Waste collection inventory routing with non-stationary stochastic demands

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TRISTAN IX

Oranjestad, Aruba, June 13–17, 2016

Outline

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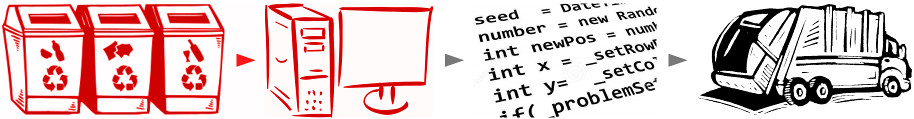
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.



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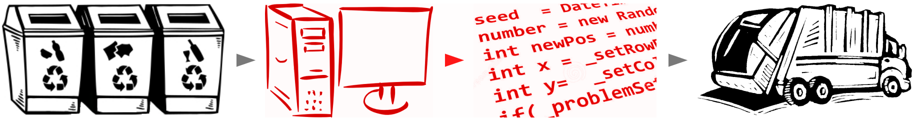

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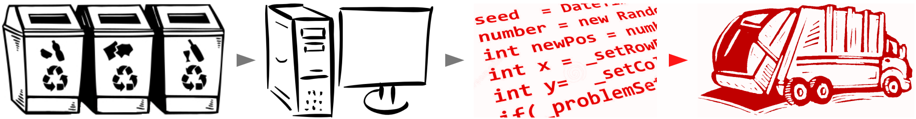
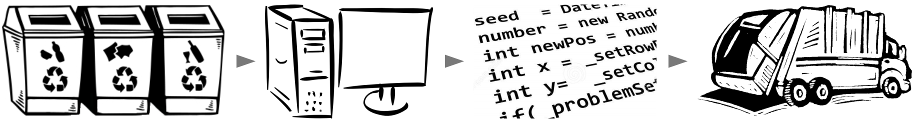


Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and route planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection thus depends on the ability to:
  - forecast container levels,
  - select the containers to collect each day,
  - and route the vehicles in an (near-)optimal way.



Problem Definition

- The setup falls within the framework of the IRP with:
  - stochastic demands,
  - order-up-to level (OU) policy,
  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).

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  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).
- The routing component includes:
  - intermediate facility visits (recycling plants),
  - heterogeneous capacitated vehicles,
  - site dependencies,
  - vehicle-to-period availabilities,
  - time windows,
  - maximum tour duration.

Outline

Related VRP Literature

- VRP with intermediate facilities (VRP-IF):
  - Bard et al. (1998a), Kim et al. (2006), Crevier et al. (2007).
- Electric and alternative fuel VRP:
  - Conrad and Figliozzi (2011), Erdoğan and Miller-Hooks (2012), Schneider et al. (2014), Schneider et al. (2015).
- Heterogeneous fixed fleet VRP:
  - Taillard (1999), Baldacci and Mingozzi (2009), Subramanian et al. (2012), Penna et al. (2013).
  - Hiermann et al. (2014) and Goeke and Schneider (2015) use some form of vehicle heterogeneity in the electric VRP.

Related Stochastic IRP Literature

- Early research on optimal replenishment policies in a stochastic setting:
  - Trudeau and Dror (1992), Jaillet et al. (2002), Bard et al. (1998b).
- Robust optimization:
  - Solyalı et al. (2012).
- Chance constraints:
  - Soysal et al. (2015), Abdollahi et al. (2014), Yu et al. (2012).
- Scenario based:
  - roll-out/branch-and-cut: Bertazzi et al. (2013), Bertazzi et al. (2015).
  - stochastic optimization: Hemmelmayr et al. (2010), Nolz et al. (2014), Adulyasak et al. (2015).

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- Dynamic probabilistic information on overflows and route failures.
- Demand forecasting model tested and validated on real data (Markov et al., 2015).
- A rich IRP with features traditionally absent or rarely considered in the IRP literature.
- ALNS algorithm performs very well on IRP benchmarks from the literature.
- Benefit of considering uncertainty in the objective function evaluated on instances derived from real data.

