A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL OF CAR OWNERSHIP, USAGE AND FUEL TYPE

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OUTLINE

- Introduction
- Background and data
- Dynamic discrete-continuous choice model
- Illustrative example
- Estimation on synthetic data
- Conclusion and future works

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INTRODUCTION

Aim of the research:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

- Motivations
  - Governmental policies to reduce carbon emissions / car usage:
    - Stockholm congestion tax
    - Independence of fossil fuels
  - Technology changes:
    - Increase of alternative-fuel vehicles
  - Economical features:
    - Financial crisis
    - Fuel price changes

⇒ Car ownership and usage vary importantly over time.

⇒ Model needed to analyze and predict impact of policies on ownership

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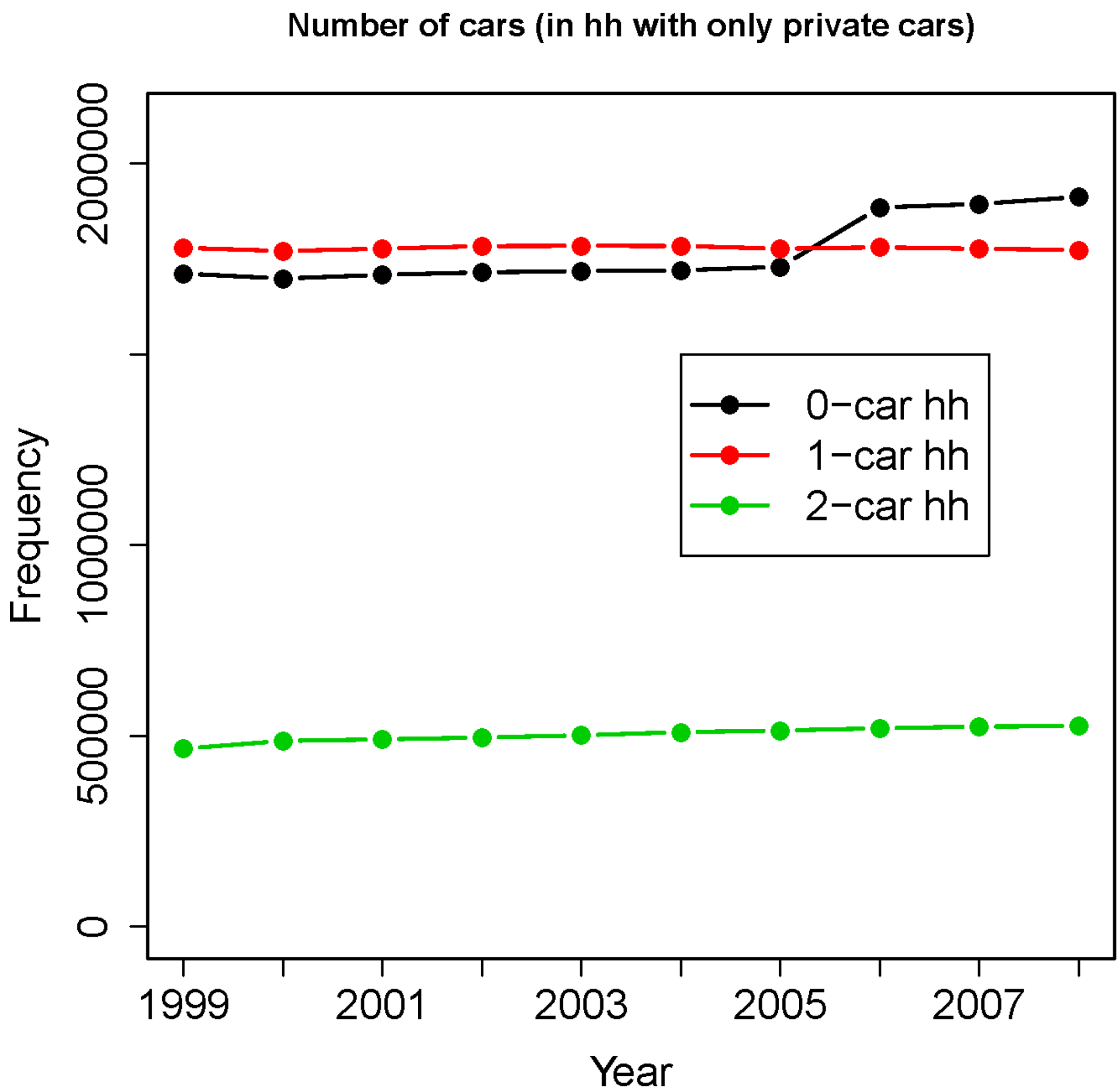
BACKGROUND AND DATA

Register data of Swedish population and car fleet:

- Data from 1998 to 2008
- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)
- All vehicles
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**

