Waste collection inventory routing with non-stationary stochastic demands

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Outline

1. Introduction
2. Related Literature
3. Formulation
4. Methodology
5. Numerical Experiments
6. Conclusion
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
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Level data is used for container selection and route planning.
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- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
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- Level data is used for container selection and route planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection thus depends on the ability to:
  - forecast container levels,
  - select the containers to collect each day,
  - and route the vehicles in an (near-)optimal way.
The setup falls within the framework of the IRP with:

- stochastic demands,
- order-up-to level (OU) policy,
- no allowed expected overflows,
- single-day backorder limit.
Introduction

Problem Definition

- The setup falls within the framework of the IRP with:
  - stochastic demands,
  - order-up-to level (OU) policy,
  - no allowed expected overflows,
  - single-day backorder limit.

- The routing component includes:
  - intermediate facility visits (recycling plants),
  - heterogeneous capacitated vehicles,
  - site dependencies,
  - vehicle-to-period availabilities,
  - time windows,
  - maximum tour duration.
Related VRP Literature

- **VRP with intermediate facilities (VRP-IF):**
  - Bard et al. (1998a), Kim et al. (2006), Crevier et al. (2007).

- **Electric and alternative fuel VRP:**

- **Heterogeneous fixed fleet VRP:**
  - Taillard (1999), Baldacci and Mingozzi (2009), Subramanian et al. (2012), Penna et al. (2013).
  - Hiermann et al. (2014) and Goeke and Schneider (2015) use some form of vehicle heterogeneity in the electric VRP.
Related Literature

Related Stochastic IRP Literature

- Early research on optimal replenishment policies in a stochastic setting:

- Robust optimization:
  - Solyalı et al. (2012).

- Chance constraints:

- Scenario based:
Contributions

- Dynamic probabilistic information on overflows and route failures.
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- Demand forecasting model tested and validated on real data (Markov et al., 2015).
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- A rich IRP with features traditionally absent or rarely considered in the IRP literature.
Contributions

- Dynamic probabilistic information on overflows and route failures.
- Demand forecasting model tested and validated on real data (Markov et al., 2015).
- A rich IRP with features traditionally absent or rarely considered in the IRP literature.
- ALNS algorithm performs very well on IRP benchmarks from the literature.
- Benefit of considering uncertainty in the objective function evaluated on instances derived from real data.
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# Nomenclature

## Sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( o )</td>
<td>origin</td>
</tr>
<tr>
<td>( \mathcal{D} )</td>
<td>set of dumps</td>
</tr>
<tr>
<td>( \mathcal{N} )</td>
<td>( {o} \cup {d} \cup \mathcal{D} \cup \mathcal{P} )</td>
</tr>
<tr>
<td>( \mathcal{T} )</td>
<td>( {0, \ldots, u} )</td>
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</table>

## Parameters

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{ij} )</td>
<td>length of arc ((i, j))</td>
</tr>
<tr>
<td>( \tau_{ijk} )</td>
<td>travel time of vehicle (k) on arc ((i, j))</td>
</tr>
<tr>
<td>( \lambda_i, \mu_i )</td>
<td>lower and upper time window bound at point (i)</td>
</tr>
<tr>
<td>( \delta_i )</td>
<td>service duration at point (i)</td>
</tr>
<tr>
<td>( \rho_{it} )</td>
<td>demand of container (i) on day (t) (random variable)</td>
</tr>
<tr>
<td>( \omega_i )</td>
<td>capacity of container (i)</td>
</tr>
<tr>
<td>( \chi )</td>
<td>container overflow cost (monetary)</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>emergency collection cost (monetary)</td>
</tr>
</tbody>
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Nomenclature

Sets

\[ \begin{align*}
o & \text{ origin} \\
\mathcal{D} & \text{ set of dumps} \\
\mathcal{N} & = \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P} \\
\mathcal{T} & = \{0, \ldots, u\} \\
\mathcal{T}^+ & = \{1, \ldots, u + 1\} \end{align*} \]