Outline

Nomenclature

Sets

o	origin	d	destination
$\mathcal{D}$	set of dumps	$\mathcal{P}$	set of containers
$\mathcal{N}$	$= \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$	$\mathcal{K}$	set of vehicles
$\mathcal{T}$	$= \{0, ..., u\}$	$\mathcal{T}^+$	$= \{1, ..., u+1\}$

Parameters

π_{ij}	length of arc (i, j)
τ_{ijk}	travel time of vehicle k on arc (i, j)
λ_i, μ_i	lower and upper time window bound at point i
δ_i	service duration at point i
ρ_{it}	demand of container i on day t (random variable)
ω_i	capacity of container i
χ	container overflow cost (monetary)
ζ	emergency collection cost (monetary)

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Parameters

σ_{it}	$= 1$ if container i is in a state of full and overflowing on day t, 0 otherwise
Ω_k	capacity of vehicle k
φ_k	daily deployment cost of vehicle k (monetary)
β_k	unit-distance running cost of vehicle k (monetary)
θ_k	unit-time running cost of vehicle k (monetary)
α_{kt}	$= 1$ if vehicle k is available on day t, 0 otherwise
α_{ik}	$= 1$ if point i is accessible by vehicle k, 0 otherwise
H	maximum tour duration

Nomenclature

Decision variables: binary

$$x_{ijkt} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses arc } (i,j) \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ikt} = \begin{cases} 1 & \text{if vehicle } k \text{ visits point } i \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{kt} = \begin{cases} 1 & \text{if vehicle } k \text{ is used on day } t \\ 0 & \text{otherwise} \end{cases}$$

Decision variables: continuous

q_{ikt}	expected pickup quantity by vehicle k at point i on day t
Q_{ikt}	expected cumulative quantity on vehicle k at point i on day t
I_{it}	expected inventory at point i at the start of day t
S_{ikt}	start-of-service time of vehicle k at point i on day t

Forecasting Model

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- The forecasting error is the standard deviation of the residuals based on a historical fit.
- Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures.
- The probabilities are dynamic and conditional, and depend on:
  - the evolution of container states over the planning horizon,
  - and the vehicle visits on each day.

Objective Function

Routing cost + Expected overflow and emergency visit cost + Expected route failure cost

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- Lower routing cost is counterbalanced by more overflows and route failures, and vice versa.
- Our goal is to minimize the expected monetary value of all components.

Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
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- Two types of container collection:
  - regular collection of container i on day t: $\exists k \in \mathcal{K} : y_{ikt} = 1$,
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.

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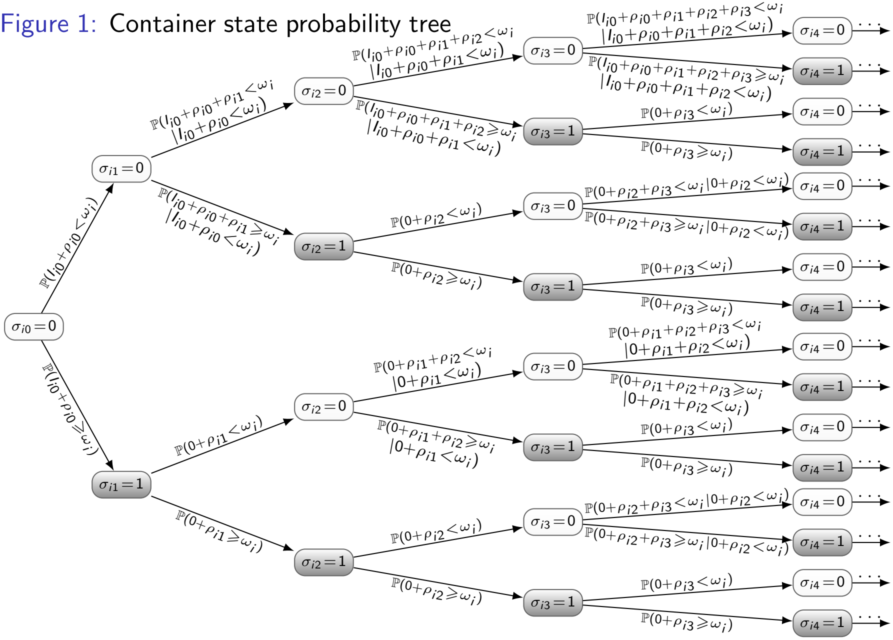
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.
- Related costs:
  - overflow cost χ: paid in state $\sigma_{it} = 1$,
  - emergency collection cost ζ: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in \mathcal{K}$.

Figure 1: Container state probability tree



Objective Function: Formulation

- Routing cost (RC):

$$\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\left(\varphi_k z_{kt}+\beta_k\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\pi_{ij}x_{ijkt}+\theta_k\left(S_{dkt}-S_{okt}\right)\right) \tag{1}$$

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- Expected overflow and emergency collection cost (EOECC):

$$\sum_{t \in \mathcal{T} \cup \mathcal{T}^+} \sum_{i \in \mathcal{P}} \left(\mathbb{P} \left(\sigma_{it} = 1 \mid \max \left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1 \right) \right) \left(\chi + \zeta - \zeta \sum_{k \in \mathcal{K}} y_{ikt} \right) \right) \tag{2}$$

