DYNAMIC DISCRETE CHOICE MODELING

FOR CAR USE, OWNERSHIP AND FUEL TYPE
BASED ON REGISTER DATA

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• Introduction
• The data
• Possible approaches
• The dynamic discrete choice model
• Value iteration results
• Conclusion and future works
Aim of the research project:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

Motivations
- Governmental policies (Source: Swedish parliament):
  - Goal of fossil-independent vehicle fleet by 2030
  - No emissions by 2050
- Technology changes:
  - Increase of alternative-fuel vehicles
  - Changes in the supply
- Company cars: represent important share of new car sales

Difficulties
- Car are durable goods modeling transactions not straightforward
- Need to account for forward-looking behavior of individuals
Current literature on car ownership and usage modeling:

- **Car ownership models in transportation literature:**
  - Mostly **static models**:
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - **Econometric literature: dynamic programming (DP) models**
  - Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

- **Other types of joint models of car ownership and usage:**
  - **Discrete-continuous model** of vehicle choice and usage based on register data (Gillingham, 2012)
  - **Duration models** and regression techniques for car holding duration and usage (De Jong, 1996)
THE DATA

Register data of Swedish population and car fleet:

- **Data from 1998 to 2008**

- **All individuals**
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)

- **All vehicles**
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**
Advantage of such detailed data:

• Can observe and analyze **demand shifts** that occurred as a response to changes in policies:
  • Changes in vehicle circulation taxes
  • Changes in fuel prices
  • Introduction of congestion pricing
    - at a national/regional level
    - at a local/regional level

• Can test the impact of **planned policies** on the demand for vehicles and car usage (e.g. fossil-independent fleet by 2030)
Aim of the project:

• Model simultaneously car ownership, usage and fuel type.  
**In details: model simultaneous choice of**

\[
\text{Transaction type} \times \left[ \begin{array}{c}
\text{Annual milage – car } c \\
\text{Private/company – car } c \\
\text{Fuel type – car } c \\
\text{New/2^{nd} hand – car } c \\
\end{array} \right] \times \# \text{ cars}
\]

• No more than 2 cars in household

• Account for forward-looking behavior of households
How can car ownership and usage be modeled?

Studied 2 approaches:

1. **Discrete-continuous dynamic programming:**
   mileage(s) considered as continuous and other variables as discrete.

2. **Dynamic discrete choice modeling:**
   all components of a choice variable are discretized.
How can car ownership and usage be modeled?

1. **Discrete-continuous DP**

   - **Endogeneous grid method (EGM)** to solve continuous choice problems (Carroll, 2006) in a fast way: avoids a numerical rootfinding procedure when the Euler equation is solved.

   - EGM **generalized for discrete-continuous** choices (Iskhakov et al, 2012). Valid when one continuous variable is considered.
POSSIBLE APPROACHES

How can car ownership and usage be modeled?

2. DDCM

- All variables considered as discrete $\Rightarrow$ mileage(s) discretized

- In transportation research, DP methods using random utility theory developed for discrete actions
  - Account for random term in utility function
  - Assumption that choices are affected by unobserved attributes, taste variations, etc.

- These DP methods lead to simple closed-form formula for the Bellman equation (e.g. Aguirregabiria and Mira, 2010)
How can car ownership and usage be modeled?

2. **DDCM**

- All variables considered as discrete \( \rightarrow \) mileage(s) discretized

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Motivations for DDCM

- In transportation engineering, usually work with random term in utility.

- Though treating mileage as continuous is more adapted, no work on discrete-continuous DP models accommodating random utility theory.

- Current DDCM were developed for a different setting:
  - Rust (1987) (and other applications) uses discrete actions and discretizes a continuous state space.
  - Here: large and discrete-continuous action space (while the state space is discrete and small).
Definition of the components of the dynamic programming model:

- **Agent**: household

- **Time step** $t$: year

- **State space** $S$

  $s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$

  
  \[ |S| = (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \]

  \[ + |Y| \times (|I_C| - 2) \times (|F| - 1) + 1 \]

  \[ + 1. \]
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+ 1.

