Metropolis-Hastings sampling of alternatives for route choice models

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August 20, 2010

Motivation

- a route choice model describes what way from an origin to a destination is chosen in a network
- universal set of alternatives is unknown and intractably large
- estimation of route choice models requires selection of a subset

Approaches to route choice set generation

- modeling of consideration sets
  - deterministic (e.g., K-SP) or stochastic (randomized SP)
  - unrealistic: fail to capture the chosen alternative
- assume that decision maker considers all alternatives
  - also unrealistic
  - sampling protocol generates operational subset
  - correct for sampling in the estimation

Sampling of alternatives

- sample $\mathcal{C}_n$ with replacement from $\mathcal{C}$ according to $\{q(i)\}_{i\in\mathcal{C}}$
- add the chosen alternative
- k_{in} is the number of times alternative i is contained in $\mathcal{C}_n$
- correct for sampling when estimating logit model

$$P(i|\mathcal{C}_n) = \frac{e^{\mu V_{in} + \ln\left(\frac{k_{in}}{b(i)}\right)}}{\sum_{j\in\mathcal{C}_n} e^{\mu V_{jn} + \ln\left(\frac{k_{jn}}{b(j)}\right)}}$$

where $\{b(i)\}_{i\in\mathcal{C}}$ is such that $q(i) = b(i)/\sum_{j\in\mathcal{C}} b(j)$

objective: sample paths according to pre-specified $\{b(i)\}_{i\in\mathcal{C}}$

Using Markov chains (MCs)

- finite state space
- discrete time $k = 0, 1, \ldots$
- at time k, process is in state i^k
- $q(i, j)$ is one-step probability to reach state j from state i
- process has a unique stationary distribution if
  - every state eventually reaches every other state
  - there is at least one state i with $q(i, i) > 0$

objective: build MC of routes with stationary distribution $\{q(i)\}_{i \in \mathcal{C}}$

Metropolis-Hastings (MH) algorithm

- given a finite state space, positive weights $\{b(i)\}_i$ and "well-mixing" proposal transition distribution $q(i,j)$, MH generates MC that converges to $q(i) = b(i)/\sum_j b(j)$

1. set iteration counter $k = 0$
2. select arbitrary initial state i^k
3. repeat beyond stationarity
    3.1 draw candidate state j from $\{q(i^k, j)\}_j$
    3.2 compute acceptance probability $\alpha(i^k, j) = \min\left(\frac{b(j)q(j,i^k)}{b(i^k)q(i^k,j)}, 1\right)$
    3.3 with probability $\alpha(i^k, j)$, let $i^{k+1} = j$; else, let $i^{k+1} = i^k$
    3.4 increase k by one

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Application of MH for route choice set generation

- state space comprises $\mathcal{C}$
- weights $b(i)$ favor plausible paths (importance sampling)
- transition distribution $q(i, j)$ creates local path modifications
  - too little variability: slow convergence
  - too much variability: random search

State space

- notation
  - Γ = a path (node sequence)
  - $\Gamma(u)$ = uth node of path Γ
  - $\Gamma(u, v)$ = sub-path from the uth to the vth node of Γ
  - $|\Gamma|$ = number of nodes in path Γ
- state = (a path Γ, two SPLICE locations, one SPLICE node)
  - SPLICE locations $u, d \in \mathbb{N}$ with $1 \leq u < d \leq |\Gamma|$
  - SPLICE node v
  - (state expansion helps to compute $q(i, j)$)

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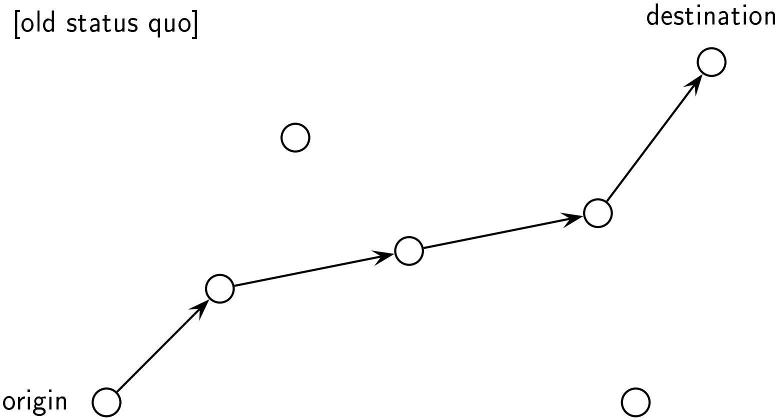
Proposal transition distribution

