Inventory routing with non-stationary stochastic demands

Iliya Markov[a], Yousef Maknoon[a], Jean-François Cordeau[b]
Sacha Varone[c], Michel Bierlaire[a]

[a]Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
École Polytechnique Fédérale de Lausanne

[b]HEC Montréal and CIRRELT

[c]Haute École de Gestion de Genève
University of Applied Sciences Western Switzerland (HES-SO)

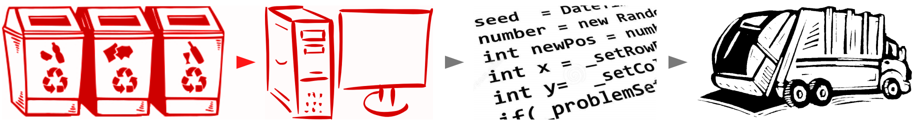
16th Swiss Transport Research Conference (STRC)
Monte Verità / Ascona, May 18–20, 2016

Outline

Outline

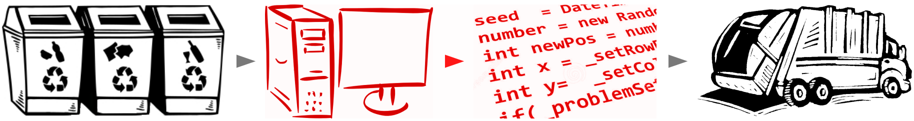
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.



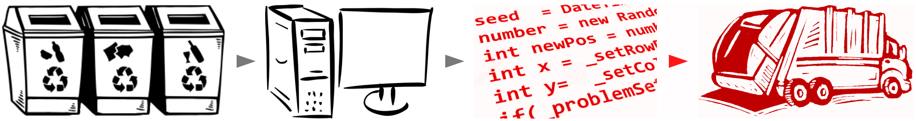
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and tour planning.



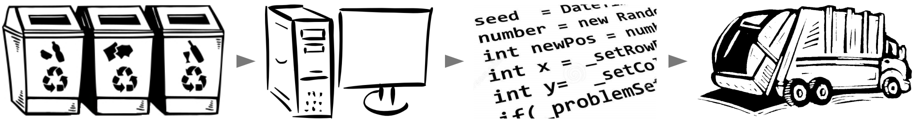
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and tour planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.



Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and tour planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection depends on the ability to:
  - forecast container levels,
  - select the containers to collect each day,
  - and route the vehicles in an (near-)optimal way.



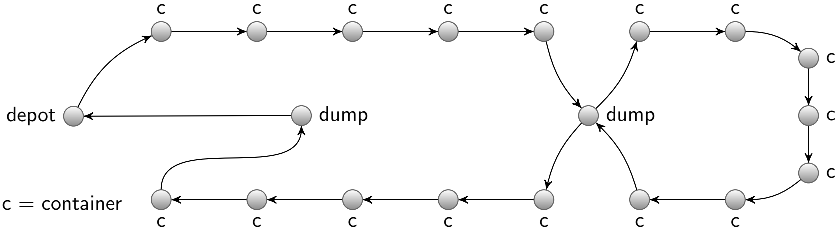
Problem Definition

- The inventory routing problem (IRP) is a multi-day problem that determines simultaneously:
  - the visit days,
  - the delivery/collection quantities,
  - the vehicle tours on each day.

Problem Definition

- The inventory routing problem (IRP) is a multi-day problem that determines simultaneously:
  - the visit days,
  - the delivery/collection quantities,
  - the vehicle tours on each day.
- The routing component in our problem is schematically represented by Figure 1:

Figure 1: Tour example



Outline

Related Stochastic IRP Literature

- Early research on optimal replenishment policies in a stochastic setting:
  - Trudeau and Dror (1992), Jaillet et al. (2002), Bard et al. (1998).
- Robust optimization:
  - Solyalı et al. (2012).
- Chance constraints:
  - Soysal et al. (2015), Abdollahi et al. (2014), Yu et al. (2012).
- Scenario based:
  - Rollout/branch-and-cut: Bertazzi et al. (2013), Bertazzi et al. (2015),
  - Stochastic programming: Hemmelmayr et al. (2010), Nolz et al. (2014), Adulyasak et al. (2015).