BACKGROUND AND DATA

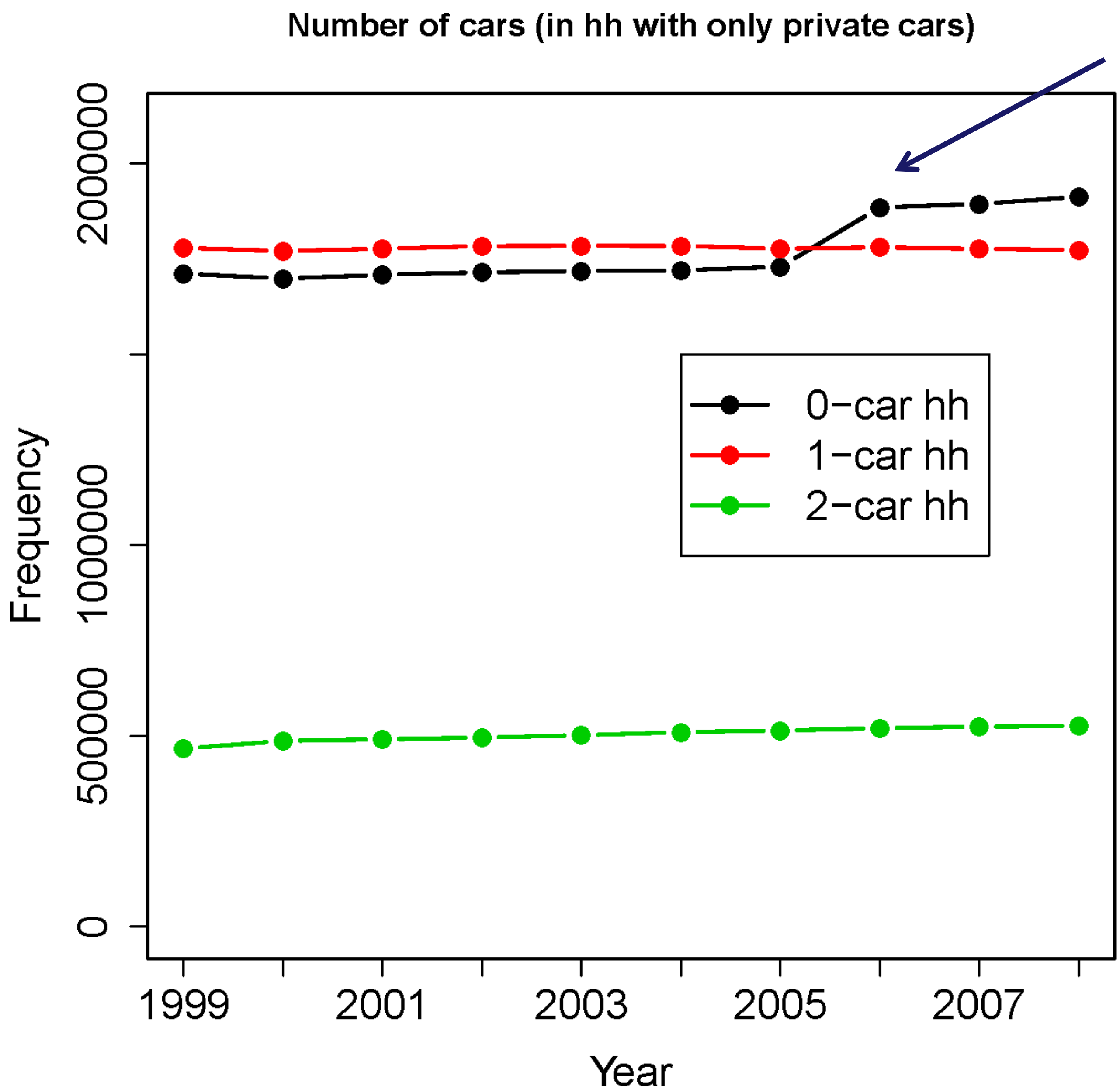
OWNERSHIP



BACKGROUND AND DATA

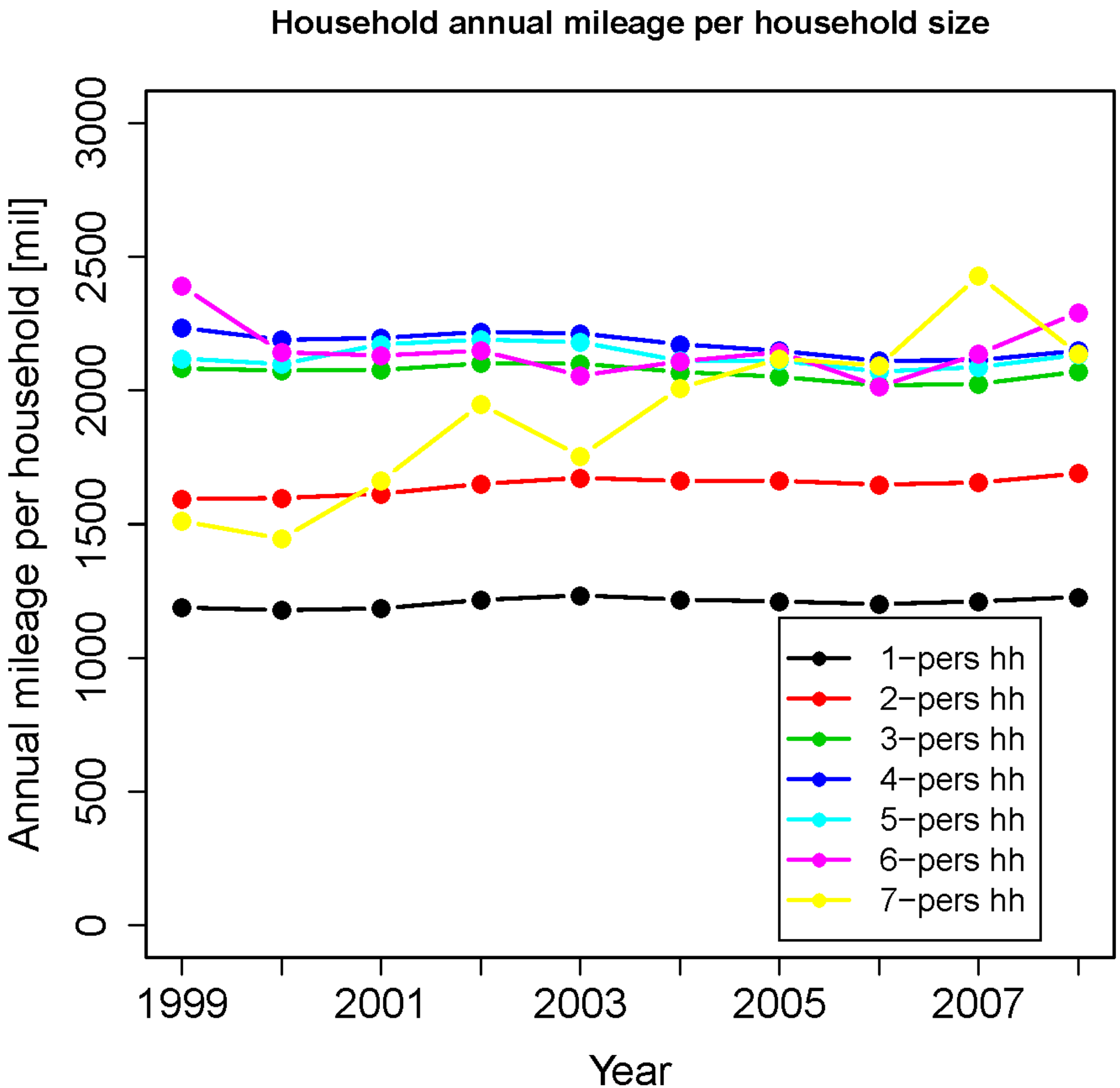
OWNERSHIP

Congestion tax Stockholm



BACKGROUND AND DATA

USAGE



BACKGROUND AND DATA

LITERATURE

- Car ownership models in transportation literature:
  - Discrete choice models (DCM) widely used, but mostly **static models**.
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - **Econometric literature**: **dynamic programming (DP)** + DCM
  - Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

- Joint models of car ownership and usage:
  - **Duration models** and regression techniques for car holding duration and usage (De Jong, 1996)
  - Vehicle type, usage and replacement decisions using **dynamic programming**, **discrete-continuous**, mixed logit (Schjerning, 2008, and Munk-Nielsen, 2012)
  - **Discrete-continuous model** of vehicle choice and usage based on register data: includes expectation of fuel prices & car future resale price (Gillingham, 2012)

Wide literature on car ownership and usage models

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BACKGROUND AND DATA

RESEARCH ISSUES

- Car are durable goods ⟹ Need to account for **forward-looking behavior** of agents
- Difficulty of modeling a **discrete-continuous choice** when jointly modeling car ownership and usage
- Many models focus on individual decisions, but choices regarding car ownership and usage made at **household level**

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BACKGROUND AND DATA

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Proposed methodology:

- Attempt to address these issues by applying **dynamic discrete-continuous choice model (DDCCM)**

Large **register data** of all **individuals** and **cars** in Sweden

BACKGROUND AND DATA

MAIN FEATURES

- In the area of **dynamic choice modeling**
  - Choices modeled at **household level**
  - **Up to two cars** allowed (only 4% households with > 2 cars in 2007)
- **Constant elasticity of substitution (CES) utility** to model annual driving distance for 2-car households
- Several **choices modeled simultaneously**