Parameters

\[ \begin{align*}
\sigma_{it} & = 1 \text{ if container } i \text{ is in a state of full and overflowing on day } t, 0 \text{ otherwise} \\
\Omega_k & \text{ capacity of vehicle } k \\
\varphi_k & \text{ daily deployment cost of vehicle } k \text{ (monetary)} \\
\beta_k & \text{ unit-distance running cost of vehicle } k \text{ (monetary)} \\
\theta_k & \text{ unit-time running cost of vehicle } k \text{ (monetary)} \\
\alpha_{kt} & = 1 \text{ if vehicle } k \text{ is available on day } t, 0 \text{ otherwise} \\
\alpha_{ik} & = 1 \text{ if point } i \text{ is accessible by vehicle } k, 0 \text{ otherwise} \\
H & \text{ maximum tour duration} \end{align*} \]
Nomenclature

Decision variables: binary

\[ x_{ijkt} = \begin{cases} 
1 & \text{if vehicle } k \text{ traverses arc } (i, j) \text{ on day } t \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_{ikt} = \begin{cases} 
1 & \text{if vehicle } k \text{ visits point } i \text{ on day } t \\
0 & \text{otherwise} 
\end{cases} \]

\[ z_{kt} = \begin{cases} 
1 & \text{if vehicle } k \text{ is used on day } t \\
0 & \text{otherwise} 
\end{cases} \]

Decision variables: continuous

\[ q_{ikt} \quad \text{expected pickup quantity by vehicle } k \text{ at point } i \text{ on day } t \]

\[ Q_{ikt} \quad \text{expected cumulative quantity on vehicle } k \text{ at point } i \text{ on day } t \]

\[ l_{it} \quad \text{expected inventory at point } i \text{ at the start of day } t \]

\[ S_{ikt} \quad \text{start-of-service time of vehicle } k \text{ at point } i \text{ on day } t \]
Demand is the amount deposited in a container on each day, and is random and non-stationary.
Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.

- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error $\varsigma$.

The forecasting error is the st. dev. of the residuals based on a historical fit. Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures. The probabilities are dynamic and conditional, and depend on:

- the evolution of container states over the planning horizon,
- and the vehicle visits on each day.
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  - the evolution of container states over the planning horizon,
  - and the vehicle visits on each day.
Objective Function

Routing cost + Expected overflow and emergency visit cost + Expected route failure cost

Lower routing cost is counterbalanced by more overflows and route failures, and vice versa. Our goal is to minimize the expected monetary value of all components.
Formulation

Objective Function

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Objective Function: Main Concepts

- **Two container states:**
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.
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  - $\sigma_{it} = 1$: full and overflowing.

- **Two types of container collection:**
  - regular collection of container $i$ on day $t$: $\exists k \in K : y_{ikt} = 1$,
  - emergency collection of container $i$ on day $t$: $\sigma_{it} = 1$ and $y_{ikt} = 0$, $\forall k \in K$. 

Related costs:
- overflow cost $\chi$: paid in state $\sigma_{it} = 1$,
- emergency collection cost $\zeta$: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0$, $\forall k \in K$. 

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- Related costs:
  - overflow cost $\chi$: paid in state $\sigma_{it} = 1$,
  - emergency collection cost $\zeta$: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in K$. 
Figure 1: Container state probability tree
Objective Function: Formulation

- Routing cost (RC):

  \[
  \sum_{t \in T} \sum_{k \in K} \left( \varphi_k z_{kt} + \beta_k \sum_{i \in N} \sum_{j \in N} \pi_{ij} x_{ijkt} + \theta_k \left( S_{dkt} - S_{okt} \right) \right)
  \]  

  (1)
Objective Function: Formulation

- **Routing cost (RC):**
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  \]  
  (1)

- **Expected overflow and emergency collection cost (EOECC):**
  \[
  \sum_{t \in T+} \sum_{i \in P} \left( \mathbb{P} (\sigma_{it} = 1 \mid \max (0, g < t: \exists k \in K: y_{ikg} = 1)) \left( \chi + \zeta - \zeta \sum_{k \in K} y_{ikt} \right) \right)
  \]  
  (2)
Objective Function: Formulation

• Expected route failure cost (ERFC):

\[
\sum_{t \in T \setminus 0} \sum_{k \in K} \sum_{S \in \mathcal{S}_{kt}} \left( C_S \mathbb{P} \left( \sum_{s \in S} l_{st} > \Omega_k \left| \max(0, g < t : y_{skg} = 1) \right. \right) \right),
\]

where

- \( \mathcal{S}_{kt} = \mathcal{S}_{kt} (y_{ikt}, \forall i \in D) \) is the set of depot-to-dump or dump-to-dump trips for vehicle \( k \) on day \( t \),
- \( S \) is the set of containers in a particular trip,
- \( C_S \) is the average routing cost of going from this set to the nearest dump and back.
Objective Function: Formulation