Contributions

- Dynamic probabilistic information on overflows and route failures.

Contributions

- Dynamic probabilistic information on overflows and route failures.
- Demand forecasting model tested and validated on real data (Markov et al., 2015).

Contributions

- Dynamic probabilistic information on overflows and route failures.
- Demand forecasting model tested and validated on real data (Markov et al., 2015).
- A rich IRP with features traditionally absent or rarely considered in the IRP literature.

Contributions

- Dynamic probabilistic information on overflows and route failures.
- Demand forecasting model tested and validated on real data (Markov et al., 2015).
- A rich IRP with features traditionally absent or rarely considered in the IRP literature.
- ALNS algorithm performs excellently on IRP benchmarks from the literature.
- Benefit of considering uncertainty in the objective function evaluated on instances derived from real data.

Outline

Basic Definitions and Ideas

- Demand is the amount deposited in a container on each day.
- It is random and non-stationary.

Basic Definitions and Ideas

- Demand is the amount deposited in a container on each day.
- It is random and non-stationary.
- The forecasting model gives:
  - The expected container demands on each day,
  - A consistent estimate of the forecasting error based on historical fit.

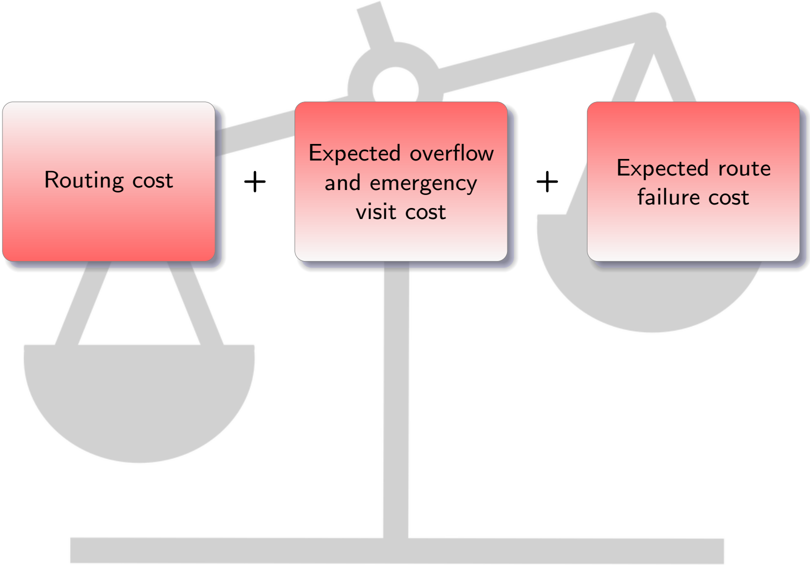
Basic Definitions and Ideas

- Demand is the amount deposited in a container on each day.
- It is random and non-stationary.
- The forecasting model gives:
  - The expected container demands on each day,
  - A consistent estimate of the forecasting error based on historical fit.
- The distribution of the forecasting error can be approximated by the normal distribution, which is used to calculate probabilities of container overflows and route failures.
- They are dynamic and conditional, and depend on:
  - The the evolution of container state scenarios on each day (overflowing vs. not full),
  - And the vehicle visits on each day.

Basic Definitions and Ideas

- Container overflow:
  - Container is full and all subsequent demand is placed beside it,
  - Overflow cost: paid on each day when there is an overflow,
  - Emergency visit cost: paid on each day when there is an overflow **and** no planned visit.
- Route failure:
  - Vehicle becomes full earlier than the next scheduled dump visit,
  - Entails the cost of visiting the closest dump.

Objective Function



Objective Function

Routing cost + Expected overflow and emergency visit cost + Expected route failure cost

- Lower routing cost is counterbalanced by more overflows and route failures, and vice versa.
- Our goal is to minimize the expected monetary value of all components.