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

THE CHOICE VARIABLE IN DETAILS

Objective

Model simultaneously car ownership, usage and fuel type.
In details: model simultaneous choice of

$$\text{Transaction type} \times \left[\text{Annual milage} - \text{car } c \times \text{Private/company car} - \text{car } c \times \text{Fuel type} - \text{car } c \times \text{New/2}^{\text{nd}} \text{ hand} - \text{car } c \right]^{\#\text{ cars}}$$

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Discrete variables

THE CHOICE VARIABLE IN DETAILS

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Continuous variables

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

ASSUMPTIONS

1. **Choice** at **household level**: up to 2 cars in household

2. **Strategic choice** of:
   - Transaction
   - Type(s) of ownership (company vs private car)
   - Fuel type(s)
   - Car state(s) (new vs 2^{nd}-hand)

⇒ Account for forward-looking behavior of households

3. **Myopic choice** of:
   - Annual mileage(s)

4. **Choice** of **mileage conditional** on choice of discrete variables

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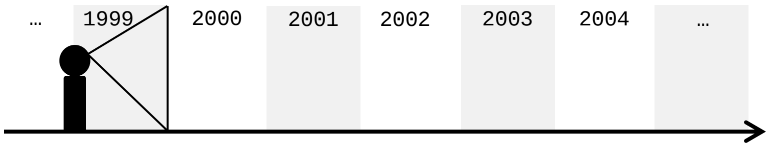
DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

ASSUMPTIONS

Myopic choice (static case)



$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}}$$

Strategic choice (dynamic case)

$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}}$$

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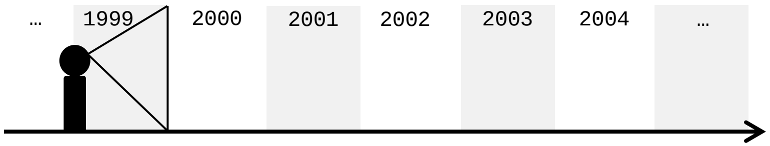

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Embeds a **choice model** into a **dynamic programming framework**

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

Components of the DDCCM:

- Agent
- Time step
- State space
- Action space
- Transition rule
- Instantaneous utility function

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Agent: household
- Time step *t*: year
- State space *S*

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})$$

Transaction type

Mileage – 1st car

Private/ company car – 1st car

Fuel type – 1st car

New/ 2nd hand – 1st car

Mileage - 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

New/ 2nd hand – 2nd car

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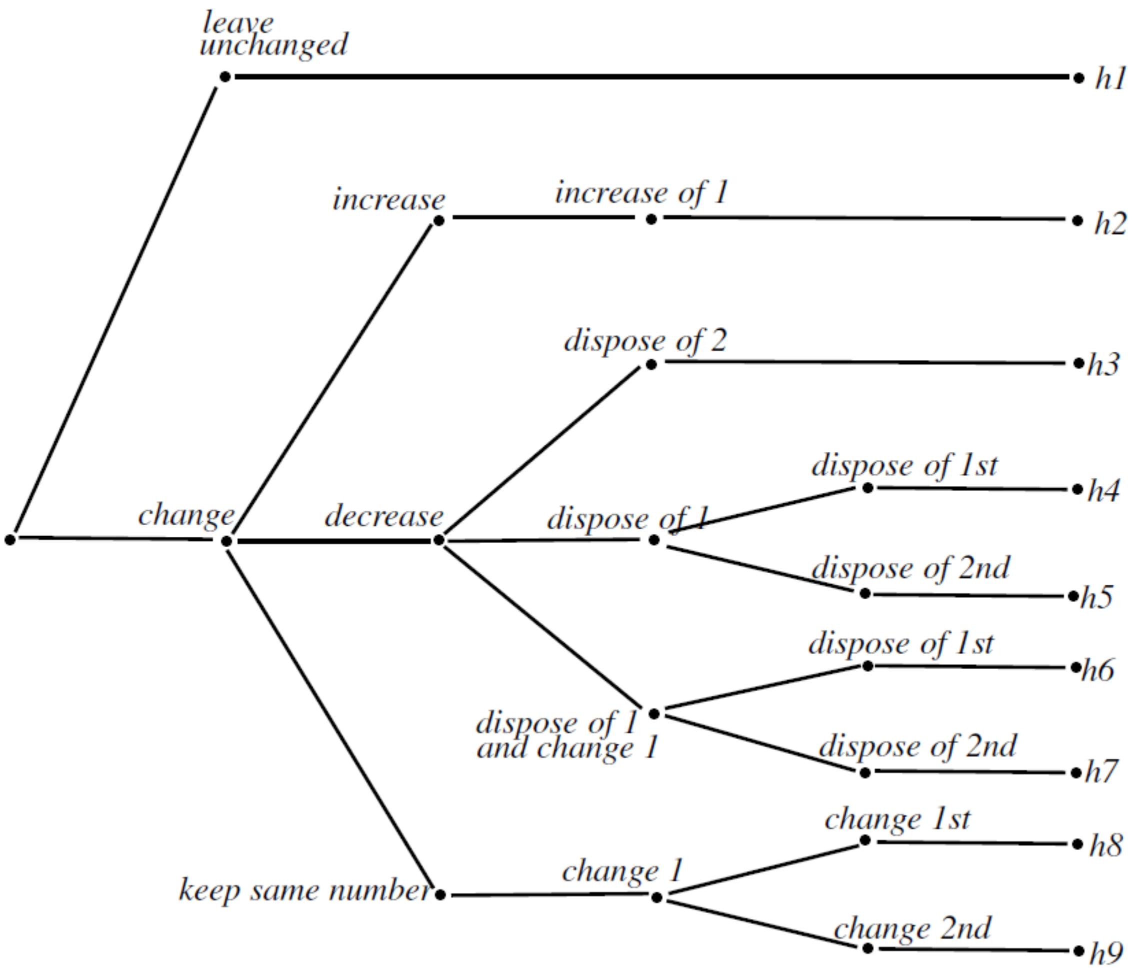
DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