- SPLICE operation
  - compute (random) path from $\Gamma(u)$ to v
  - compute (random) path from v to $\Gamma(d)$
  - replace $\Gamma(u, d)$ by that sequence
- SHUFFLE operation
  - re-sample (uniformly) splice locations u and d
  - re-sample splice node v near to to $\Gamma(u, d)$
- randomly select one procedure

$$q(i,j) = \gamma q_{SPLICE}(i,j) + (1-\gamma) q_{SHUFFLE}(i,j)$$

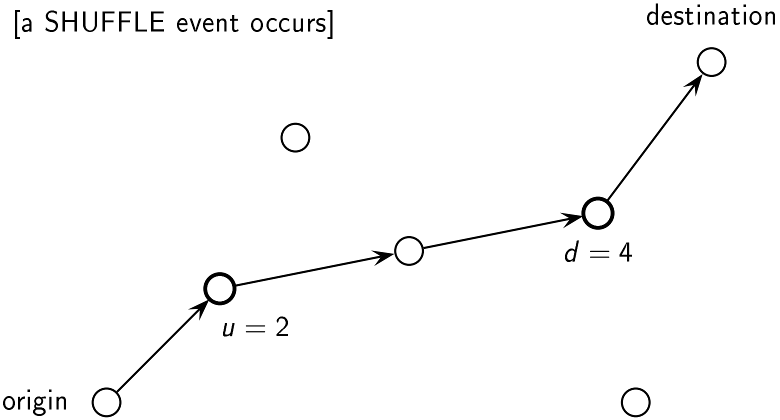
with $0 < \gamma < 1$

Illustrative example



Illustrative example

[a SHUFFLE event occurs]



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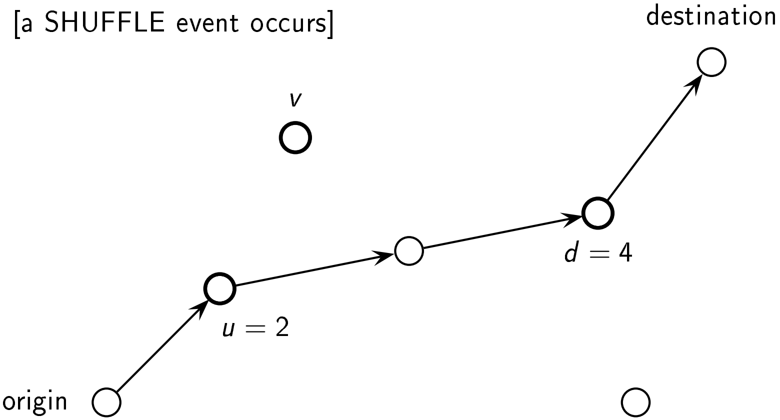

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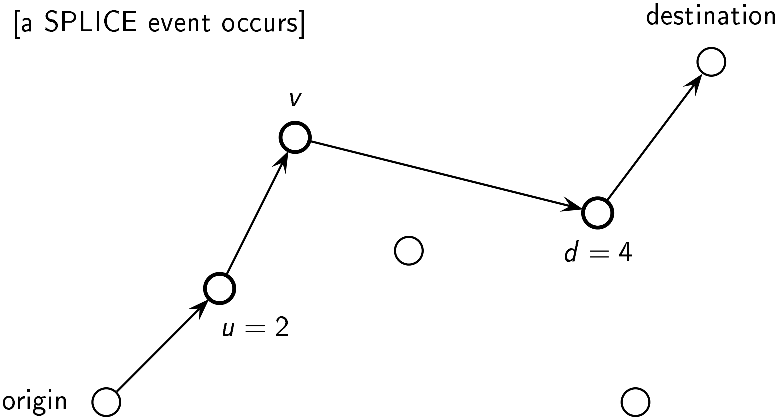

