Comparative analysis of hedonic rents and maximum bids in a land-use simulation context

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Outline

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2. Bid-auction approach
3. Bid-choice equivalence
4. Hedonic rent models
5. Simulation experiment
6. Simulation results
7. Conclusions / Discussion
Motivation

- Evolution of land use (location choice) models:
  - Aggregated → Disaggregated
  - Equilibrium → Dynamic microsimulation
  - Bid-auction / Choice

- Bid approach: consistent with economic theory. Usually implemented in equilibrium models

- Choice approach: easier to implement. Hedonic rents

- Hedonic rent models take some simplifying assumptions
Bid-auction approach

- Willingness to pay of household $h$ for a residential unit $i$ can expressed in the form of a bid: $B_{hi}$
- Probability of household $h$ being the best bidder for location $i$:
  \[ P_{h/i} = \frac{\exp(\mu B_{hi})}{\sum_g \exp(\mu B_{gi})} \]
- Rent: expected maximum bid:
  \[ r_i = \frac{1}{\mu} \ln \left( \sum_g \exp(\mu B_{gi}) \right) \]
Bid-choice equivalence

- Choice approach assumes that households are price takers
- The utility (consumer surplus) can be written as:
  \[ V_{hi} = B_{hi} - r_i \]

  \[ P_{i/h} = \frac{\exp(\mu(B_{hi} - r_i))}{\sum_j \exp(\mu(B_{hj} - r_j))} \]

- If prices are the outcome of an auction, the location distribution is the same for the bid and choice approaches (Martínez 1992, 2000)
Hedonic rents

- Assumption: rents can be described as a function of the location attributes \( (z_i) \)… if a market equilibrium has been reached (Rosen, 1974)

- In general
  \[
  r_i = \sum_k \alpha_k z_{ik} \quad \rightarrow \quad \alpha_k = \frac{\partial r_i}{\partial z_{ik}}
  \]

- From the bid approach:
  \[
  \frac{\partial r_i}{\partial z_{ik}} = \sum_h \left( \frac{\partial \left( \ln \left( \sum_g \exp(B_{gi}) \right) \right)}{\partial B_{hi}} \cdot \frac{\partial B_{hi}}{\partial z_{ik}} \right) \quad \rightarrow \quad \frac{\partial r_i}{\partial z_{ik}} = \sum_h \left( P_{h/i} \cdot \beta_{hk} \right)
  \]
Simulation experiment

Objective
Compare rents obtained from:

– Maximum bid (logsum)
– Different specifications of hedonic rent models
Simulation experiment

- Synthetic city with:
  - 10 zones ($i$)
  - 3 types of residential units ($v$)
  - 3 types of household ($h$)

\[ B_{hvi} = \beta_{hz} z_i + \beta_{hy} y_v + \beta_{hH_1} H_{1i} + \beta_{hH_3} H_{3i} + b_h \]

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<th>parameter</th>
<th>$h = 1$</th>
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<td>$\beta_{hz}$</td>
<td>1.5</td>
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<td>$\beta_{hy}$</td>
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<td>$\beta_{hH_1}$</td>
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<td>$\beta_{hH_3}$</td>
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Simulation experiment

- Synthetic city with:
  - 10 zones ($i$)
  - 3 types of residential units ($v$)
  - 3 types of household ($h$)

$$B_{hvi} = \beta_{hz} z_i + \beta_{hy} y_v + \beta_{hH_1} H_{1i} + \beta_{hH_3} H_{3i} + b_h$$

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Simulation experiment

- $b_h$ represents adjustments in the bid accounting for:
  - Rich households realizing that they don’t have to bid their full willingness to pay
  - Poor households realizing that, in order to locate somewhere, they have to increase their bid

- Equivalent to ensure that all households are located somewhere

\[
H_h = \sum_{vi} S_{vi} P_{h/vi} \quad \longrightarrow \quad b_h = -\ln \left( \sum_{vi} S_{vi} \exp (B_{hvi} - r_{vi}) \right) \quad \forall h
\]
Simulation experiment

- Simulation of location choices following bid approach

\[ P_{h/\text{vi}} = \frac{H_h \exp(B_{hvi})}{\sum_g H_g \exp(B_{gvi})} \]

\[ r_{vi} = \ln \left( \sum_g H_g \exp(B_{gvi}) \right) \]

