Recent developments in route choice modeling

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Outline

- Introduction
  - Problem description
  - Existing models
- Subnetwork approach
- Latent route choice model
- Future work
Route choice problem

Given a transportation network composed of nodes, links, origin and destinations.
For a given transportation mode and origin-destination pair, which is the chosen route?
Applications

- Intelligent transportation systems
- GPS navigation
- Transportation planning
Issues

• The choice set is unknown
• There are many (feasible) alternatives available
• The alternatives are often highly correlated due to overlapping paths
• Choice data is difficult to obtain
Existing Approaches

- Assumption: Travelers use the shortest (with regard to any arbitrary generalized cost) route among all
  - Behaviorally unrealistic
- Random utility models (discrete choice models)
Existing Approaches - MNL

- Random terms are assumed to be i.i.d. Extreme Value

\[ P(i|C_n) = \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}} \]

Alternatives are assumed to be independent. This assumption is (in general) not valid in a route choice context due to overlapping paths.
Existing Approaches

Travel time is the only considered attribute and

\[ V_1 = V_2 = V_3 = T \] then

\[ P(1\mid\{1, 2, 3\}) = P(2\mid\{1, 2, 3\}) = P(3\mid\{1, 2, 3\}) = \frac{1}{3} \]

- Unrealistic path choice probabilities for correlated alternatives (overlapping paths)
Existing Approaches

- Few models explicitly capturing correlation have been used on large-scale route choice problems
  - C-Logit (Cascetta et al., 1996)
  - Path Size Logit (Ben-Akiva and Bierlaire, 1999)
  - Link-Nested Logit (Vovsha and Bekhor, 1998)
  - Logit Kernel model adapted to route choice situation (Bekhor et al., 2002)
- Probit model (Daganzo, 1977) permits an arbitrarily covariance structure specification but cannot be applied in a large-scale route choice context
Existing Approaches

- Link based path-multilevel logit model (Marzano and Papola, 2005)
  - Illustrated on simple examples and not estimated on real data
Subnetwork approach
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?
Subnetworks

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- Which are the behaviorally important decisions?
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?

- Which are the behaviorally important decisions?
- Our hypothesis: choice of specific parts of the network (e.g. main roads, city center)
- Concept: subnetwork
Subnetworks

- Subnetwork approach designed to be behaviorally realistic and convenient for the analyst
- Subnetwork component is a set of links corresponding to a part of the network which can be easily labeled
- Paths sharing a subnetwork component are assumed to be correlated even if they are not physically overlapping
Subnetworks - Methodology

- Factor analytic specification of an error component model (based on model presented in Bekhor et al., 2002)

\[ U_n = \beta^T X_n + F_n T \zeta_n + \nu_n \]

- \( F_n (J \times Q) \): factor loadings matrix
- \((f_n)_{iq} = \sqrt{l_{niq}}\)
- \( T_{(Q \times Q)} = \text{diag} (\sigma_1, \sigma_2, \ldots, \sigma_Q) \)
- \( \zeta_n (Q \times 1) \): vector of i.i.d. \( \text{N}(0,1) \) variates
- \( \nu(J \times 1) \): vector of i.i.d. Extreme Value distributed variates
Subnetworks - Example

![Subnetworks Diagram]

- Path 1
- Path 2
- Path 3

$S_a$

$S_b$
Subnetworks - Example

\[ U_1 = \beta^T X_1 + \sqrt{l_{1a}} \sigma_a \zeta_a + \sqrt{l_{1b}} \sigma_b \zeta_b + \nu_1 \]
\[ U_2 = \beta^T X_2 + \sqrt{l_{2a}} \sigma_a \zeta_a + \nu_2 \]
\[ U_3 = \beta^T X_3 + \sqrt{l_{3b}} \sigma_b \zeta_b + \nu_3 \]

\[
\mathbf{FTT}^T \mathbf{FT} =
\begin{bmatrix}
    l_{1a} \sigma_a^2 + l_{1b} \sigma_b^2 & \sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & \sqrt{l_{1b}} \sqrt{l_{3b}} \sigma_b^2 \\
    \sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & l_{2a} \sigma_a^2 & 0 \\
    \sqrt{l_{3b}} \sqrt{l_{1b}} \sigma_b^2 & 0 & l_{3b} \sigma_b^2
\end{bmatrix}
\]
Empirical Results

