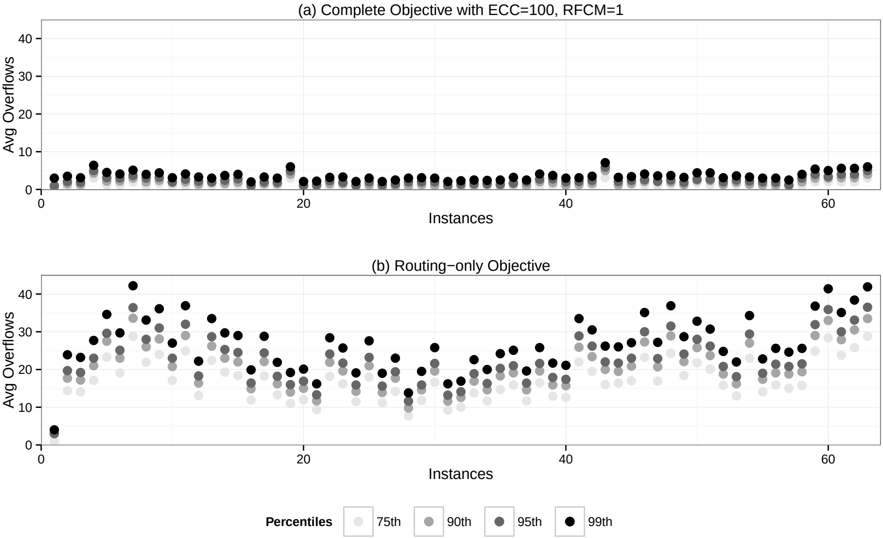
Probabilistic Policies

Table 2: Container Overflows and Route Failures

			Avg Num Overflows				Avg Num Route Failures			
Objective	ECC	RFCM	75th Perc.	90th Perc.	95th Perc.	99th Perc.	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Complete	100.00	1.00	0.98	1.78	2.40	3.58	0.03	0.03	0.04	0.05
Complete	100.00	0.50	0.99	1.78	2.39	3.55	0.04	0.05	0.05	0.07
Complete	100.00	0.25	0.97	1.80	2.38	3.56	0.04	0.05	0.06	0.10
Complete	100.00	0.00	0.94	1.77	2.33	3.54	0.08	0.10	0.12	0.16
Complete	50.00	1.00	1.26	2.19	2.82	4.14	0.05	0.05	0.05	0.05
Complete	50.00	0.50	1.28	2.19	2.84	4.16	0.06	0.07	0.08	0.09
Complete	50.00	0.25	1.28	2.18	2.83	4.15	0.04	0.06	0.07	0.10
Complete	50.00	0.00	1.31	2.23	2.85	4.18	0.07	0.09	0.10	0.12
Complete	25.00	1.00	1.48	2.46	3.14	4.58	0.05	0.05	0.05	0.07
Complete	25.00	0.50	1.48	2.46	3.14	4.58	0.05	0.07	0.07	0.10
Complete	25.00	0.25	1.51	2.50	3.18	4.61	0.04	0.07	0.07	0.09
Complete	25.00	0.00	1.54	2.51	3.19	4.64	0.08	0.10	0.10	0.12
Routing-only	0.00	0.00	16.97	20.45	22.56	26.70	0.01	0.03	0.04	0.05

Probabilistic Policies

Figure 3: Average Number of Overflows for All Instances



Alternative Policies

- An alternative practical policy is the use of artificially low capacities in the solution process:
  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
  - Truck Effective Capacity (TEC): the fraction of the usable truck capacity,
  - tests for values of 1.00, 0.90 and 0.75.

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  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
  - Truck Effective Capacity (TEC): the fraction of the usable truck capacity,
  - tests for values of 1.00, 0.90 and 0.75.
- The simulation experiments are wrt the original capacities.
- The objective is always routing-only.