Transaction types



DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Transition rule: deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Transition rule: deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.

- Instantaneous utility function:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = \underbrace{v(s_t, a_t^C, a_t^D, x_t, \theta)}_{\text{Deterministic term}} + \underbrace{\varepsilon_D(a_t^D)}_{\text{Random term for discrete choices}}$$

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a_t^C, a_t^D, x_t, \theta) = \underbrace{v_t^D(s_t, a_t^D, x_t, \theta)}_{\text{Utility for discrete actions}} + \underbrace{v_t^C(s_t, a_t^D, a_t^C, x_t, \theta)}_{\text{Utility for continuous actions}}$$

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Instantaneous utility function
  - Utility for continuous actions:
    Constant elasticity of substitution (CES) utility function (e.g. Zabalza, 1983):

$$v_t^C(s_t, a_t^D, a_t^C, x_t, \theta) = (m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho})^{-1/\rho}$$

    - Captures substitution patterns between the choice of both annual driving distances
    - ρ = elasticity of substitution
    - α = share parameter
  - Formulation could be extended to introduce randomness in α.

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

MODEL ESTIMATION

- Parameters obtained by maximizing likelihood:

$$\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^{D} | s_{n,t}, x_{n,t}, \theta)$$

- Optimization algorithm: Rust's **nested fixed point algorithm (NFXP)** (Rust, 1987):

  - **Outer optimization algorithm:** search algorithm to obtain parameters maximizing likelihood

  - **Inner value iteration algorithm:** solves the dynamic programming problem for each parameter trial

- Plan to investigate variants of NFXP to speed up computational time (e.g. swapped algorithm from Aguirregabiria and Mira, 2002)

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

Outer algorithm:

- Standard estimation procedure (as for static discrete choice models)
- Here: BHHH algorithm

Inner algorithm:

Two steps

1. Finding the optimal value(s) of annual mileage conditional on the discrete choices
2. Finding the expected discounted utility of future choices (= value function)

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

INNER ALGORITHM

1. Finding the optimal value(s) of mileage

- Maximization of the continuous utility: $\max_{m_{1,t},m_{2,t}} v_t^C$

  s.t. $p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \text{Inc}_t$

- Find analytical solutions: $m^*_{1,t}$ and $m^*_{2,t}$

$$m^*_{2,t} = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))}\alpha^{(-1/(\rho+1))}}$$

$$m^*_{1,t} = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m^*_{2,t}$$

- Optimal continuous utility $v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)$

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

INNER ALGORITHM

2. Finding the expected discounted utility of future choices (= value function)

- Logsum formula used in the completely discrete case (DDCM)
  (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)

- Logsum can be applied here given the key assumptions:
  - Choice of mileage(s) is conditional on discrete actions
  - Choice of mileage(s) is myopic

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \right\}$$

- Iterate on Bellman equation to find integrated value function $\overline{V}$

ILLUSTRATIVE EXAMPLE

Assumptions for the example:

- Deterministic utility function

$$v_t^D(s_t, a_t^D, x_t, \theta) = \mathrm{C}(s_t) + \tau(a_t^D) + \beta_{\mathrm{Age}}(a_t^D, s_t) \cdot \max(\mathrm{Age1}_t, \mathrm{Age2}_t)$$