- The approach has been tested on three datasets: Boston (Ramming, 2001), Switzerland, and Borlänge.
- Deterministic choice set generation
  - Link elimination
- GPS data from 24 individuals
  - 2978 observations, 2179 origin-destination pairs
- Borlänge network
  - 3077 nodes and 7459 links
- BIOGEME (biogeme.epfl.ch, Bierlaire, 2003) has been used for all model estimations.
Borlänge Road Network
## Subnetwork Components

<table>
<thead>
<tr>
<th></th>
<th>R.50 S</th>
<th>R.50 N</th>
<th>R.70 S</th>
<th>R.70 N</th>
<th>R.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component length [m]</td>
<td>5255</td>
<td>4966</td>
<td>11362</td>
<td>7028</td>
<td>1733</td>
</tr>
<tr>
<td>Nb. of Observations</td>
<td>173</td>
<td>153</td>
<td>261</td>
<td>366</td>
<td>209</td>
</tr>
<tr>
<td>Weighted Nb. of</td>
<td>36</td>
<td>88</td>
<td>65</td>
<td>73</td>
<td>116</td>
</tr>
<tr>
<td>Observations (N_q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
N_q = \sum_{o \in O} \frac{L_o}{L_q}
\]
Model Specifications

- Six different models: MNL, PSL, EC\(_1\), EC\(_1'\), EC\(_2\) and EC\(_2'\)
- EC\(_1\) and EC\(_1'\) have a simplified correlation structure
- EC\(_1'\) and EC\(_2'\) do not include a Path Size attribute
- Deterministic part of the utility

\[ V_i = \beta_{PS} \ln(PS_i) + \beta_{EstimatedTime} EstimatedTime_i + \beta_{NbSpeedBumps} NbSpeedBumps_i + \beta_{NbLeftTurns} NbLeftTurns_i + \beta_{AvgLinkLength} AvgLinkLength_i \]
Estimation Results

- Parameter estimates for explanatory variables are stable across the different models
- Path size parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>EC₁</th>
<th>EC₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Size</td>
<td>-0.28</td>
<td>-0.49</td>
<td>-0.53</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.33</td>
<td>-0.53</td>
<td>-0.56</td>
</tr>
<tr>
<td>Rob. T-test 0</td>
<td>-4.05</td>
<td>-5.61</td>
<td>-5.91</td>
</tr>
</tbody>
</table>

- All covariance parameters estimates in the different models are significant except the one associated with R.50 S
## Estimation Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Nb. σ Estimates</th>
<th>Nb. Estimated Parameters</th>
<th>Final L-L</th>
<th>Adjusted Rho-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNL</td>
<td>-</td>
<td>12</td>
<td>-4186.07</td>
<td>0.152</td>
</tr>
<tr>
<td>PSL</td>
<td>-</td>
<td>13</td>
<td>-4174.72</td>
<td>0.154</td>
</tr>
<tr>
<td>EC₁ (with PS)</td>
<td>1</td>
<td>14</td>
<td>-4142.40</td>
<td>0.161</td>
</tr>
<tr>
<td>EC₁</td>
<td>1</td>
<td>13</td>
<td>-4165.59</td>
<td>0.156</td>
</tr>
<tr>
<td>EC₂ (with PS)</td>
<td>5</td>
<td>18</td>
<td>-4136.92</td>
<td>0.161</td>
</tr>
<tr>
<td>EC₂</td>
<td>5</td>
<td>17</td>
<td>-4162.74</td>
<td>0.156</td>
</tr>
</tbody>
</table>

1000 pseudo-random draws for Maximum Simulated Likelihood estimation

2978 observations

Null log likelihood: -4951.11

BIOGEME (biogeme.epfl.ch) has been used for all model estimations.
Forecasting Results

- Comparison of the different models in terms of their performance of predicting choice probabilities
- Five subsamples of the dataset
  - Observations corresponding to 80% of the origin destination pairs (randomly chosen) are used for estimating the models
  - The models are applied on the observations corresponding to the other 20% of the origin destination pairs
- Comparison of final log-likelihood values
Forecasting Results