Alternative Policies

Table 3: Cost and KPI

Objective	CEC	TEC	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)	Avg Collected Volume (L)	Liters Per Unit Cost	Liters Per Unit Routing Cost
Routing-only	1.00	1.00	425.48	0.00	0.00	25,311.81	59.49	59.49
Routing-only	1.00	0.90	426.94	0.00	0.00	25,233.43	59.10	59.10
Routing-only	1.00	0.75	428.02	0.00	0.00	25,371.43	59.28	59.28
Routing-only	0.90	1.00	488.76	0.00	0.00	31,532.12	64.51	64.51
Routing-only	0.90	0.90	489.20	0.00	0.00	31,611.40	64.62	64.62
Routing-only	0.90	0.75	491.91	0.00	0.00	31,732.72	64.51	64.51
Routing-only	0.75	1.00	564.83	0.00	0.00	44,134.12	78.14	78.14
Routing-only	0.75	0.90	570.32	0.00	0.00	44,084.86	77.30	77.30
Routing-only	0.75	0.75	575.98	0.00	0.00	44,079.24	76.53	76.53

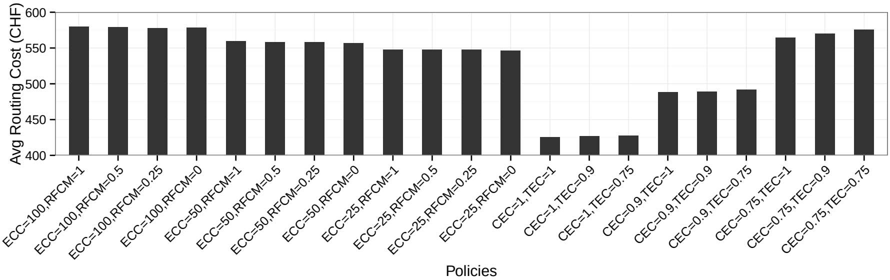
Alternative Policies

Table 4: Container Overflows and Route Failures

			Avg Num Overflows				Avg Num Route Failures			
Objective	CEC	TEC	75th Perc.	90th Perc.	95th Perc.	99th Perc.	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Routing-only	1.00	1.00	16.97	20.45	22.58	26.72	0.01	0.03	0.03	0.04
Routing-only	1.00	0.90	17.02	20.51	22.65	26.80	0.00	0.00	0.00	0.00
Routing-only	1.00	0.75	16.91	20.40	22.54	26.65	0.00	0.00	0.00	0.00
Routing-only	0.90	1.00	10.32	13.14	14.85	18.29	0.02	0.02	0.02	0.02
Routing-only	0.90	0.90	10.30	13.09	14.81	18.24	0.00	0.00	0.00	0.00
Routing-only	0.90	0.75	10.32	13.09	14.85	18.28	0.00	0.00	0.00	0.00
Routing-only	0.75	1.00	4.24	6.08	7.27	9.68	0.03	0.03	0.03	0.03
Routing-only	0.75	0.90	4.24	6.06	7.26	9.68	0.00	0.00	0.00	0.00
Routing-only	0.75	0.75	4.22	6.04	7.26	9.67	0.00	0.00	0.00	0.00

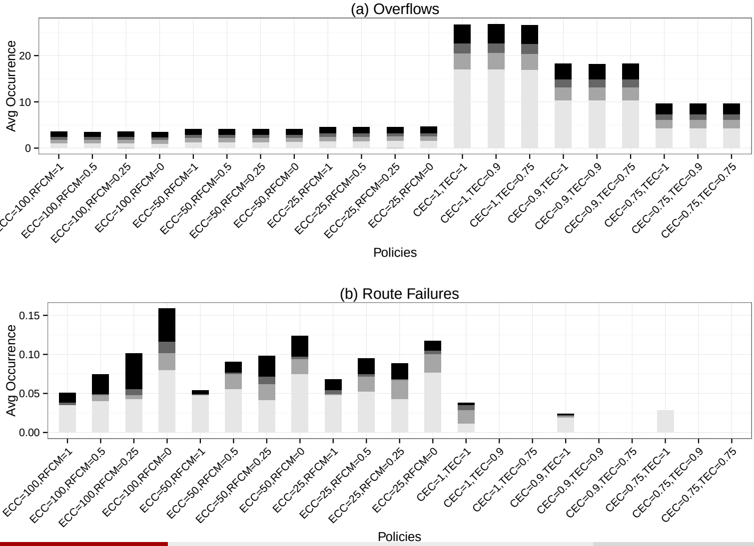
Policy Comparison

Figure 4: Comparison of Routing Cost



Policy Comparison

Figure 5: Comparison of Container Overflows and Route Failures



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Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- Computational experiments on real-data instances demonstrate:
  - the relevance of the probabilistic information captured in the objective,
  - the superiority of the probabilistic approach in comparison to alternative policies.