2-car households
Definition of the components of the dynamic programming model:

- **Agent**: household
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$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

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+ 1. \text{ 0-car households}
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Definition of the components of the dynamic programming model:

- **Agent**: household
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$$+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)$$

$$+ 1. = 1407 \Rightarrow \text{relatively small state space}$$
Definition of the components of the dynamic programming model:

- **Action space** $A$

\[ a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_1, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_2, \tilde{r}_{2,t}) \]
Definition of the components of the dynamic programming model:

- **Action space** \( A \)

**Transaction type:** details
### Definition of the components of the dynamic programming model:

- **Action space** $A$

<table>
<thead>
<tr>
<th>Transaction name</th>
<th>0 car</th>
<th>1 car</th>
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<tbody>
<tr>
<td>$h1$: leave unchanged</td>
<td>1</td>
<td>4</td>
<td>16</td>
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<tr>
<td>$h2$: increase 1</td>
<td>72</td>
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<tr>
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**Size of the action space assuming:**
- 4 levels mileage
- 3 levels company car
- 3 levels fuel type
- 2 levels new/2nd hand

**Maximum size of action space**
Definition of the components of the dynamic programming model:

- **Instantaneous utility:** \( u(s_t, a_t, x_t, \theta) = v(s_t, a_t, x_t, \theta) + \varepsilon(a_t) \)

- **Transition rule:** deterministic rule: each state \( s_{t+1} \) can be inferred exactly once \( s_t \) and \( a_t \) are known.

**Example:**

If \( s_t = [2,1,2,0,0,0] \) and \( a_t = [1,2,0,0,0,1,3,0,0] \),

then \( s_{t+1} = [3,1,2,0,3,0] \).
Resolution of the dynamic programming problem:

Value iteration

- Value function:

\[ V(s_t, x_t, \theta) = \max_{a_t \in A} \{ u(s_t, a_t, x_t, \theta) + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \} \]

- Using the integrated value function \( \bar{V}(s_t, x_t, \theta) = \int V(s_t, x_t, \varepsilon_t) dG_{\varepsilon}(\varepsilon_t) \)
  - Another update rule that has a closed-form expression can be defined
  - Obtained from the expected maximum utility

\[ \bar{V}(s_t, x_t, \theta) = \log \sum_{a_t \in A} \exp \{ u(s_t, a_t, x_t, \theta) + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \} \]
Resolution of the DDCM problem:

- Parameters obtained by maximizing likelihood:

\[
\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T} P(a_{n,t} | s_{n,t}, x_{n,t}, \theta) f(s_{n,t} | s_{n,t-1}, a_{n,t-1})
\]

- Optimization algorithm is Rust's **nested fixed point algorithm** (Rust, 1987):
  
  - **Outer optimization algorithm**: search algorithm to obtain parameters maximizing likelihood
  
  - **Inner value iteration algorithm**: solves the DP problem for each parameter trial
Preliminary results for the value iteration algorithm:

Inputs:
- **Size state space = 1407**
  - Max age = 5
  - Company car levels = 3
  - Number of fuel types = 3
- **Size action space = max 745**
  - Number of transaction types = 9
  - Number of mileage levels = 4
  - Number of levels of new/old = 2

- **Utility function contains:**
  - Transaction-dependent parameters for fuel type
  - Transaction-dependent parameters for age of oldest car
- **Parameters of DP problem:**
  - Discount factor $\beta = 0.7$
  - Stopping criterion $\varepsilon = 0.01$
Preliminary results for the value iteration algorithm:

Program:

• Prototype in Matlab
• 15 hours on 8-core server

Graph:

• \( \bar{V} \) vs age of oldest of 2 cars
Preliminary results for the value iteration algorithm:

Program:

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Graph:

• Assume an initial action $a = [3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ 
  $u(a)$ vs age of oldest car
Preliminary results for the value iteration algorithm:

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Graph:
• Assume an initial action
  \( a = [3, 1, 0, 0, 0, 0, 0, 0, 0] \)
  \( P(a) \) vs age of oldest car
**Conclusion and Future Works**

**Conclusion:**
- First results of value function
- Shows feasibility of problem

**Future works:**
- Speed up computation of integrated value function $\rightarrow$ C++
- Sequential choice of two vehicles if computational time still too large
- Exploratory analysis to specify instantaneous utility
- Implement outer loop (maximization algorithm)
- Scenario testing:
  - Validation of policy measures taken during the years available in the data
  - Test policy measures that are planned to be applied in future years
Thanks!