Constraints

- Collection policy:
  - Order-up-to (OU),
  - No expected overflows over the planning horizon,
  - An overflow on day 0 is out of our control,
  - But the container must be collected on day 0 (single-day backorder limit).

Constraints

- Collection policy:
  - Order-up-to (OU),
  - No expected overflows over the planning horizon,
  - An overflow on day 0 is out of our control,
  - But the container must be collected on day 0 (single-day backorder limit).
- We also need to ensure/enforce the rich features of the routing component:
  - Point accessibilities,
  - Vehicle availabilities,
  - Vehicle capacities,
  - Time windows,
  - Maximum tour duration.

Outline

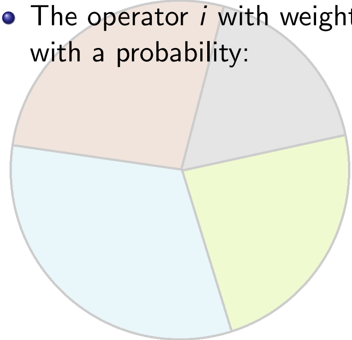
Adaptive Large Neighborhood Search (ALNS)

- A meta-heuristic framework in which a number of fairly simple destroy and repair operators compete in modifying the current solution.

Adaptive Large Neighborhood Search (ALNS)

- A meta-heuristic framework in which a number of fairly simple destroy and repair operators compete in modifying the current solution.
- At each iteration, a destroy-repair operator couple is drawn based on past performance.
- The operator i with weight ω_i is drawn from the destroy (repair) pool with a probability:

$$\mathbb{P}(i) = \frac{\omega_i}{\sum_{j \in \mathcal{O}} \omega_j} \quad (1)$$

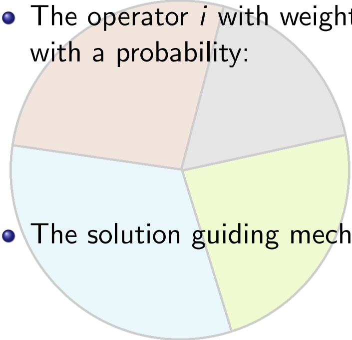


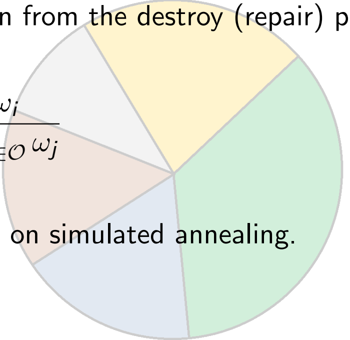
Adaptive Large Neighborhood Search (ALNS)

- A meta-heuristic framework in which a number of fairly simple destroy and repair operators compete in modifying the current solution.
- At each iteration, a destroy-repair operator couple is drawn based on past performance.
- The operator i with weight ω_i is drawn from the destroy (repair) pool with a probability:

$$\mathbb{P}(i) = \frac{\omega_i}{\sum_{j \in \mathcal{O}} \omega_j} \tag{1}$$

- The solution guiding mechanism relies on simulated annealing.





The Operators

Destroy operators:

- Remove ρ containers randomly.
- Remove ρ worst containers.
- Shaw removals (Shaw, 1997).
- Empty a random day.
- Empty a random vehicle.
- Remove a random dump.
- Remove the worst dump.
- Remove consecutive visits.

Repair operators:

- Insert ρ containers randomly.
- Insert ρ containers in the best way.
- Shaw insertions (Shaw, 1997).
- Swap ρ random containers.
- Insert a dump randomly.
- Swap random dumps.
- Replace a random dump.
- Reorder dumps DP operator.