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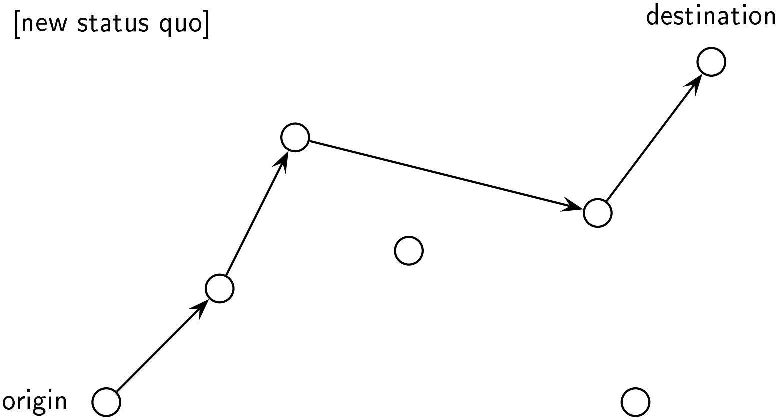

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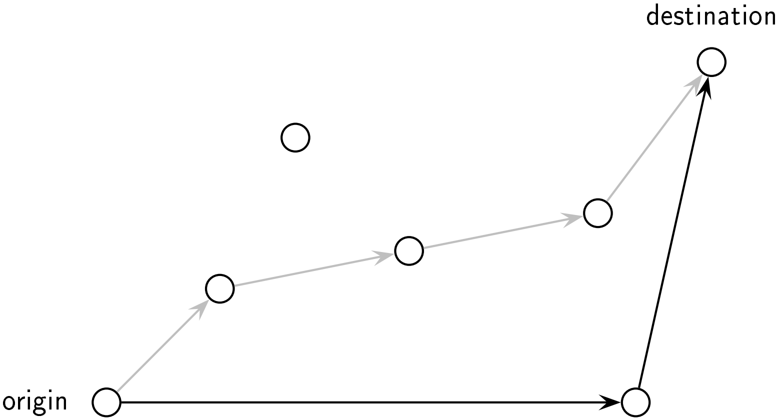
Discussion

- this *eventually* draws paths from *any* distribution
- computationally
  - feasible path in every iteration
  - need to reach and identify stationarity
  - strong auto-correlation of subsequent paths
- behaviorally
  - explorative travel behavior?
  - occasional intermediate destinations?
- iterated DTA simulations (such as MATSim)
  - an all-day plan is a generalized path
  - small plan choice set for computational reasons
  - challenge: capture distribution in simulated conditions

Shortest path SPLICE reaches every path



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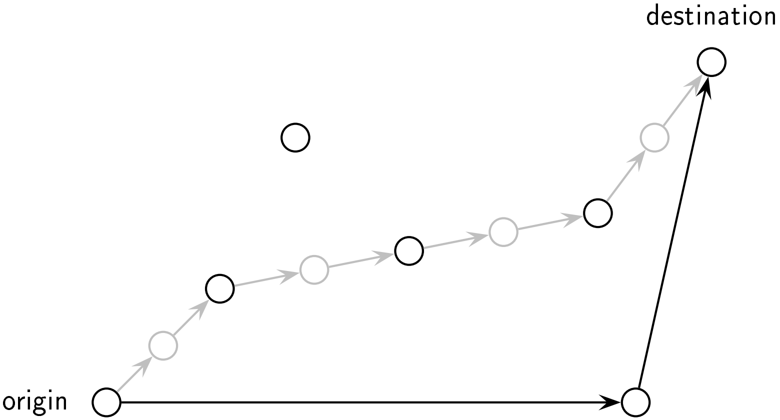