Objective Function: Formulation

- Expected route failure cost (ERFC):

$$\sum_{t \in \mathcal{T} \setminus 0} \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathscr{S}_{kt}} \left(C_{\mathcal{S}} \, \mathbb{P} \left(\sum_{s \in \mathcal{S}} I_{st} > \Omega_k \,\middle|\, \max(0, g < t \colon y_{skg} = 1) \right) \right), \qquad (3)$$

where

- $\mathscr{S}_{kt} = \mathscr{S}_{kt}(y_{ikt}, \forall i \in \mathcal{D})$ is the set of depot-to-dump or dump-to-dump trips for vehicle k on day t,
- $\mathcal{S}$ is the set of containers in a particular trip,
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- The objective function becomes

$$z(\cdot) = \text{RC} + \text{EOECC} + \text{ERFC} \tag{4}$$

and is non-linear, thus resulting in an MINLP.

Constraints: Basic routing

$$\sum_{j\in\mathcal{N}} x_{ojkt} = \alpha_{kt} z_{kt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \qquad (5)$$

$$\sum_{i\in\mathcal{D}} x_{idkt} = \alpha_{kt} z_{kt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \qquad (6)$$

$$y_{ikt} = \sum_{j\in\mathcal{N}} x_{ijkt} = \sum_{j\in\mathcal{N}} x_{jikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \qquad (7)$$

$$\sum_{k\in\mathcal{K}} y_{ikt} \leqslant 1, \qquad \forall t \in \mathcal{T}, i \in \mathcal{P} \qquad (8)$$

$$y_{ikt} \leqslant \alpha_{ik}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \qquad (9)$$

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$$I_{it} = I_{i(t-1)} - \sum_{k\in\mathcal{K}} q_{ik(t-1)} + \mathbb{E}(\rho_{i(t-1)}), \qquad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \qquad (11)$$

$$I_{it} \leqslant \omega_i, \qquad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \qquad (12)$$

$$I_{i0} - \omega_i \leqslant \omega_i \sum_{k\in\mathcal{K}} y_{ik0}, \qquad \forall i \in \mathcal{P} \qquad (13)$$

$$q_{ikt} \leqslant M y_{ikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \qquad (14)$$

$$q_{ikt} \leqslant I_{it}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \qquad (15)$$

$$q_{ikt} \geqslant I_{it} - M(1 - y_{ikt}), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \qquad (16)$$

Constraints: Inventory balance

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Constraints: Capacity related

$$q_{ikt} \leqslant Q_{ikt} \leqslant \Omega_k, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{17}$$

$$Q_{ikt} = 0, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{18}$$

$$Q_{ikt} + q_{jkt} \leqslant Q_{jkt} + \Omega_k \left(1 - x_{ijkt}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \tag{19}$$

$$S_{ikt} + \delta_i + \tau_{ijk} \leqslant S_{jkt} + \left(\mu_i + \delta_i + \tau_{ijk}\right)\left(1 - x_{ijkt}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \tag{20}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leqslant S_{ikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\} \tag{21}$$

$$S_{jkt} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{N} \setminus \{o\} \tag{22}$$

$$0 \leqslant S_{dkt} - S_{okt} \leqslant \mathrm{H} \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{23}$$

$$x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i, j \in \mathcal{N} \tag{24}$$

$$q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geqslant 0, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \tag{25}$$

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Outline

Adaptive Large Neighborhood Search (ALNS)

- Solved by ALNS with the following operators:

Destroy operators:

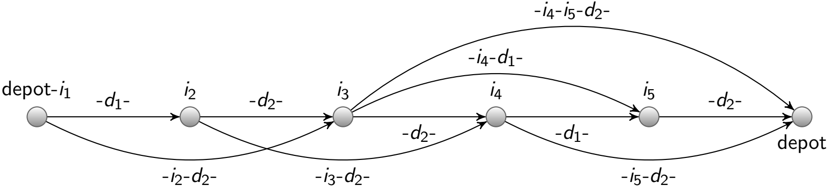
- remove ρ containers randomly,
- remove ρ worst containers,
- Shaw removals (Shaw, 1997),
- empty a random day,
- empty a random vehicle,
- remove a random dump,
- remove the worst dump,
- remove consecutive visits.

Repair operators:

- insert ρ containers randomly,
- insert ρ containers in the best way,
- Shaw insertions (Shaw, 1997),
- swap ρ random containers,
- insert a dump randomly,
- swap random dumps,
- replace a random dump,
- reorder dumps DP operator.

Reorder dumps DP Operator (Hemmelmayr et al., 2013)

Figure 2: Feasibility graph of the *reorder dumps* DP operator



- Preserves/restores vehicle capacity feasibility.
- Removes all dump visits and reinserts them in a locally optimal way solving a shortest path problem using the Bellman-Ford algorithm.
- Followed by local search improvement using 2-opt.