Constant for households with at least one car

Transaction costs

Transaction-dependant parameters relative to age of oldest car

- Chose arbitrary values for parameters

ILLUSTRATIVE EXAMPLE

		β_{Age}			τ
Transaction name	Case	0 car	1 car	2 cars	all households
h_1: leave unchanged		0	-1	-1	0
h_2: increase 1		0	0	-	-3
h_3: dispose 2		-	-	1	0
h_4: dispose 1st	1st car is oldest	-	1.5	1.5	0
	2nd car is oldest	-	-	0	0
h_5: dispose 2nd	1st car is oldest	-	-	0	0
	2nd car is oldest	-	-	1.5	0
h_6: dispose 1st and change 2nd		-	-	0	-4
h_7: dispose 2nd and change 1st		-	-	0	-4
h_8: change 1st	1st car is oldest	-	1.5	1.5	-4
	2nd car is oldest	-	-	0	-4
h_9: change 2nd	1st car is oldest	-	-	0	-4
	2nd car is oldest	-	-	1.5	-4

C = 5, for 1- or 2-car households

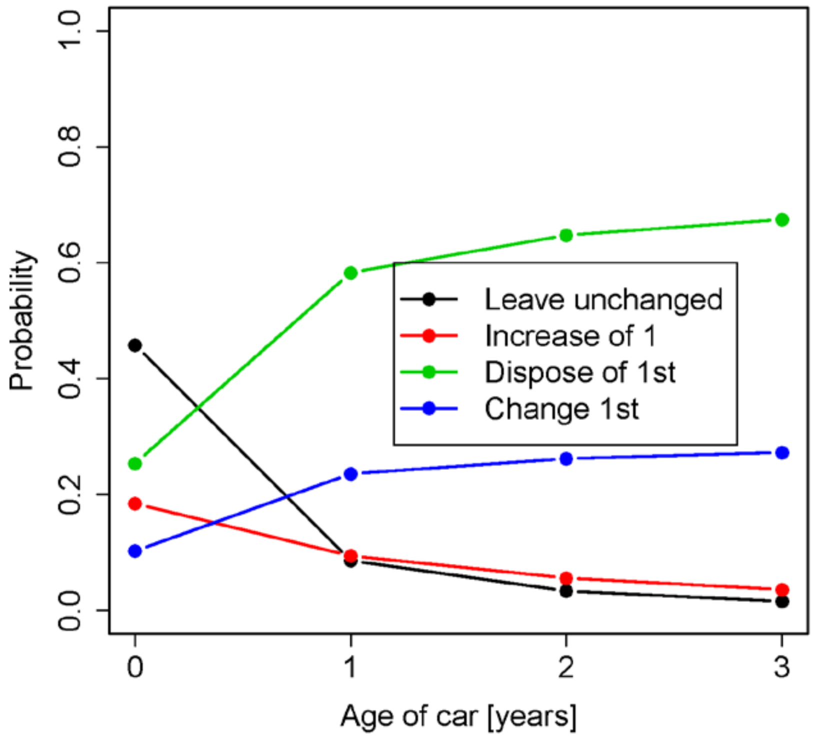
ILLUSTRATIVE EXAMPLE

Assumptions for the example:

- Visualize choice probabilities for one observation:
  - 1-car household
  - Annual income = 530’000 SEK (≈ 60’800 €)
  - 8% expenses on fuel

ILLUSTRATIVE EXAMPLE

Static



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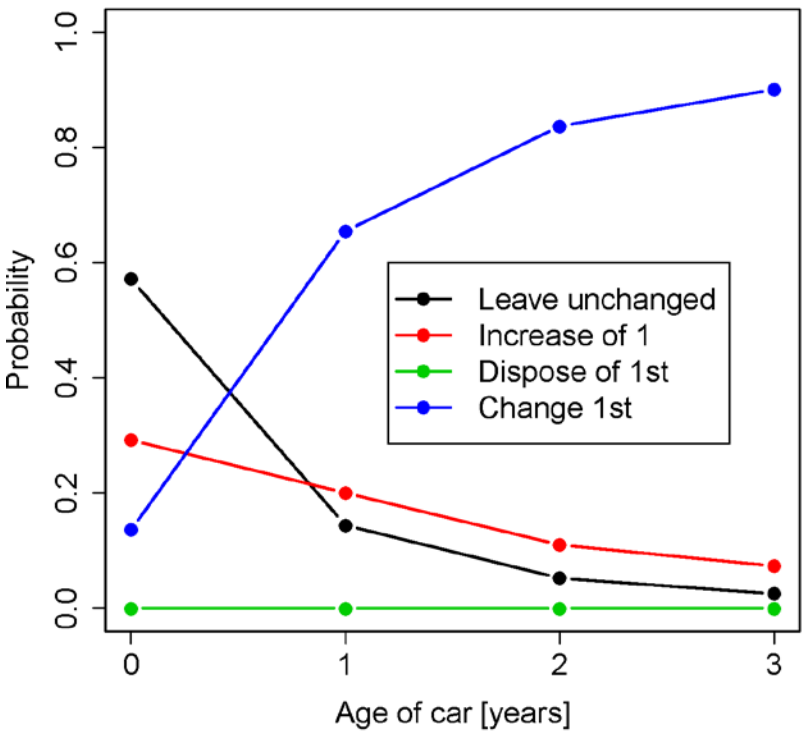