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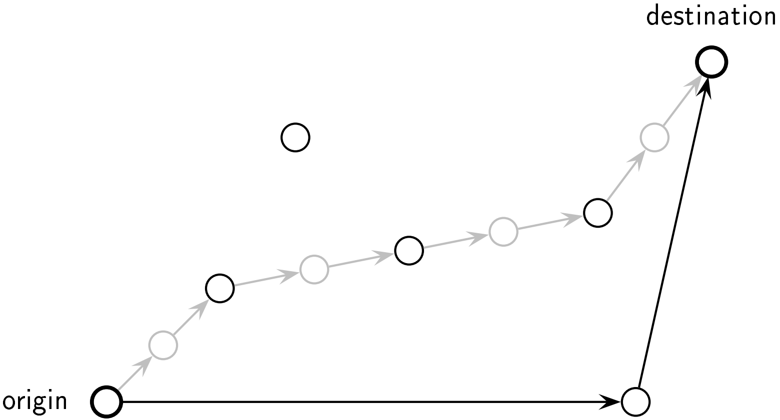

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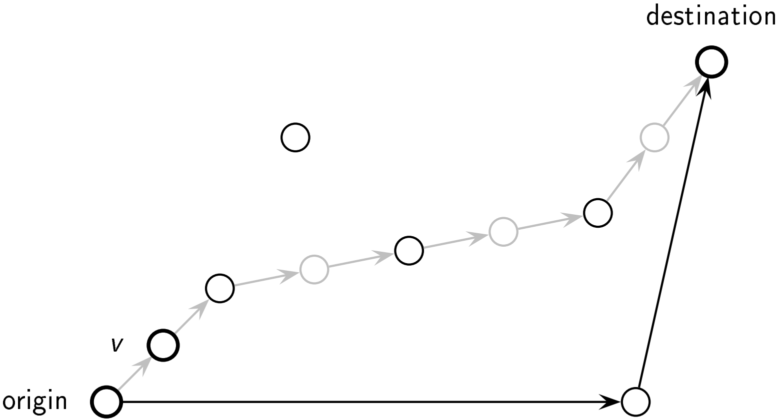

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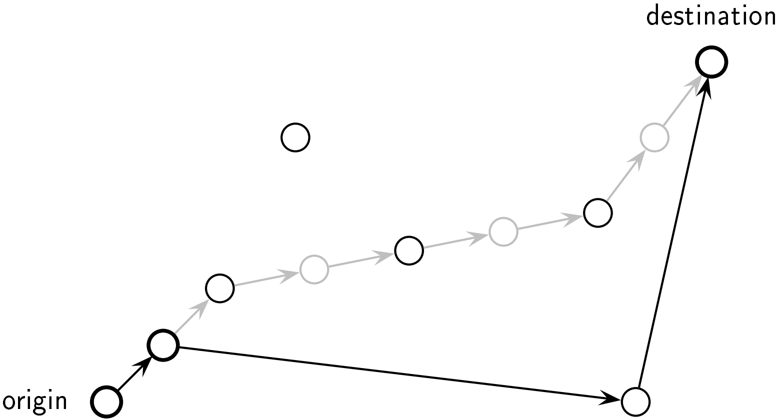

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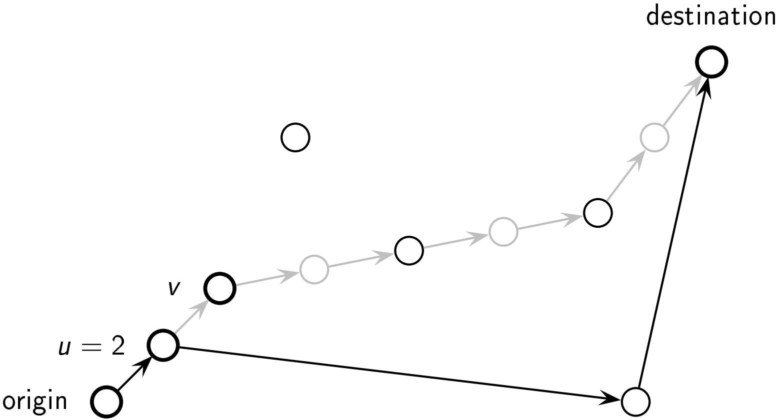