The Search Strategy

- Accepting intermediate infeasible solutions facilitates the exploration of the search space of tightly constrained problems.
- We allow the following feasibility violations of the solution s:
  - $V^{\Omega}(s)$: vehicle capacity violation
  - $V^{\mu}(s)$: time window violation
  - $V^{\mathrm{H}}(s)$: duration violation
  - $V^{\omega}(s)$: container capacity violation
  - $V^{0}(s)$: backorder limit violation
  - $V^{\alpha}(s)$: accessibility violation

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  - $V^{\omega}(s)$: container capacity violation
  - $V^{0}(s)$: backorder limit violation
  - $V^{\alpha}(s)$: accessibility violation
- The solution representation during the search is:

$$f(s) = z(s) + L^{\Omega} V^{\Omega}(s) + L^{\mu} V^{\mu} + L^{\mathrm{H}} V^{\mathrm{H}}(s) + L^{\omega} V^{\omega}(s) + L^{0} V^{0}(s) + L^{\alpha} V^{\alpha}(s) \qquad (26)$$

with the penalties L^{Ω} through L^{α} dynamically adjusted during the search to encourage or discourage infeasible solutions.

Outline

Archetti et al. (2007) Instances

- First classical IRP testbed.
- 160 instances in total.
- 5 to 50 customers.
- 3 or 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

Archetti et al. (2007) Instances

- First classical IRP testbed.
- 160 instances in total.
- 5 to 50 customers.
- 3 or 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

- Optimal solutions (branch-and-cut) by Archetti et al. (2007).
- Heuristic solutions by Archetti et al. (2012), Coelho et al. (2012a), Coelho et al. (2012b), etc...
- We solve each instance 10 times and report best and average results.

Archetti et al. (2007) Instances

Table 1: Results on instances with high inventory holding cost

		ALNS fast version			ALNS slow version		
u	n	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Runtime(s.)	Best Gap(%)	Avg Gap(%)
3	5	8	0.00	0.00	32	0.00	0.00
3	10	14	0.00	0.00	59	0.00	0.00
3	15	22	0.00	0.00	93	0.00	0.00
3	20	36	0.00	0.01	149	0.00	0.00
3	25	53	0.00	0.06	221	0.00	0.01
3	30	77	0.00	0.27	318	0.00	0.06
3	35	108	0.01	0.15	440	0.00	0.04
3	40	149	0.12	0.48	602	0.01	0.23
3	45	199	0.17	0.47	808	0.10	0.25
3	50	276	0.15	0.52	1074	0.07	0.25
6	5	14	0.00	0.00	55	0.00	0.00
6	10	28	0.00	0.01	113	0.00	0.00
6	15	53	0.00	0.07	198	0.00	0.01
6	20	81	0.04	0.14	331	0.01	0.08
6	25	128	0.19	0.64	513	0.10	0.38
6	30	189	0.08	0.70	772	0.07	0.38
Average		90	0.05	0.22	361	0.02	0.11

Archetti et al. (2007) Instances

Table 2: Results on instances with low inventory holding cost

		ALNS fast version			ALNS slow version		
u	n	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Runtime(s.)	Best Gap(%)	Avg Gap(%)
3	5	7	0.00	0.00	30	0.00	0.00
3	10	14	0.00	0.00	55	0.00	0.00
3	15	22	0.00	0.00	89	0.00	0.00
3	20	34	0.00	0.04	141	0.00	0.01
3	25	52	0.00	0.17	210	0.00	0.04
3	30	71	0.02	0.56	295	0.00	0.14
3	35	101	0.01	0.53	423	0.00	0.18
3	40	140	0.37	1.20	567	0.12	0.48
3	45	191	0.59	1.71	751	0.26	1.03
3	50	247	0.30	1.52	1009	0.25	1.00
6	5	13	0.00	0.00	54	0.00	0.00
6	10	28	0.00	0.02	109	0.00	0.01
6	15	49	0.00	0.03	188	0.00	0.00
6	20	77	0.08	0.26	315	0.05	0.15
6	25	121	0.25	1.29	493	0.24	0.65
6	30	182	0.67	1.90	726	0.07	1.06
Average		84	0.14	0.58	341	0.06	0.30

Archetti et al. (2012) Instances

- 60 instances in total.
- 50, 100 and 200 customers.
- 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

Archetti et al. (2012) Instances

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- Solved by Archetti et al. (2012) using a hybrid heuristic algorithm.
- For the moment we have solved the 50-customer instances 10 times and provide best and average results.