- **Expected route failure cost (ERFC):**
  \[
  \sum_{t \in T \setminus 0} \sum_{k \in K} \sum_{S \in S_{kt}} \left( C_S \mathbb{P} \left( \sum_{s \in S} l_{st} > \Omega_k \left| \max(0, g < t: y_{skg} = 1) \right. \right) \right),
  \]
  \begin{align*}
  \text{where} \\
  - S_{kt} = S_{kt}(y_{ikt}, \forall i \in D) \text{ is the set of depot-to-dump or dump-to-dump} \\
  \text{trips for vehicle } k \text{ on day } t, \\
  - S \text{ is the set of containers in a particular trip,} \\
  - C_S \text{ is the average routing cost of going from this set to the nearest dump and back.}
  \end{align*}

- The objective function becomes
  \[
  z(\cdot) = RC + EOECC + ERFC
  \]
  and is non-linear, thus resulting in an MINLP.
Constraints: Basic routing

\[ \sum_{j \in \mathcal{N}} x_{o j k t} = \alpha_{k t} z_{k t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \]  
\[ \sum_{i \in \mathcal{D}} x_{i d k t} = \alpha_{k t} z_{k t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \]  
\[ y_{i k t} = \sum_{j \in \mathcal{N}} x_{i j k t} = \sum_{j \in \mathcal{N}} x_{j i k t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]  
\[ \sum_{k \in \mathcal{K}} y_{i k t} \leq 1, \quad \forall t \in \mathcal{T}, i \in \mathcal{P} \]  
\[ y_{i k t} \leq \alpha_{i k}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]  
\[ \sum_{i \in \mathcal{N}} x_{i j k t} = \sum_{i \in \mathcal{N}} x_{j i k t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{D} \cup \mathcal{P} \]  
\[ l_{i t} = l_{i (t-1)} - \sum_{k \in \mathcal{K}} q_{i k (t-1)} + \mathbb{E}(\rho_{i (t-1)}), \quad \forall t \in \mathcal{T}^{+}, i \in \mathcal{P} \]  
\[ l_{i t} \leq \omega_{i}, \quad \forall t \in \mathcal{T}^{+}, i \in \mathcal{P} \]  
\[ l_{i 0} - \omega_{i} \leq \omega_{i} \sum_{k \in \mathcal{K}} y_{i k 0}, \quad \forall i \in \mathcal{P} \]  
\[ q_{i k t} \leq M y_{i k t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]  
\[ q_{i k t} \leq l_{i t}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]  
\[ q_{i k t} \geq l_{i t} - M (1 - y_{i k t}), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]
Constraints: Inventory balance

\[
\sum_{j \in \mathcal{N}} x_{ojkt} = \alpha_{kt} z_{kt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} 
\]

\[
\sum_{i \in \mathcal{D}} x_{idkt} = \alpha_{kt} z_{kt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} 
\]

\[
y_{ikt} = \sum_{j \in \mathcal{N}} x_{ijkt} = \sum_{j \in \mathcal{N}} x_{jikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} 
\]

\[
\sum_{k \in \mathcal{K}} y_{ikt} \leq 1, \quad \forall t \in \mathcal{T}, i \in \mathcal{P} 
\]

\[
y_{ikt} \leq \alpha_{ik}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} 
\]

\[
\sum_{i \in \mathcal{N}} x_{ijkt} = \sum_{i \in \mathcal{N}} x_{jikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{D} \cup \mathcal{P} 
\]

\[
l_{it} = l_{i(t-1)} - \sum_{k \in \mathcal{K}} q_{ikt(t-1)} + \mathbb{E}(\rho_{i(t-1)}), \quad \forall t \in \mathcal{T}^+, i \in \mathcal{P} 
\]

\[
l_{it} \leq \omega_i, \quad \forall t \in \mathcal{T}^+, i \in \mathcal{P} 
\]

\[
l_{i0} - \omega_i \leq \omega_i \sum_{k \in \mathcal{K}} y_{ik0}, \quad \forall i \in \mathcal{P} 
\]

\[
q_{ikt} \leq \omega_{y_{ikt}}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} 
\]