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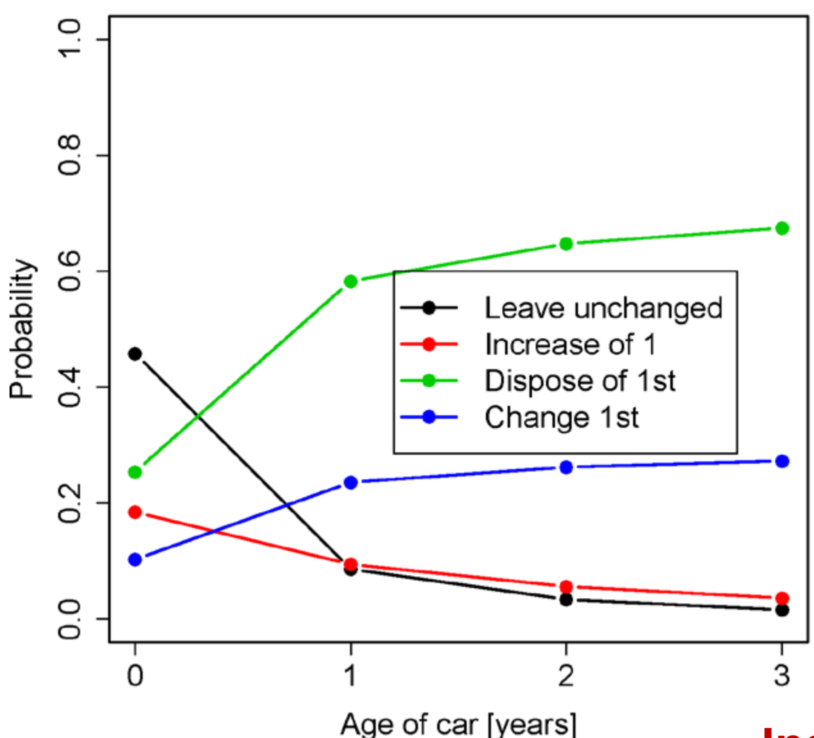

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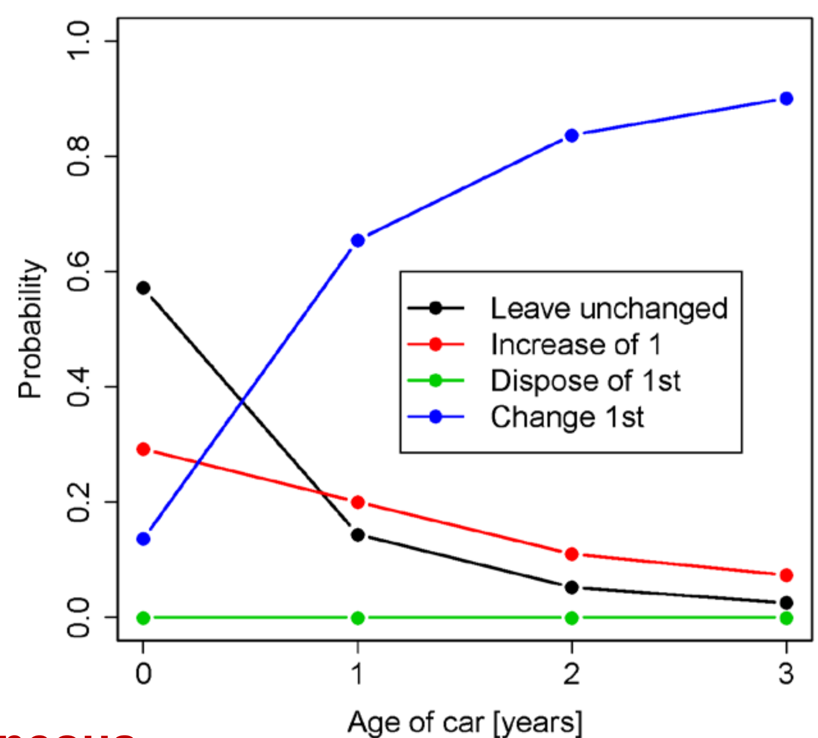

Instantaneous utility

$$P(a^D_{n,t} | s_{n,t}, x_{n,t}, \theta) = \frac{\boxed{v^D_{n,t} + v^{C*}_{n,t}} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f}{\sum_{\tilde{a}^D_{n,t}} \left\{ v^D_{n,t} + v^{C*}_{n,t} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f \right\}}$$