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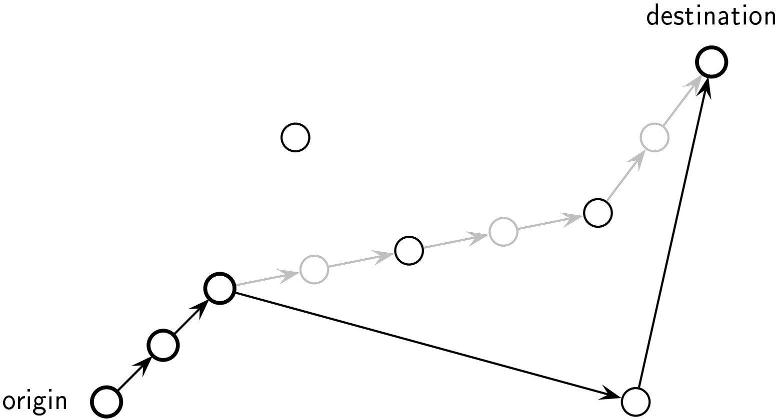

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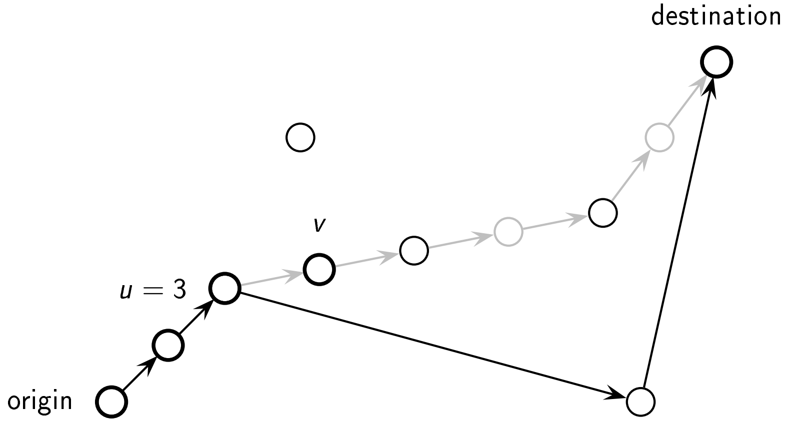

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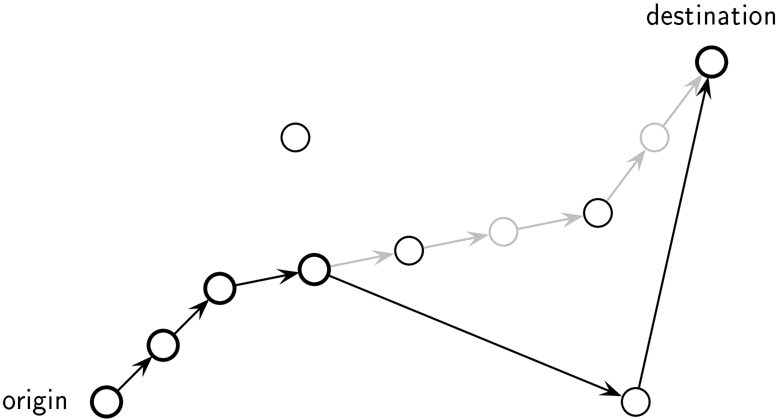