\[
q_{ikt} \leq l_{it}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} 
\]

\[
q_{ikt} \geq l_{it} - M(1 - y_{ikt}), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} 
\]
Constraints: Capacity related

\[ q_{ikt} \leq Q_{ikt} \leq \Omega_k, \quad \forall t \in T, k \in K, i \in P \]  
\[ Q_{ikt} = 0, \quad \forall t \in T, k \in K, i \in \mathcal{N} \setminus \mathcal{P} \]  
\[ Q_{ikt} + q_{jkt} \leq Q_{jkt} + \Omega_k (1 - x_{ijkt}), \quad \forall t \in T, k \in K, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \]  
\[ S_{ikt} + \delta_i + \tau_{ijk} \leq S_{jkt} + (\mu_i + \delta_i + \tau_{ijk}) (1 - x_{ijkt}), \quad \forall t \in T, k \in K, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \]  
\[ \lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leq S_{ikt}, \quad \forall t \in T, k \in K, i \in \mathcal{N} \setminus \{d\} \]  
\[ S_{jkt} \leq \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \quad \forall t \in T, k \in K, j \in \mathcal{N} \setminus \{o\} \]  
\[ 0 \leq S_{dkt} - S_{okt} \leq H, \quad \forall t \in T, k \in K \]  
\[ x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \quad \forall t \in T, k \in K, i, j \in \mathcal{N} \]  
\[ q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geq 0, \quad \forall t \in T, k \in K, i \in \mathcal{N} \]
Formulation

Constraints: Time related

\[ q_{ikt} \leq Q_{ikt} \leq \Omega_k, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \]  
(17)

\[ Q_{ikt} = 0, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \]  
(18)

\[ Q_{ikt} + q_{jkt} \leq Q_{jkt} + \Omega_k (1 - x_{ijkt}), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \]  
(19)

\[ S_{ikt} + \delta_i + \tau_{ijk} \leq S_{jkt} + (\mu_i + \delta_i + \tau_{ijk})(1 - x_{ijkt}), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \]  
(20)

\[ \lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leq S_{ikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\} \]  
(21)

\[ S_{jkt} \leq \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{N} \setminus \{o\} \]  
(22)

\[ 0 \leq S_{dkt} - S_{okt} \leq H, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \]  
(23)

\[ x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i, j \in \mathcal{N} \]  
(24)

\[ q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geq 0, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \]  
(25)
Constraints: Domain

\( q_{ikt} \leq Q_{ikt} \leq \Omega_k, \quad \forall t \in T, k \in K, i \in P \) \hspace{1cm} (17)

\( Q_{ikt} = 0, \quad \forall t \in T, k \in K, i \in N \setminus P \) \hspace{1cm} (18)

\( Q_{ikt} + q_{jkt} \leq Q_{jkt} + \Omega_k (1 - x_{ijkt}), \quad \forall t \in T, k \in K, i \in N \setminus \{d\}, j \in P \) \hspace{1cm} (19)

\( S_{ikt} + \delta_i + \tau_{ijk} \leq S_{jkt} + (\mu_i + \delta_i + \tau_{ijk})(1 - x_{ijkt}), \quad \forall t \in T, k \in K, i \in N \setminus \{d\}, j \in N \setminus \{o\} \) \hspace{1cm} (20)

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\( 0 \leq S_{dkt} - S_{okt} \leq H \), \hspace{1cm} (23)

\( x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \quad \forall t \in T, k \in K, i, j \in N \) \hspace{1cm} (24)

\( q_{ikt}, Q_{ikt}, l_{it}, S_{ikt} \geq 0, \quad \forall t \in T, k \in K, i \in N \) \hspace{1cm} (25)
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Adaptive Large Neighborhood Search (ALNS)

Solved by ALNS with the following operators:

**Destroy operators:**
- remove $\rho$ containers randomly.
- remove $\rho$ worst containers.
- Shaw removals (Shaw, 1997).
- empty a random day.
- empty a random vehicle.
- remove a random dump.
- remove the worst dump.
- remove consecutive visits.