Archetti et al. (2012) Instances

Table 3: Results on 50-customer instances with high inventory holding cost

		ALNS				
Instance	Archetti et al. (2012)	Runtime(s.)	Best Cost	Avg Cost	Best Gap(%)	Avg Gap(%)
abs1n50	31,147.82	670	30,708.05	30,809.31	-1.41	-1.09
abs2n50	30,192.51	676	30,226.23	30,271.07	0.11	0.26
abs3n50	30,420.96	667	30,388.68	30,515.79	-0.11	0.31
abs4n50	31,898.84	671	32,103.17	32,213.62	0.64	0.99
abs5n50	29,518.68	666	29,646.74	29,797.79	0.43	0.95
abs6n50	32,394.50	652	32,336.81	32,420.63	-0.18	0.08
abs7n50	30,165.00	661	30,222.28	30,269.23	0.19	0.35
abs8n50	26,416.46	652	26,409.83	26,537.19	-0.03	0.46
abs9n50	30,671.88	656	30,543.31	30,630.53	-0.42	-0.13
abs10n50	32,362.01	635	31,937.51	32,065.85	-1.31	-0.92
Average	30,518.87	661	30,452.26	30,553.10	-0.21	0.13

Archetti et al. (2012) Instances

Table 4: Results on 50-customer instances with low inventory holding cost

		ALNS				
Instance	Archetti et al. (2012)	Runtime(s.)	Best Cost	Avg Cost	Best Gap(%)	Avg Gap(%)
abs1n50	10,409.13	611	10,377.36	10,449.91	-0.31	0.39
abs2n50	10,881.35	643	10,927.83	11,014.20	0.43	1.22
abs3n50	10,767.39	622	10,702.05	10,924.09	-0.61	1.46
abs4n50	10,656.21	632	10,711.86	10,875.98	0.52	2.06
abs5n50	10,234.60	624	10,332.55	10,458.54	0.96	2.19
abs6n50	10,533.63	620	10,388.66	10,485.72	-1.38	-0.45
abs7n50	10,460.82	626	10,388.08	10,497.06	-0.70	0.35
abs8n50	10,411.20	623	10,683.31	10,771.40	2.61	3.46
abs9n50	10,305.69	610	10,416.97	10,472.96	1.08	1.62
abs10n50	10,470.63	598	10,047.06	10,153.50	-4.05	-3.03
Average	10,513.07	621	10,497.57	10,610.33	-0.14	0.93

Instances Based on Real Data

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Maximum tour duration of 4 hours.
- Time windows from 8h00 to 12h00.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

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- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

- We solve each instance 10 times and provide best and average results.
- We simulate the forecasting error realizations and evaluate the relevance of the probability information captured by the objective function.

Real Data: The Relevant Costs

- Truck related:
  - vehicle per day: 100 CHF,
  - vehicle per km: 2.95 CHF,
  - driver per hour: 40 CHF.
- Container related:
  - overflow cost χ: 100 CHF,
  - emergency collection cost ζ: 100 CHF, 50 CHF, 25 CHF.
- Route failure related:
  - cost of visiting the nearest dump from a cluster $C_{\mathcal{S}}$, multiplied by a *route failure cost multiplier* (RFCM): 1.00, 0.50, 0.25.

Two Problem Types

- Routing-only:
  - Optimizes the routing cost only in the objective function, disregarding all probability information.
  - In other words, it ignores the risk of container overflows and route failures.
- Complete:
  - Optimizes the complete objective function as previously defined.

Real Data: Cost Comparison

Table 5: Basic results for real data instances

Type	Emergency Cost	RFCM	Runtime (s.)	Avg Num Tours	Avg Num Containers	Avg Num Dump Visits	Best Cost (CHF)	Avg Cost (CHF)	Gap Avg-Best (%)
Complete	100.00	1.00	781.71	1.96	43.44	2.31	664.76	679.54	2.22
Complete	100.00	0.50	862.13	1.96	43.43	2.30	664.82	678.84	2.11
Complete	100.00	0.25	806.52	1.95	43.52	2.28	664.34	677.81	2.03
Complete	50.00	0.50	812.67	1.91	41.22	2.21	650.55	662.28	1.80
Complete	50.00	0.25	809.76	1.91	41.19	2.19	650.72	661.88	1.71
Complete	25.00	0.50	789.00	1.90	39.56	2.14	641.79	652.04	1.60
Complete	25.00	0.25	789.40	1.90	39.57	2.15	641.42	651.85	1.63
Routing-only	0.00	0.00	725.46	1.83	16.77	1.87	422.64	425.08	0.58