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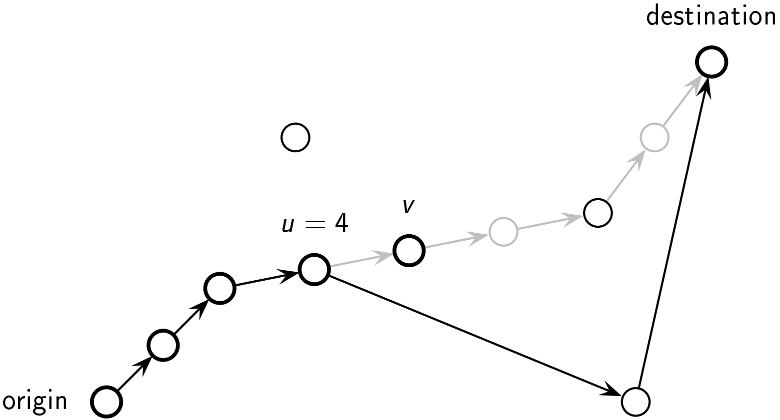

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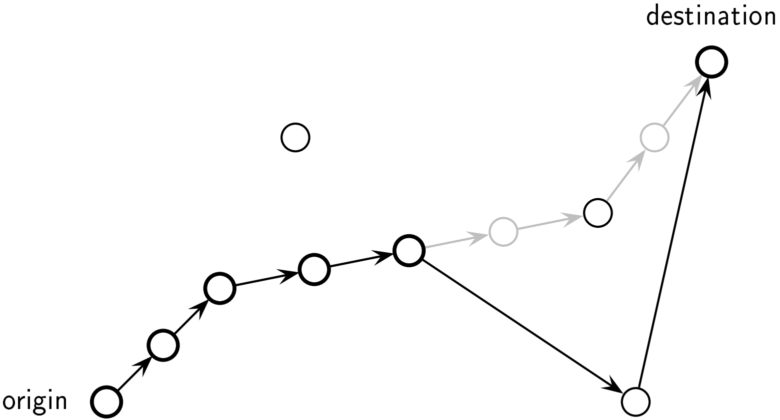

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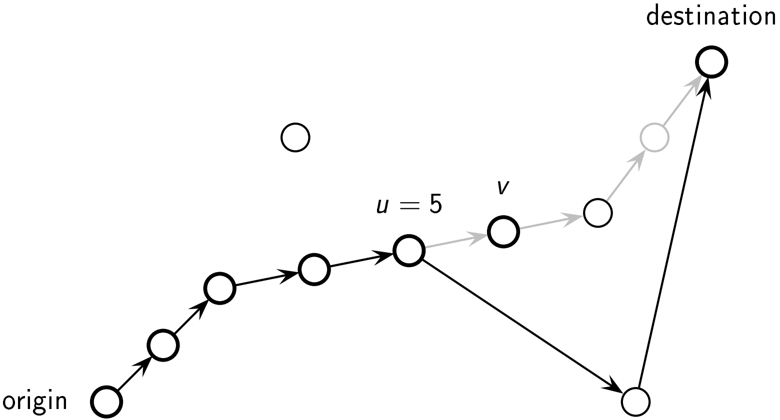

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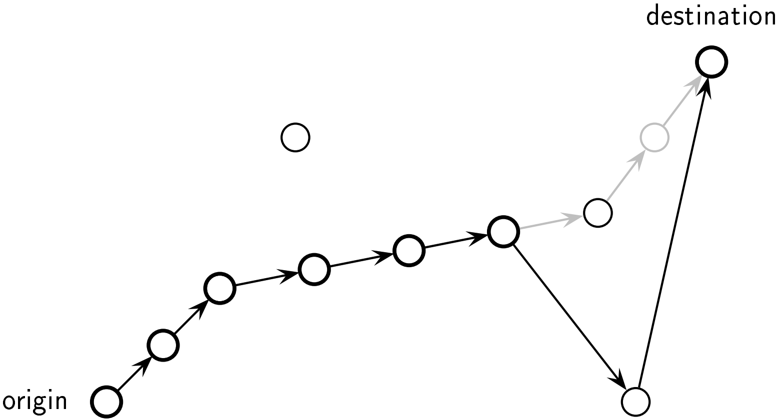