Outline

Archetti et al. (2007) Instances

Table 1: Results on high cost instances

		ALNS Fast version			ALNS Slow version		
H	n	Runtime(s.)	Min Gap(%)	Avg Gap(%)	Runtime(s.)	Min Gap(%)	Avg Gap(%)
3	5	8	0.00	0.00	32	0.00	0.00
3	10	14	0.00	0.00	59	0.00	0.00
3	15	22	0.00	0.00	93	0.00	0.00
3	20	36	0.00	0.01	149	0.00	0.00
3	25	53	0.00	0.06	221	0.00	0.01
3	30	77	0.00	0.27	318	0.00	0.06
3	35	108	0.01	0.15	440	0.00	0.04
3	40	149	0.12	0.48	602	0.01	0.23
3	45	199	0.17	0.47	808	0.10	0.25
3	50	276	0.15	0.52	1074	0.07	0.25
6	5	14	0.00	0.00	55	0.00	0.00
6	10	28	0.00	0.01	113	0.00	0.00
6	15	53	0.00	0.07	198	0.00	0.01
6	20	81	0.04	0.14	331	0.01	0.08
6	25	128	0.19	0.64	513	0.10	0.38
6	30	189	0.08	0.70	772	0.07	0.38
Average		90	0.05	0.22	361	0.02	0.11

Archetti et al. (2007) Instances

Table 2: Results on low cost instances

		ALNS Fast version			ALNS Slow version		
H	n	Runtime(s.)	Min Gap(%)	Avg Gap(%)	Runtime(s.)	Min Gap(%)	Avg Gap(%)
3	5	7	0.00	0.00	30	0.00	0.00
3	10	14	0.00	0.00	55	0.00	0.00
3	15	22	0.00	0.00	89	0.00	0.00
3	20	34	0.00	0.04	141	0.00	0.01
3	25	52	0.00	0.17	210	0.00	0.04
3	30	71	0.02	0.56	295	0.00	0.14
3	35	101	0.01	0.53	423	0.00	0.18
3	40	140	0.37	1.20	567	0.12	0.48
3	45	191	0.59	1.71	751	0.26	1.03
3	50	247	0.30	1.52	1009	0.25	1.00
6	5	13	0.00	0.00	54	0.00	0.00
6	10	28	0.00	0.02	109	0.00	0.01
6	15	49	0.00	0.03	188	0.00	0.00
6	20	77	0.08	0.26	315	0.05	0.15
6	25	121	0.25	1.29	493	0.24	0.65
6	30	182	0.67	1.90	726	0.07	1.06
Average		84	0.14	0.58	341	0.06	0.30

Archetti et al. (2012) Instances

Table 3: Results on high cost 50-customer instances

Instance	Runtime(s.)	Min Cost	Avg Cost	Min Gap(%)	Avg Gap(%)
abs1n50	670	30,708.05	30,809.31	-1.41	-1.09
abs2n50	676	30,226.23	30,271.07	0.11	0.26
abs3n50	667	30,388.68	30,515.79	-0.11	0.31
abs4n50	671	32,103.17	32,213.62	0.64	0.99
abs5n50	666	29,646.74	29,797.79	0.43	0.95
abs6n50	652	32,336.81	32,420.63	-0.18	0.08
abs7n50	661	30,222.28	30,269.23	0.19	0.35
abs8n50	652	26,409.83	26,537.19	-0.03	0.46
abs9n50	656	30,543.31	30,630.53	-0.42	-0.13
abs10n50	635	31,937.51	32,065.85	-1.31	-0.92
Average	661	30,452.26	30,553.10	-0.21	0.13

Archetti et al. (2012) Instances

Table 4: Results on low cost 50-customer instances

Instance	Runtime(s.)	Min Cost	Avg Cost	Min Gap(%)	Avg Gap(%)
abs1n50	611	10,377.36	10,449.91	-0.31	0.39
abs2n50	643	10,927.83	11,014.20	0.43	1.22
abs3n50	622	10,702.05	10,924.09	-0.61	1.46
abs4n50	632	10,711.86	10,875.98	0.52	2.06
abs5n50	624	10,332.55	10,458.54	0.96	2.19
abs6n50	620	10,388.66	10,485.72	-1.38	-0.45
abs7n50	626	10,388.08	10,497.06	-0.70	0.35
abs8n50	623	10,683.31	10,771.40	2.61	3.46
abs9n50	610	10,416.97	10,472.96	1.08	1.62
abs10n50	598	10,047.06	10,153.50	-4.05	-3.03
Average	621	10,497.57	10,610.33	-0.14	0.93

Instances Based on Real Data

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Vehicle-related costs:
  - Per day: 100 CHF,
  - Per km: 2.95 CHF,
  - Per hour: 40 CHF.
- Container-related costs:
  - Overflow cost: 100 CHF,
  - Emergency collection cost: 100 CHF.