Real Data: Cost Comparison

Table 6: Cost breakdown and KPI for real data instances

Type	Emergency Cost	RFCM	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)	Avg Collected Volume (L)	Liters per Unit Cost	Liters per Unit Routing Cost
Complete	100.00	1.00	579.78	99.73	0.03	47,234.59	69.51	81.47
Complete	100.00	0.50	579.46	99.33	0.05	47,225.62	69.57	81.50
Complete	100.00	0.25	577.84	99.93	0.04	47,455.19	70.01	82.13
Complete	50.00	0.50	558.37	103.82	0.09	45,852.89	69.24	82.12
Complete	50.00	0.25	558.47	103.35	0.07	45,949.94	69.42	82.28
Complete	25.00	0.50	548.10	103.83	0.11	44,653.66	68.48	81.47
Complete	25.00	0.25	547.75	104.05	0.06	44,678.38	68.54	81.57
Routing-only	0.00	0.00	425.08	0.00	0.00	25,286.94	59.49	59.49

Real Data: Occurrence of Rare Events

Table 7: Average number of overflows at various percentiles for real data instances for 10,000 simulations

Type	Emergency Cost	RFCM	75th perc.	90th perc.	95th perc.	99th perc.
Complete	100.00	1.00	0.98	1.78	2.40	3.58
Complete	100.00	0.50	0.99	1.78	2.39	3.55
Complete	100.00	0.25	0.97	1.80	2.38	3.56
Complete	50.00	0.50	1.28	2.19	2.84	4.16
Complete	50.00	0.25	1.28	2.18	2.83	4.15
Complete	25.00	0.50	1.48	2.46	3.14	4.58
Complete	25.00	0.25	1.51	2.50	3.18	4.61
Routing-only	0.00	0.00	16.97	20.45	22.56	26.70

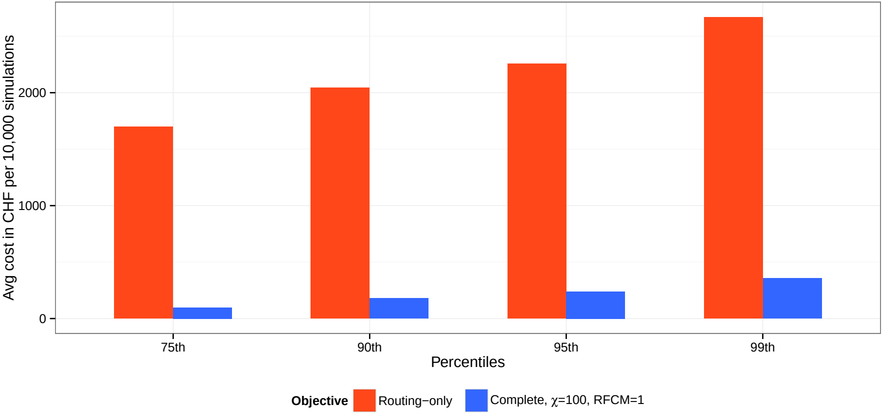
Real Data: Occurrence of Rare Events

Table 8: Average number of route failures at various percentiles for real data instances for 10,000 simulations

Type	Emergency Cost	RFCM	75th perc.	90th perc.	95th perc.	99th perc.
Complete	100.00	1.00	0.03	0.03	0.04	0.05
Complete	100.00	0.50	0.04	0.05	0.05	0.07
Complete	100.00	0.25	0.04	0.05	0.06	0.10
Complete	50.00	0.50	0.06	0.07	0.08	0.09
Complete	50.00	0.25	0.04	0.06	0.07	0.10
Complete	25.00	0.50	0.05	0.07	0.07	0.10
Complete	25.00	0.25	0.04	0.07	0.07	0.09
Routing-only	0.00	0.00	0.01	0.03	0.04	0.05

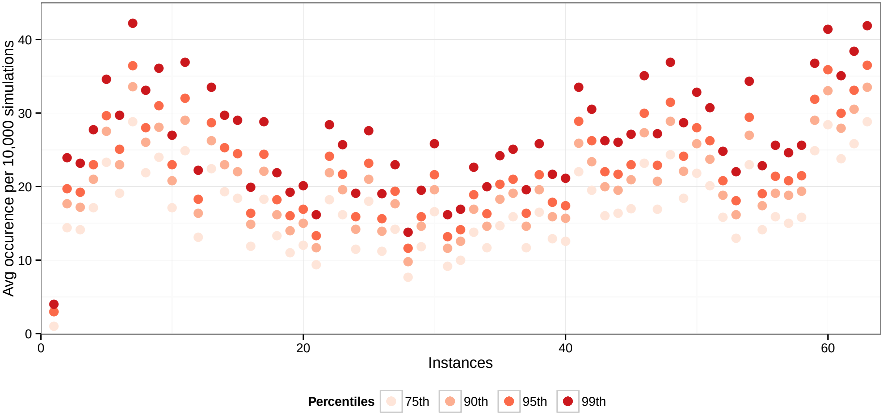
Real Data: Taking Advantage of Probability Information