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- Computational experiments on real-data instances demonstrate:
  - the relevance of the probabilistic information captured in the objective,
  - the superiority of the probabilistic approach in comparison to alternative policies.
- Future research directions:
  - generalizations for solving other problems,
  - chance constraints/robust optimization,
  - value of stochastic information.

Thank you.

Questions?

A1: Average Number of Collections by Day

Table 5: Average Number of Collections by Day

Type	EC	RFCM	day $t=0$	day $t=1$	day $t=2$	day $t=3$	day $t=4$	day $t=5$	day $t=6$	day $t=7$
Complete	100.00	1.00	60	4	15	53	49	—	—	—
Complete	100.00	0.50	60	6	17	54	56	—	—	—
Complete	100.00	0.25	60	5	16	56	52	—	—	—
Complete	100.00	0.00	60	4	14	53	53	—	—	—
Complete	50.00	1.00	59	6	25	56	44	—	—	—
Complete	50.00	0.50	59	7	18	57	44	—	—	—
Complete	50.00	0.25	59	6	20	54	37	—	—	—
Complete	50.00	0.00	59	6	23	55	43	—	—	—
Complete	25.00	1.00	57	8	27	54	31	—	—	—
Complete	25.00	0.50	57	8	24	56	26	—	—	—
Complete	25.00	0.25	57	8	24	55	29	—	—	—
Complete	25.00	0.00	57	9	28	54	34	—	—	—
Routing-only	0.00	0.00	53	60	45	7	3	—	—	—

A2: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $\mathcal{T}$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.
- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).

A2: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $\mathcal{T}$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.
- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).
- We hypothesize that the solution cost of a DSIRP is bounded:
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- Tests on 41 instances, each covering two weeks of white glass collections in the canton of Geneva, Switzerland in 2014, 2015, or 2016.

A2: Rolling Horizon Approach

Table 6: Analysis of Rolling Horizon DSIRP Bounds

Instance	Static IRP with True Demand	Rolling DSIRP with Forecast Demand	Static SIRP with Forecast Demand	Instance	Static IRP with True Demand	Rolling DSIRP with Forecast Demand	Static SIRP with Forecast Demand
Inst_1	276.44	582.89	665.19	Inst_22	429.20	531.04	607.63
Inst_2	448.67	784.55	854.49	Inst_23	241.44	551.58	690.62
Inst_3	307.95	653.60	819.79	**Inst_24**	**547.92**	**758.84**	**748.71**
Inst_4	266.15	574.23	700.36	Inst_25	446.31	618.80	696.75
Inst_5	454.61	682.24	824.57	Inst_26	442.38	589.53	695.11
Inst_6	485.30	677.92	764.86	Inst_27	441.36	589.07	707.30
Inst_7	268.65	569.11	649.57	Inst_28	468.46	616.53	738.58
Inst_8	429.56	585.42	681.23	Inst_29	436.25	575.25	701.73
Inst_9	442.34	599.24	659.30	Inst_30	414.41	677.65	690.37
Inst_10	448.70	564.04	650.88	Inst_31	442.87	544.75	668.51
Inst_11	467.88	549.61	670.36	Inst_32	255.32	612.44	694.35
Inst_12	**449.20**	**674.53**	**626.18**	Inst_33	460.04	677.54	808.74
Inst_13	254.66	556.94	629.93	Inst_34	505.55	682.90	711.62
Inst_14	276.60	585.77	683.65	**Inst_35**	**490.37**	**989.21**	**785.51**
Inst_15	431.08	548.56	790.39	Inst_36	454.60	646.95	805.95
Inst_16	529.60	635.37	701.64	Inst_37	465.31	607.52	746.64
Inst_17	423.07	578.84	662.76	Inst_38	520.38	721.23	815.21
Inst_18	458.18	595.36	680.75	Inst_39	243.94	613.96	705.10
Inst_19	448.66	524.63	611.56	Inst_40	450.94	624.76	759.97
Inst_20	418.12	520.30	653.18	Inst_41	403.01	575.80	688.24
Inst_21	**276.32**	**791.63**	**626.29**				

Note: The four instances for which the hypothesized bounds do not hold are shown in bold.

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