**Repair operators:**
- insert $\rho$ containers randomly.
- insert $\rho$ containers in the best way.
- Shaw insertions (Shaw, 1997).
- swap $\rho$ random containers.
- insert a dump randomly.
- swap random dumps.
- replace a random dump.
- reorder dumps DP operator.
Reorder dumps DP Operator (Hemmelmayr et al., 2013)

Figure 2: Feasibility graph of the *reorder dumps* DP operator

- Preserves/restores vehicle capacity feasibility.
- Removes all dump visits and reinserts them in a locally optimal way solving a shortest path problem using the Bellman-Ford algorithm.
- Followed by local search improvement using 2-opt.
The Search Strategy

- Accepting intermediate infeasible solutions facilitates the exploration of the search space of tightly constrained problems.

- We allow the following feasibility violations of the solution $s$:
  - $V^\Omega(s)$: vehicle capacity violation
  - $V^\mu(s)$: time window violation
  - $V^H(s)$: duration violation
  - $V^\omega(s)$: container capacity violation
  - $V^0(s)$: backorder limit violation
  - $V^\alpha(s)$: accessibility violation
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  - $V^H(s)$: duration violation
  - $V^\omega(s)$: container capacity violation
  - $V^0(s)$: backorder limit violation
  - $V^\alpha(s)$: accessibility violation

- The solution representation during the search is:

$$f(s) = z(s) + L^\Omega V^\Omega(s) + L^\mu V^\mu + L^H V^H(s) + L^\omega V^\omega(s) + L^0 V^0(s) + L^\alpha V^\alpha(s)$$ \hspace{1cm} (26)

with the penalties $L^\Omega$ through $L^\alpha$ dynamically adjusted during the search to encourage or discourage infeasible solutions.
Numerical Experiments

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Archetti et al. (2007) Instances

- First classical IRP testbed.
- 160 instances in total.
- 5 to 50 customers.
- 3 or 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.
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- Single vehicle.
- Low and high inventory holding costs.

- Heuristic solutions by Archetti et al. (2012), Coelho et al. (2012a), Coelho et al. (2012b), etc...
- We solve each instance 10 times and report best and average results.
Archetti et al. (2007) Instances

Table 1: Results on instances with high inventory holding cost

<table>
<thead>
<tr>
<th></th>
<th>ALNS fast version</th>
<th></th>
<th>ALNS slow version</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Runtime(s.)</td>
<td>Best Gap(%)</td>
<td>Avg Gap(%)</td>
<td>Runtime(s.)</td>
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<td>0.00</td>
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<td>22</td>
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<td>0.00</td>
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<td>0.01</td>
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<td>276</td>
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<td>0.14</td>
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<td>0.64</td>
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<td>189</td>
<td>0.08</td>
<td>0.70</td>
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<tr>
<td>6</td>
<td></td>
<td>90</td>
<td>0.05</td>
<td>0.22</td>
</tr>
</tbody>
</table>
### Table 2: Results on instances with low inventory holding cost

<table>
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<tr>
<th>$u$</th>
<th>$n$</th>
<th>ALNS fast version</th>
<th>ALNS slow version</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Runtime(s.)</td>
<td>Best Gap(%)</td>
</tr>
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<td>5</td>
<td>7</td>
<td>0.00</td>
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<td>10</td>
<td>14</td>
<td>0.00</td>
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<tr>
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<td>15</td>
<td>22</td>
<td>0.00</td>
</tr>
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<td>20</td>
<td>34</td>
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<td>40</td>
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<td>0.37</td>
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<td>191</td>
<td>0.59</td>
</tr>
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<td>247</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
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<td>0.00</td>
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<tr>
<td>6</td>
<td>10</td>
<td>28</td>
<td>0.00</td>
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<tr>
<td>6</td>
<td>15</td>
<td>49</td>
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<td>6</td>
<td>20</td>
<td>77</td>
<td>0.08</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>121</td>
<td>0.25</td>
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<td>182</td>
<td>0.67</td>
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<tr>
<td></td>
<td></td>
<td>Average 84</td>
<td>0.14</td>
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</tbody>
</table>
Archetti et al. (2012) Instances

- 60 instances in total.
- 50, 100 and 200 customers.
- 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.
Archetti et al. (2012) Instances

- 60 instances in total.
- 50, 100 and 200 customers.
- 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

Solved by Archetti et al. (2012) using a hybrid heuristic algorithm.