Figure 3: Average cost percentiles of container overflows



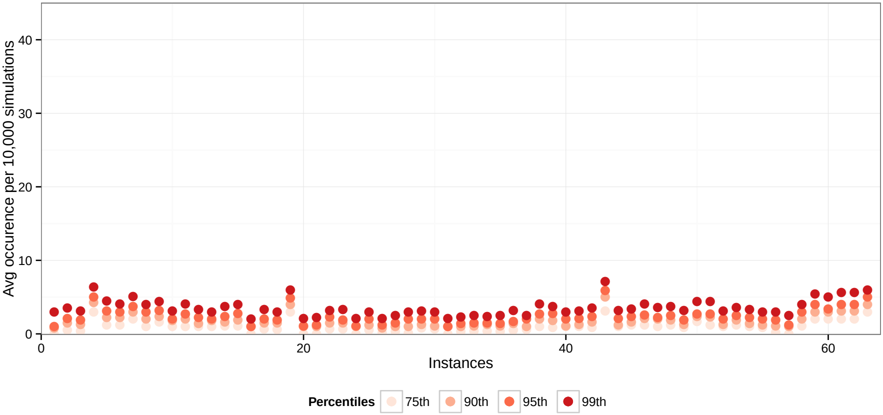
Real Data: Taking Advantage of Probability Information

Figure 4: Container overflow percentiles for routing-only objective



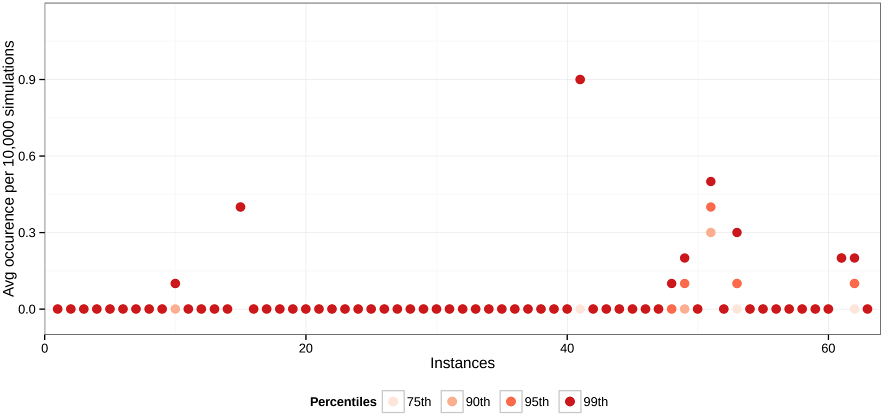
Real Data: Taking Advantage of Probability Information

Figure 5: Container overflow percentiles for complete objective, χ=100, RFCM=1



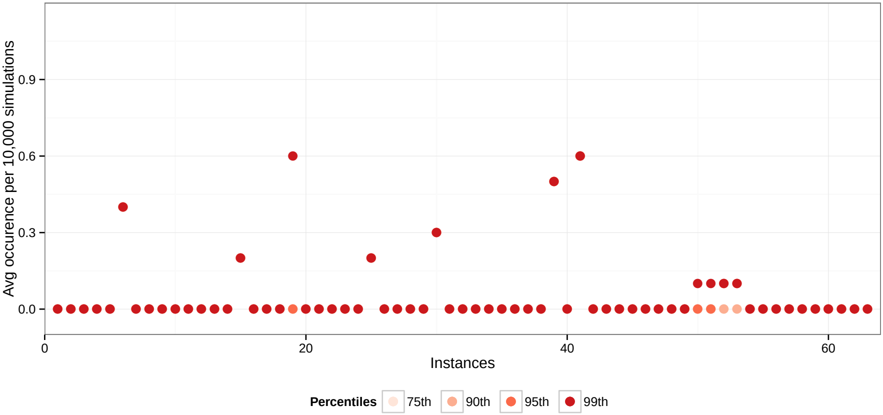
Real Data: Taking Advantage of Probability Information

Figure 6: Route failure percentiles for routing-only objective



Real Data: Taking Advantage of Probability Information

Figure 7: Route failure percentiles for complete objective, χ=100, RFCM=1



Outline

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces very good results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces very good results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.
- Future research directions:
  - decomposition methods,
  - scenario generation,
  - robust optimization,
  - location-routing, open tours, online re-optimization, multiple flows...

Thank you.

Questions?

Abdollahi, M., Arvan, M., Omidvar, A., and Ameri, F. (2014). A simulation optimization approach to apply value at risk analysis on the inventory routing problem with backlogged demand. *International Journal of Industrial Engineering Computations*, 5(4):603–620.