Instances Based on Real Data

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Vehicle-related costs:
  - Per day: 100 CHF,
  - Per km: 2.95 CHF,
  - Per hour: 40 CHF.
- Container-related costs:
  - Overflow cost: 100 CHF,
  - Emergency collection cost: 100 CHF.
- Two types of problem:
  - Routing-only: Considers no overflow and route failure risk,
  - Complete: Considers full objective with the above costs.

Real Data: Cost Comparison

Table 5: Cost breakdown for real data instances

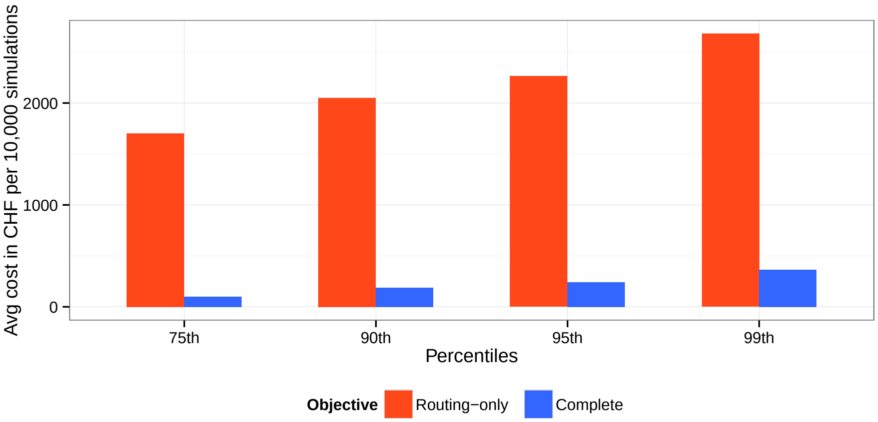
	Avg Cost (CHF)	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)
Routing-only	430.61	430.61	0.00	0.00
Complete	693.66	588.16	105.44	0.06

Table 6: Performance indicators for real data instances

	Avg Collected Volume (L)	Liters per Unit Cost	Liters per Unit Routing Cost
Routing-only	25,106.81	58.31	58.31
Complete	47,364.96	68.28	80.53

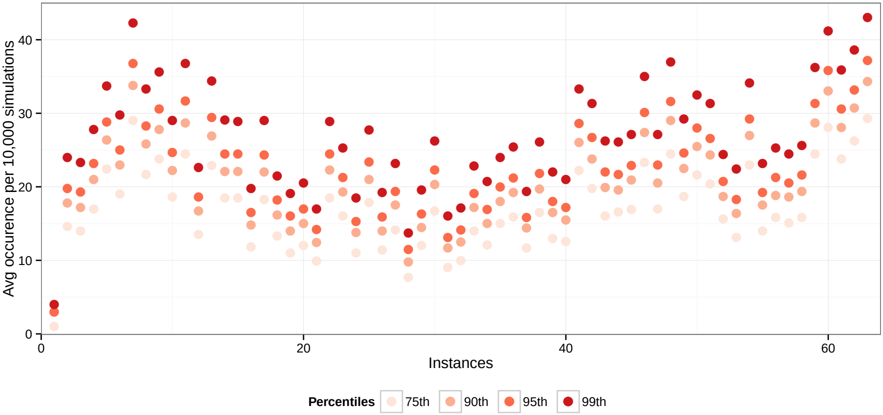
Real Data: Taking Advantage of Probability Information