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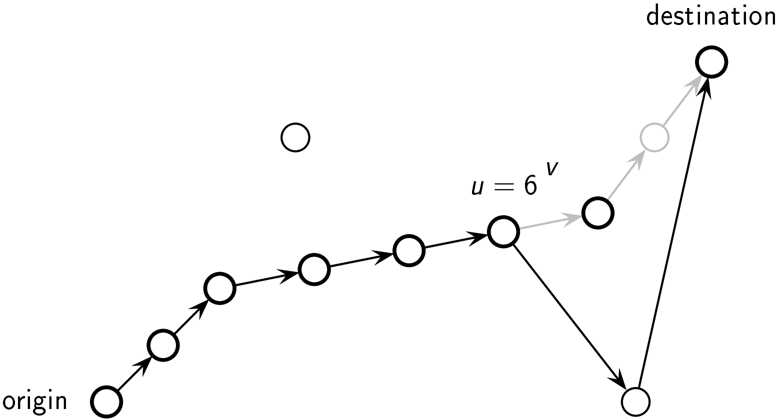

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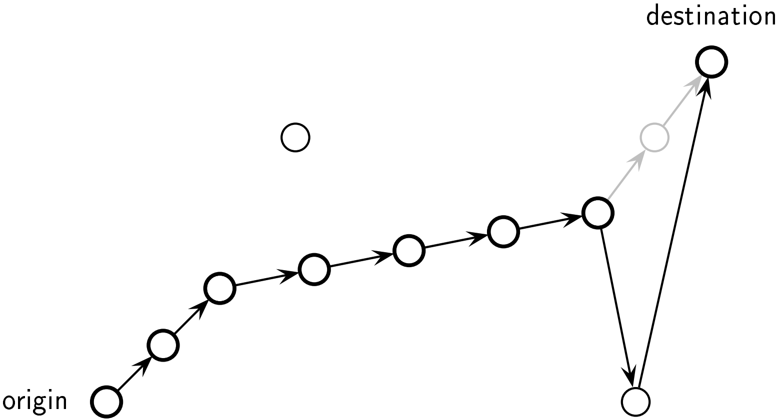

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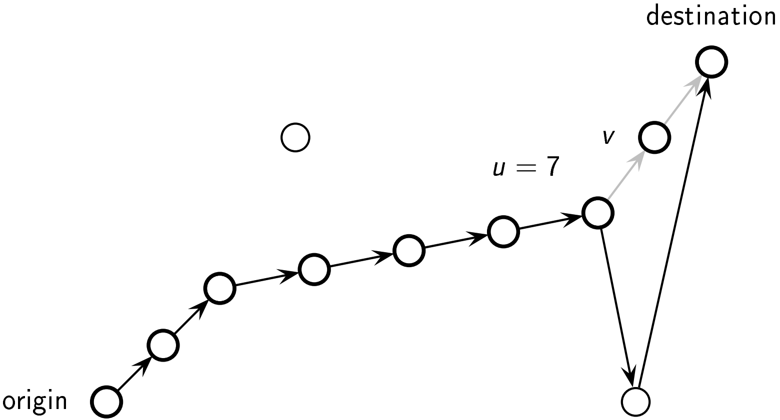

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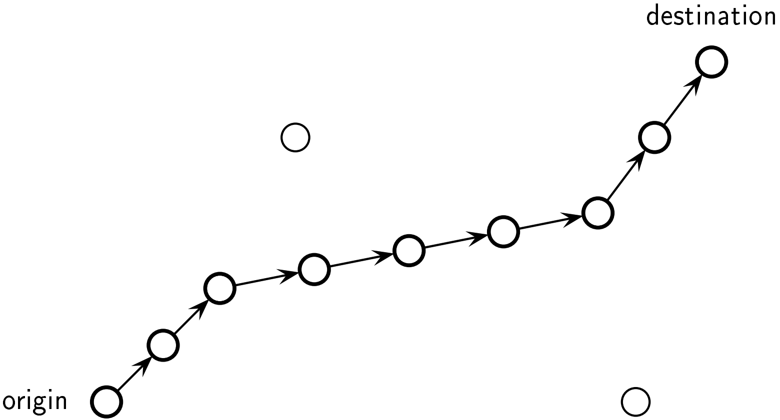

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