For the moment we have solved the 50-customer instances 10 times and provide best and average results.
Archetti et al. (2012) Instances

Table 3: Results on 50-customer instances with high inventory holding cost

<table>
<thead>
<tr>
<th>Instance</th>
<th>Archetti et al. (2012)</th>
<th>Runtime(s.)</th>
<th>Best Cost</th>
<th>Avg Cost</th>
<th>Best Gap(%)</th>
<th>Avg Gap(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>abs1n50</td>
<td>31,147.82</td>
<td>670</td>
<td>30,708.05</td>
<td>30,809.31</td>
<td>-1.41</td>
<td>-1.09</td>
</tr>
<tr>
<td>abs2n50</td>
<td>30,192.51</td>
<td>676</td>
<td>30,226.23</td>
<td>30,271.07</td>
<td>0.11</td>
<td>0.26</td>
</tr>
<tr>
<td>abs3n50</td>
<td>30,420.96</td>
<td>667</td>
<td>30,388.68</td>
<td>30,515.79</td>
<td>-0.11</td>
<td>0.31</td>
</tr>
<tr>
<td>abs4n50</td>
<td>31,898.84</td>
<td>671</td>
<td>32,103.17</td>
<td>32,213.62</td>
<td>0.64</td>
<td>0.99</td>
</tr>
<tr>
<td>abs5n50</td>
<td>29,518.68</td>
<td>666</td>
<td>29,646.74</td>
<td>29,797.79</td>
<td>0.43</td>
<td>0.95</td>
</tr>
<tr>
<td>abs6n50</td>
<td>32,394.50</td>
<td>652</td>
<td>32,336.81</td>
<td>32,420.63</td>
<td>-0.18</td>
<td>0.08</td>
</tr>
<tr>
<td>abs7n50</td>
<td>30,165.00</td>
<td>661</td>
<td>30,222.28</td>
<td>30,269.23</td>
<td>0.19</td>
<td>0.35</td>
</tr>
<tr>
<td>abs8n50</td>
<td>26,416.46</td>
<td>652</td>
<td>26,409.83</td>
<td>26,537.19</td>
<td>-0.03</td>
<td>0.46</td>
</tr>
<tr>
<td>abs9n50</td>
<td>30,671.88</td>
<td>656</td>
<td>30,543.31</td>
<td>30,630.53</td>
<td>-0.42</td>
<td>-0.13</td>
</tr>
<tr>
<td>abs10n50</td>
<td>32,362.01</td>
<td>635</td>
<td>31,937.51</td>
<td>32,065.85</td>
<td>-1.31</td>
<td>-0.92</td>
</tr>
<tr>
<td>Average</td>
<td>30,518.87</td>
<td>661</td>
<td>30,452.26</td>
<td>30,553.10</td>
<td>-0.21</td>
<td>0.13</td>
</tr>
</tbody>
</table>
## Archetti et al. (2012) Instances

### Table 4: Results on 50-customer instances with low inventory holding cost

<table>
<thead>
<tr>
<th>Instance</th>
<th>Archetti et al. (2012)</th>
<th>Runtime(s.)</th>
<th>Best Cost</th>
<th>Avg Cost</th>
<th>Best Gap(%)</th>
<th>Avg Gap(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>abs1n50</td>
<td>10,409.13</td>
<td>611</td>
<td>10,377.36</td>
<td>10,449.91</td>
<td>-0.31</td>
<td>0.39</td>
</tr>
<tr>
<td>abs2n50</td>
<td>10,881.35</td>
<td>643</td>
<td>10,927.83</td>
<td>11,014.20</td>
<td>0.43</td>
<td>1.22</td>
</tr>
<tr>
<td>abs3n50</td>
<td>10,767.39</td>
<td>622</td>
<td>10,702.05</td>
<td>10,924.09</td>
<td>-0.61</td>
<td>1.46</td>
</tr>
<tr>
<td>abs4n50</td>
<td>10,656.21</td>
<td>632</td>
<td>10,711.86</td>
<td>10,875.98</td>
<td>0.52</td>
<td>2.06</td>
</tr>
<tr>
<td>abs5n50</td>
<td>10,234.60</td>
<td>624</td>
<td>10,332.55</td>
<td>10,458.54</td>
<td>0.96</td>
<td>2.19</td>
</tr>
<tr>
<td>abs6n50</td>
<td>10,533.63</td>
<td>620</td>
<td>10,388.66</td>
<td>10,485.54</td>
<td>-1.38</td>
<td>-0.45</td>
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<tr>
<td>abs7n50</td>
<td>10,460.82</td>
<td>626</td>
<td>10,388.08</td>
<td>10,497.06</td>
<td>-0.70</td>
<td>0.35</td>
</tr>
<tr>
<td>abs8n50</td>
<td>10,411.20</td>
<td>623</td>
<td>10,683.31</td>
<td>10,771.40</td>
<td>2.61</td>
<td>3.46</td>
</tr>
<tr>
<td>abs9n50</td>
<td>10,305.69</td>
<td>610</td>
<td>10,416.97</td>
<td>10,472.96</td>
<td>1.08</td>
<td>1.62</td>
</tr>
<tr>
<td>abs10n50</td>
<td>10,470.63</td>
<td>598</td>
<td>10,047.06</td>
<td>10,153.50</td>
<td>-4.05</td>
<td>-3.03</td>
</tr>
<tr>
<td>Average</td>
<td>10,513.07</td>
<td>621</td>
<td>10,497.57</td>
<td>10,610.33</td>
<td>-0.14</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Instances Based on Real Data