Adulyasak, Y., Cordeau, J.-F., and Jans, R. (2015). Benders decomposition for production routing under demand uncertainty. *Operations Research*, 63(4):851–867.

Archetti, C., Bertazzi, L., Hertz, A., and Speranza, M. G. (2012). A hybrid heuristic for an inventory routing problem. *INFORMS Journal on Computing*, 24(1):101–116.

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.

Baldacci, R. and Mingozzi, A. (2009). A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming*, 120(2):347–380.

Bard, J. F., Huang, L., Dror, M., and Jaillet, P. (1998a). A branch and cut algorithm for the VRP with satellite facilities. *IIE Transactions*, 30(9):821–834.

Bard, J. F., Huang, L., Jaillet, P., and Dror, M. (1998b). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32(2):189–203.

Bertazzi, L., Bosco, A., Guerriero, F., and Laganà, D. (2013). A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, 27:89–107.

Bertazzi, L., Bosco, A., and Laganà, D. (2015). Managing stochastic demand in an inventory routing problem with transportation procurement. *Omega*, 56:112–121.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012a). Consistency in multi-vehicle inventory-routing. *Transportation Research Part C: Emerging Technologies*, 24:270–287.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012b). The inventory-routing problem with transshipment. *Computers & Operations Research*, 39(11):2537–2548.

Conrad, R. G. and Figliozzi, M. A. (2011). The recharging vehicle routing problem. In Doolen, T. and Aken, E. V., editors, *Proceedings of the 2011 Industrial Engineering Research Conference*, Reno, NV, USA.

Crevier, B., Cordeau, J.-F., and Laporte, G. (2007). The multi-depot vehicle routing problem with inter-depot routes. *European Journal of Operational Research*, 176(2):756–773.

Erdoğan, S. and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):100–114.

Goeke, D. and Schneider, M. (2015). Routing a mixed fleet of electric and conventional vehicles. *European Journal of Operational Research*, 245(1):81–99.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Rath, S. (2013). A heuristic solution method for node routing based solid waste collection problems. *Journal of Heuristics*, 19(2):129–156.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research*, 202(3):686–695.

Hiermann, G., Puchinger, J., and Hartl, R. F. (2014). The electric fleet size and mix vehicle routing problem with time windows and recharging stations. Working paper, Austrian Institute of Technology and University of Vienna, Austria.

Jaillet, P., Bard, J. F., Huang, L., and Dror, M. (2002). Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science*, 36(3):292–300.

Kim, B. I., Kim, S., and Sahoo, S. (2006). Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12):3624–3642.

Markov, I., de Lapparent, M., Bierlaire, M., and Varone, S. (2015). Modeling a waste disposal process via a discrete mixture of count data models. In *Proceedings of the 15th Swiss Transport Research Conference (STRC)*, April 17–19, 2015, Ascona, Switzerland.

Nolz, P. C., Absi, N., and Feillet, D. (2014). A stochastic inventory routing problem for infectious medical waste collection. *Networks*, 63(1):82–95.

Penna, P. H. V., Subramanian, A., and Ochi, L. S. (2013). An iterated local search heuristic for the heterogeneous fleet vehicle routing problem. *Journal of Heuristics*, 19(2):201–232.

Schneider, M., Stenger, A., and Goeke, D. (2014). The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science*, 48(4):500–520.

Schneider, M., Stenger, A., and Hof, J. (2015). An adaptive VNS algorithm for vehicle routing problems with intermediate stops. *OR Spectrum*, 37(2):353–387.

Shaw, P. (1997). A new local search algorithm providing high quality solutions to vehicle routing problems. Technical report, APES Group, Department of Computer Sciences, University of Strathclyde, Glasgow, Scotland.

Solyalı, O., Cordeau, J.-F., and Laporte, G. (2012). Robust inventory routing under demand uncertainty. *Transportation Science*, 46(3):327–340.

Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., and van der Vorst, J. G. (2015). Modeling an inventory routing problem for perishable products with environmental considerations and demand uncertainty. *International Journal of Production Economics*, 164:118–133.

Subramanian, A., Penna, P. H. V., Uchoa, E., and Ochi, L. S. (2012). A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research*, 221(2):285–295.

Taillard, É. D. (1999). A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO - Operations Research*, 33(1):1–14.

Trudeau, P. and Dror, M. (1992). Stochastic inventory routing: Route design with stockouts and route failures. *Transportation Science*, 26(3):171–184.

Yu, Y., Chu, C., Chen, H., and Chu, F. (2012). Large scale stochastic inventory routing problems with split delivery and service level constraints. *Annals of Operations Research*, 197(1):135–158.