Figure 2: Cost percentiles of container overflows



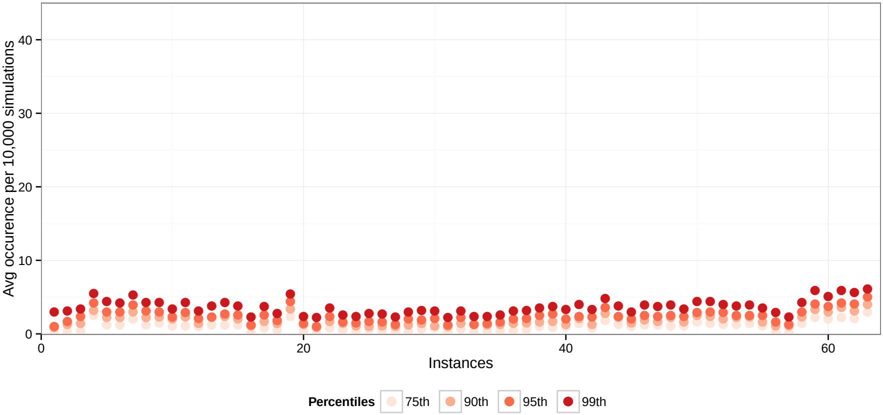
Real Data: Taking Advantage of Probability Information

Figure 3: Container overflow percentiles for routing-only objective



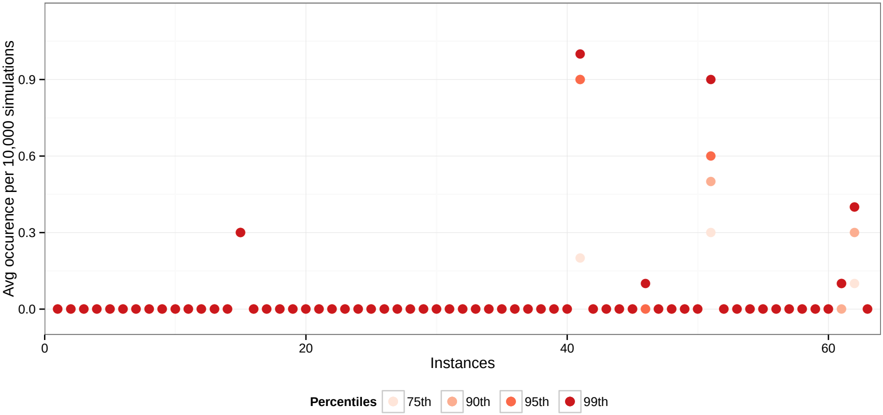
Real Data: Taking Advantage of Probability Information

Figure 4: Container overflow percentiles for complete objective



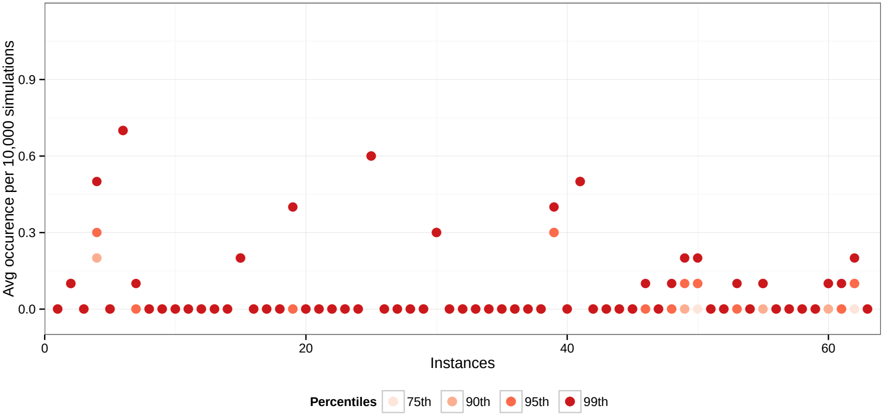
Real Data: Taking Advantage of Probability Information