- Same specification of deterministic utility function for all models
- Same interpretation of these models as for those estimated on the complete dataset
- Coefficient and covariance parameter values are stable across models
Forecasting Results

Recent developments in route choice modeling – p.25/48
Conclusion - Subnetworks

- Models based on subnetworks are designed for route choice modeling of realistic size
- Correlation on subnetwork is explicitly captured within a factor analytic specification of an Error Component model
- Estimation and prediction results clearly shows the superiority of the Error Component models compared to PSL and MNL
Conclusion - Subnetworks

- The subnetwork approach is flexible and the model complexity can be controlled by the analyst
- Paper to appear in Transportation Research Part B
A latent route choice model
Mobility Pricing

漫画内容:

左上角对话框：“How's the congestion charge trial going?”
中间上对话框：“Not well.”
中间下对话框：“Isn't it keeping people off the road?”
中间下对话框：“Yes...”
右上对话框：“...but not in the way we'd like...”
背景图：一辆越野车陷入泥泞中。
Swiss Mobility Pricing Project

- A part of a major study on various mobility pricing scenarios in Switzerland
- A collaboration with ETH Zurich and USI Lugano
- Revealed Preferences (RP) and Stated Preferences (SP) data has been collected
- RP data concern long distance route choice by car
  - Route descriptions are approximative
  - Route choices are latent
Objective

- Estimate route choice models based on latent chosen routes
- Literature on latent choice models
  - Ben-Akiva et al. (1984), label path approach
  - Ben-Akiva and Lerman (1985), destination choice
  - Toledo et al. (2003), Ben-Akiva et al. (2006) lane choice
Observations

- Exact descriptions of chosen routes are difficult and expensive to obtain
- The concept of path and network as we need for modeling is abstract for respondents
- Here, a chosen route is described by a sequence of cities and locations
- *Aggregate observations* (several paths in the network can correspond to the same observation)
Observations

- Better quality of the observations
- Travelers do not need to refer to the network used by the analyst
- Exact origin-destination pairs are not necessarily known
- Exact route is not known
Observations - Example

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Modeling Approach

• Several possible modeling approaches
  • Construction of paths from the aggregate observations
    - Involves subjective judgments and generate noise
  • Alternatives in the model are aggregates instead of physical paths
    - Estimated model is of little use in practice
• Our approach: compute the likelihood of an aggregate observation for a classical route choice model
Modeling Approach

- Probability of an aggregate observation $i$:

$$P(i) = \sum_{s \in S_i} P(s|S_i) \sum_{r \in C_s} \delta_{ri} P(r|C_s)$$

- $s$: origin-destination pair
- $S_i$: set of all origin-destination pairs for observation $i$
- $r$: route
- $C_s$: set of all routes for origin-destination pair $s$
- $\delta_{ri} = \begin{cases} 1 & \text{if } r \text{ corresponds to } i \\ 0 & \text{otherwise} \end{cases}$
Modeling Approach

- Probability of an aggregate observation $i$:

\[ P(i) = \sum_{s \in S_i} P(s|S_i) \sum_{r \in C_s} \delta_{ri} P(r|C_s) \]

- $P(s|S_i)$ can be modeled in several ways
- $P(r|C_s)$: route choice model that is identifiable if
  1. at least one of the routes in $C_s$ crosses the observed zones, and
  2. at least one route in $C_s$ does not cross the observed zones.

- This type of models can be estimated with BIOGEME
Empirical Results

- Simplified Swiss network (39411 links and 14841 nodes)
- RP data collection through telephone interviews
- Long distance car travel
- The chosen routes are described with the origin and destination cities as well as 1 to 3 cities or locations that the route pass by
- 940 observations available after data cleaning and verification
Empirical Results
Empirical Results

- This application is one of few presented in the literature that are based on RP data.
- The network is to our knowledge the largest one used for evaluation of route choice modeling approaches.
Empirical Results

- No information available on the exact origin destination pairs

\[ P(s|i) = \frac{1}{|S_i|} \quad \forall s \in S_i \]

- Two origin-destination pairs are randomly chosen for each observation
Empirical Results