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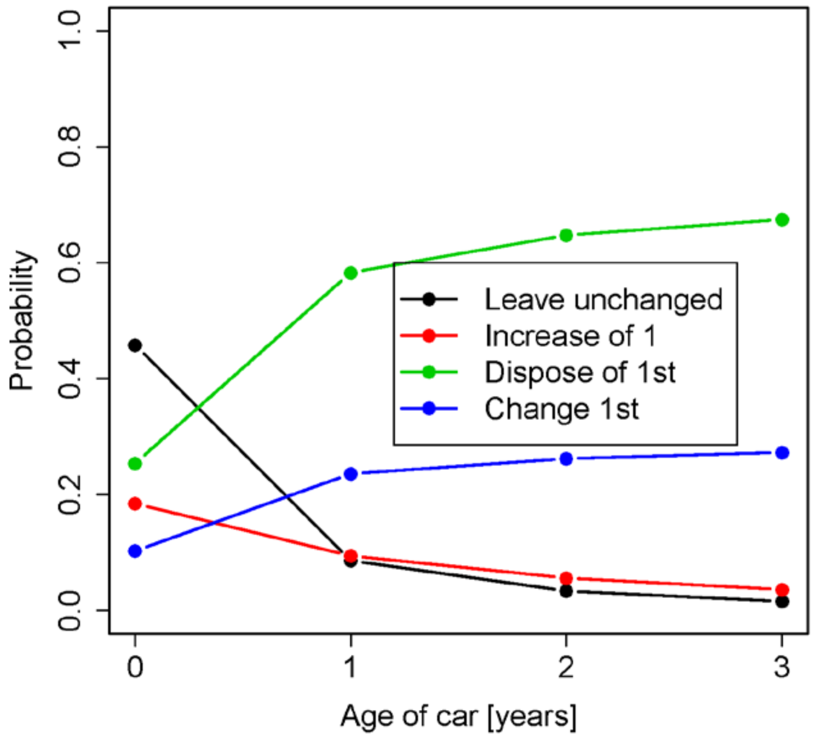

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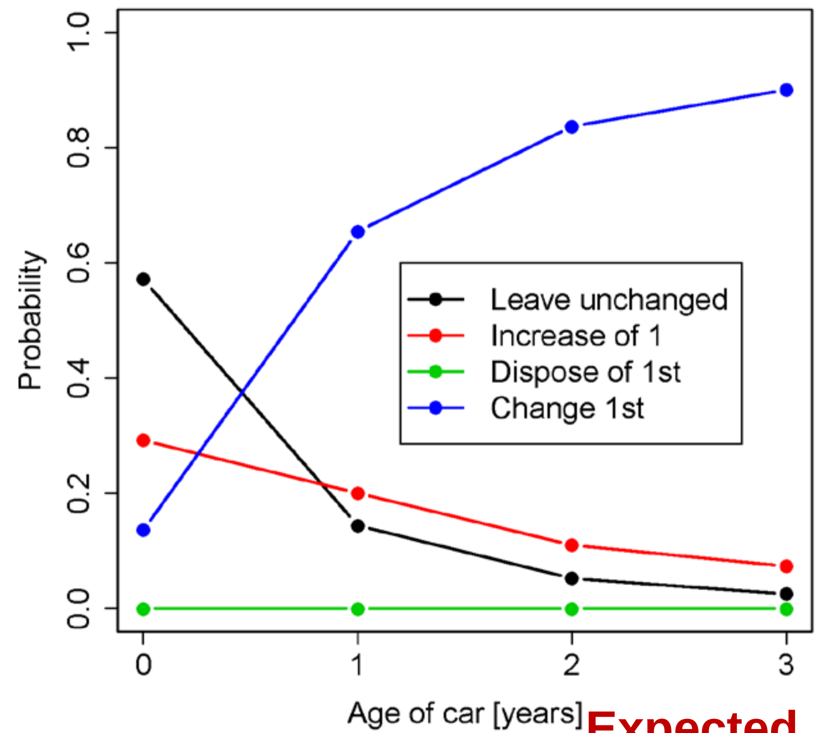

Expected discounted utility

$$P(a^D_{n,t} | s_{n,t}, x_{n,t}, \theta) = \frac{v^D_{n,t} + v^{C*}_{n,t} + \boxed{\beta \sum_{s_{n,t+1} \in S} \bar{V} f}}{\sum_{\tilde{a}^D_{n,t}} \left\{ v^D_{n,t} + v^{C*}_{n,t} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f \right\}}$$

ESTIMATION ON SYNTHETIC DATA

Approach to validate the model framework

- Generate 100 observations based on distributions of variables in the Swedish register data
- Generate choice (for each observation) based on postulated parameters
- Estimation of model
- Approach validated once postulated parameters are retrieved

CONCLUSION AND FUTURE WORKS

Conclusion:

- Methodology to model choice of car ownership and usage dynamically
- Example of application shows feasibility of approach
- Currently validating the approach (using synthetic data)

Next steps:

- Model estimation on register data
- Scenario testing:
  - Validation of policy measures taken during the years available in the data
  - Test policy measures that are planned to be applied in future years

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Thanks!