Figure 5: Route failure percentiles for routing-only objective



Real Data: Taking Advantage of Probability Information

Figure 6: Route failure percentiles for complete objective



Real Data: Explaining Overflows, Comparison

Table 7: Driving factors for the occurrence of container overflows

(a) Regressions on forecasting error

	75th percentile		90th percentile		95th percentile		99th percentile	
Objective	coefficient	R^2	coefficient	R^2	coefficient	R^2	coefficient	R^2
Routing-only	0.16***	0.51	0.18***	0.52	0.19***	0.51	0.21***	0.51
Complete	0.02***	0.49	0.02***	0.53	0.03***	0.52	0.03***	0.57

(b) Regressions on number of containers in instance

	75th percentile		90th percentile		95th percentile		99th percentile	
Objective	coefficient	R^2	coefficient	R^2	coefficient	R^2	coefficient	R^2
Routing-only	0.34***	0.25	0.37***	0.25	0.41***	0.26	0.47***	0.27
Complete	0.02**	0.08	0.02**	0.07	0.03**	0.09	0.03*	0.06

Significance codes: *** 99%, ** 95%, * 90%

Outline

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces excellent results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces excellent results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate the practical relevance of our approach.
- Future research directions:
  - Decomposition methods,
  - Scenario generation,
  - Chance constraints,
  - Location-routing, open tours, online re-optimization, multiple flows...

Thank you.

Questions?

Abdollahi, M., Arvan, M., Omidvar, A., and Ameri, F. (2014). A simulation optimization approach to apply value at risk analysis on the inventory routing problem with backlogged demand. *International Journal of Industrial Engineering Computations*, 5(4):603–620.

Adulyasak, Y., Cordeau, J.-F., and Jans, R. (2015). Benders decomposition for production routing under demand uncertainty. *Operations Research*, 63(4):851–867.

Archetti, C., Bertazzi, L., Hertz, A., and Speranza, M. G. (2012). A hybrid heuristic for an inventory routing problem. *INFORMS Journal on Computing*, 24(1):101–116.

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.

Bard, J. F., Huang, L., Jaillet, P., and Dror, M. (1998). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32(2):189–203.

Bertazzi, L., Bosco, A., Guerriero, F., and Laganà, D. (2013). A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, 27:89–107.

Bertazzi, L., Bosco, A., and Laganà, D. (2015). Managing stochastic demand in an inventory routing problem with transportation procurement. *Omega*, 56:112–121.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research*, 202(3):686–695.

Jaillet, P., Bard, J. F., Huang, L., and Dror, M. (2002). Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science*, 36(3):292–300.

Markov, I., de Lapparent, M., Bierlaire, M., and Varone, S. (2015). Modeling a waste disposal process via a discrete mixture of count data models. In *Proceedings of the 15th Swiss Transport Research Conference (STRC)*, April 17–19, 2015, Ascona, Switzerland.

Nolz, P. C., Absi, N., and Feillet, D. (2014). A stochastic inventory routing problem for infectious medical waste collection. *Networks*, 63(1):82–95.

Pisinger, D. and Ropke, S. (2007). A general heuristic for vehicle routing problems. *Computers & Operations Research*, 34(8):2403–2435.

Ropke, S. and Pisinger, D. (2006). An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science*, 40(4):455–472.

Shaw, P. (1997). A new local search algorithm providing high quality solutions to vehicle routing problems. Technical report, APES Group, Department of Computer Sciences, University of Strathclyde, Glasgow, Scotland.

Solyalı, O., Cordeau, J.-F., and Laporte, G. (2012). Robust inventory routing under demand uncertainty. *Transportation Science*, 46(3):327–340.

Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., and van der Vorst, J. G. (2015). Modeling an inventory routing problem for perishable products with environmental considerations and demand uncertainty. *International Journal of Production Economics*, 164:118–133.

Trudeau, P. and Dror, M. (1992). Stochastic inventory routing: Route design with stockouts and route failures. *Transportation Science*, 26(3):171–184.

Yu, Y., Chu, C., Chen, H., and Chu, F. (2012). Large scale stochastic inventory routing problems with split delivery and service level constraints. *Annals of Operations Research*, 197(1):135–158.