- In each period:
  1. A fraction of the households relocate
  2. All households adjust their bids
  3. Rents are recalculated

- 2 scenarios:
  a) Constant income distribution
  b) Increment of high income / decrease of low income
Simulation results (a)

- Logsum rents by zone
Simulation results (b)

- Logsum rents by zone
Simulation experiment

- Hedonic rent models to compare:
  - “naive”:
    \[ r_{vi} = c + \alpha_z z_i + \alpha_y y_v + \alpha_{H1} H_{1i} + \alpha_{H3} H_{3i} \]
  - Pseudo-logsum:
    \[ r_{vi} = c + \sum_h P_{h/vi} \left( \alpha_{hz} z_i + \alpha_{hy} y_v + \alpha_{hH1} H_{1i} + \alpha_{hH3} H_{3i} \right) \]
  - Pseudo-logsum2:
    \[ r_{vi} = c + \sum_h \frac{H_{hi}}{H_i} \left( \alpha_{hz} z_i + \alpha_{hy} y_v + \alpha_{hH1} H_{1i} + \alpha_{hH3} H_{3i} \right) \]
Simulation experiment

- Estimation over data generated for period 1
- “naive”
- pseudo-logsum

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<th>parameter</th>
<th>estimate</th>
<th>std-error</th>
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<td>( c )</td>
<td>8.555</td>
<td>0.011</td>
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<tr>
<td>( \alpha_z )</td>
<td>0.825</td>
<td>0.005</td>
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<td>( \alpha_y )</td>
<td>0.927</td>
<td>0.004</td>
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<tr>
<td>( \alpha_{H1} )</td>
<td>1.007</td>
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<td>( \alpha_{H3} )</td>
<td>-0.822</td>
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\( R^2 = 0.991 \)

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<td>( c )</td>
<td>8.776 (3.21E-05 )</td>
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<tr>
<td>( \alpha_{h,z} )</td>
<td>1.232 (2.09E-04)</td>
<td>0.634 (1.09E-04)</td>
<td>0.302 (1.56E-04)</td>
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<tr>
<td>( \alpha_{h,t} )</td>
<td>1.241 (1.79E-04)</td>
<td>0.629 (1.30E-04)</td>
<td>0.306 (2.03E-04)</td>
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<tr>
<td>( \alpha_{h,H1} )</td>
<td>1.238 (5.03E-04)</td>
<td>0.630 (3.16E-04)</td>
<td>0.305 (3.25E-04)</td>
</tr>
<tr>
<td>( \alpha_{h,H3} )</td>
<td>-1.244 (1.26E-03)</td>
<td>-0.626 (6.48E-04)</td>
<td>-0.308 (1.90E-04)</td>
</tr>
</tbody>
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\( R^2 = 0.995 \)
Simulation results (a)

- Average rents

![Graph showing average rents over periods for different models: logsum, pseudo-logsum, pseudo-logsum2, and hedonic.](image)
Simulation results (a)

- Rents by housing unit type
  - “Big” (type 1)
  - “Small” (type 3)
Simulation results (a)

- Rents by zone
  - High income (zone 5)
  - Low income (zone 1)
Simulation results (b)

- Average rents

![Graph showing simulation results for average rents over periods.](image-url)
Simulation results (b)

- Rents by housing unit type
  - “Big” (type 1)
  - “Small” (type 3)
Simulation results (b)

- Rents by zone
  - High income (zone 5)
  - Low income (zone 1)
Conclusions / Discussion

- Maximum bid and hedonic approaches generate different results
- Maximum bid approach naturally captures heterogeneity in households preferences
- It is hard to reproduce maximum bids using hedonic rent models
- Hedonic models are insensitive to changes in general market conditions (like income distribution)
Conclusions / Discussion

- Adjustment of the willingness to pay ($b_h$) is not explicitly modeled in most models, however, any assumption of location of all households requires some adjustment in the prices.
- Is it possible to directly replace a hedonic rent model by the expected maximum bid (logsum)?
- Further work:
  - Analysis with real data
  - Combination of logsum with hedonic approach