- 46 routes per choice set are generated with a choice set generation algorithm
- After choice set generation 780 observations are available
  - 160 observations were removed because either all or none of the generated routes crossed the observed zones
- We estimate Path Size Logit (Ben-Akiva and Bierlaire, 1999) and Subnetwork (Frejinger and Bierlaire, 2006) models
Empirical Results - Subnetwork

- Subnetwork: main motorways in Switzerland
- Correlation among routes is explicitly modeled on the subnetwork
- Combined with a Path Size attribute
- Linear-in-parameters utility specifications
<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>In(path size) based on free-flow time</td>
<td>1.04 (0.134) 7.81</td>
<td>1.10 (0.141) 7.78</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Freeway free-flow time 0-30 min</td>
<td>-7.12 (0.877) -8.12</td>
<td>-7.45 (0.984) -7.57</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-7.12</td>
<td>-7.04</td>
</tr>
<tr>
<td>Freeway free-flow time 30min - 1 hour</td>
<td>-1.69 (0.875) -1.93</td>
<td>-2.26 (1.03) -2.19</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-1.69</td>
<td>-2.14</td>
</tr>
<tr>
<td>Freeway free-flow time 1 hour +</td>
<td>-4.98 (0.772) -6.45</td>
<td>-5.64 (1.00) -5.61</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-4.98</td>
<td>-5.33</td>
</tr>
<tr>
<td>CN free-flow time 0-30 min</td>
<td>-6.03 (0.882) -6.84</td>
<td>-6.25 (0.975) -6.41</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-6.03</td>
<td>-5.91</td>
</tr>
<tr>
<td>CN free-flow time 30 min +</td>
<td>-1.87 (0.331) -5.64</td>
<td>-2.16 (0.384) -5.63</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-1.87</td>
<td>-2.04</td>
</tr>
<tr>
<td>Main free-flow travel time 10 min +</td>
<td>-2.03 (0.502) -4.05</td>
<td>-2.46 (0.624) -3.95</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.03</td>
<td>-2.33</td>
</tr>
<tr>
<td>Small free-flow travel time</td>
<td>-2.16 (0.685) -3.16</td>
<td>-2.75 (0.804) -3.42</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.16</td>
<td>-2.60</td>
</tr>
<tr>
<td>Proportion of time on freeways</td>
<td>-2.2 (0.812) -2.71</td>
<td>-2.31 (0.865) -2.67</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.2</td>
<td>-2.18</td>
</tr>
<tr>
<td>Proportion of time on CN 0 fixed</td>
<td>0 fixed</td>
<td>0 fixed</td>
</tr>
<tr>
<td>Proportion of time on main</td>
<td>-4.43 (0.752) -5.88</td>
<td>-4.40 (0.800) -5.51</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-4.43</td>
<td>-4.16</td>
</tr>
<tr>
<td>Proportion of time on small</td>
<td>-6.23 (0.992) -6.28</td>
<td>-6.02 (1.03) -5.83</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-6.23</td>
<td>-5.69</td>
</tr>
<tr>
<td>Covariance parameter</td>
<td>0.217 (0.0543) 4.00</td>
<td>0.205</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Empirical Results

<table>
<thead>
<tr>
<th></th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance parameter</td>
<td></td>
<td>0.217</td>
</tr>
<tr>
<td>(Rob. Std. Error) Rob. T-test</td>
<td>(0.0543) 4.00</td>
<td></td>
</tr>
<tr>
<td>Number of simulation draws</td>
<td>-</td>
<td>1000</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-1164.850</td>
<td>-1161.472</td>
</tr>
<tr>
<td>Adjusted rho square</td>
<td>0.145</td>
<td>0.147</td>
</tr>
<tr>
<td>Sample size: 780, Null log-likelihood:</td>
<td>-1375.851</td>
<td></td>
</tr>
</tbody>
</table>
Empirical Results

- All parameters have their expected signs and are significantly different from zero
- The values and significance level are stable across the two models
- The subnetwork model is significantly better than the Path Size Logit (PSL) model
Conclusion - Latent route choice

- Aggregate observations are convenient to report paths
- They can be used for estimating route choice models
- Care must be taken about the level of aggregation
- Parameters of the RP model are significant and meaningful
- Available in Biogeme / Bioroute
Future work

- Choice set generation
  - Stochastic path generation algorithm
- Analysis of sensitivity of the modeling results regarding the choice set definition