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Maximum tour duration of 4 hours.
- Time windows from 8h00 to 12h00.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.
Instances Based on Real Data

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- Time windows from 8h00 to 12h00.
- Planning horizon of 7 days.
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- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

- We solve each instance 10 times and provide best and average results.
- We simulate the forecasting error realizations and evaluate the relevance of the probability information captured by the objective function.
Real Data: The Relevant Costs

- **Truck related:**
  - vehicle per day: 100 CHF,
  - vehicle per km: 2.95 CHF,
  - driver per hour: 40 CHF.

- **Container related:**
  - overflow cost: 100 CHF,
  - emergency collection cost: 100 CHF.

- **Route failure related:**
  - cost of visiting the nearest dump from a cluster (as defined).
Two Problem Types

- **Routing-only:**
  - Optimizes the routing cost only in the objective function, disregarding all probability information.
  - In other words, it ignores the risk of container overflows and route failures.

- **Complete:**
  - Optimizes the complete objective function as previously defined.
### Table 5: Basic results for real data instances

<table>
<thead>
<tr>
<th></th>
<th>Runtime (s.)</th>
<th>Avg Num Tours</th>
<th>Avg Num Containers</th>
<th>Avg Num Dump Visits</th>
<th>Best Cost (CHF)</th>
<th>Avg Cost (CHF)</th>
<th>Gap Best (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routing-only</td>
<td>302.69</td>
<td>1.85</td>
<td>16.76</td>
<td>1.87</td>
<td>425.97</td>
<td>430.61</td>
<td>1.16</td>
</tr>
<tr>
<td>Complete</td>
<td>268.76</td>
<td>1.98</td>
<td>43.33</td>
<td>2.27</td>
<td>668.37</td>
<td>693.66</td>
<td>3.75</td>
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</table>

### Table 6: Cost breakdown and KPI for real data instances

<table>
<thead>
<tr>
<th></th>
<th>Avg Routing Cost (CHF)</th>
<th>Avg Overflow Cost (CHF)</th>
<th>Avg Rte Failure Cost (CHF)</th>
<th>Avg Collected Volume (L)</th>
<th>Liters per Unit Cost</th>
<th>Liters per Unit Routing Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routing-only</td>
<td>430.61</td>
<td>0.00</td>
<td>0.00</td>
<td>25,106.81</td>
<td>58.31</td>
<td>58.31</td>
</tr>
<tr>
<td>Complete</td>
<td>588.16</td>
<td>105.44</td>
<td>0.06</td>
<td>47,364.96</td>
<td>68.28</td>
<td>80.53</td>
</tr>
</tbody>
</table>
Real Data: Taking Advantage of Probability Information

Figure 3: Cost percentiles of container overflows
Figure 4: Container overflow percentiles for routing-only objective
Real Data: Taking Advantage of Probability Information

Figure 5: Container overflow percentiles for complete objective
**Figure 6:** Route failure percentiles for routing-only objective
Real Data: Taking Advantage of Probability Information

Figure 7: Route failure percentiles for complete objective
Outline

1. Introduction
2. Related Literature
3. Formulation
4. Methodology
5. Numerical Experiments
6. Conclusion
Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces very good results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.
Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces very good results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.
- Future research directions:
  - decomposition methods,
  - scenario generation,
  - robust optimization,
  - location-routing, open tours, online re-optimization, multiple flows...
Thank you.

Questions?


