Dynamic network loading: a stochastic differentiable model that derives link state distributions

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July 19, 2011

Motivation

- stochastic queueing model of traffic flow
  - modeling of risk-averse travel behavior
  - analysis of network breakdowns
  - computation of expected values
  - ...
- specified through differentiable equations
  - linearization of network loading map
  - optimization, calibration, OD estimation
  - ...

Outline

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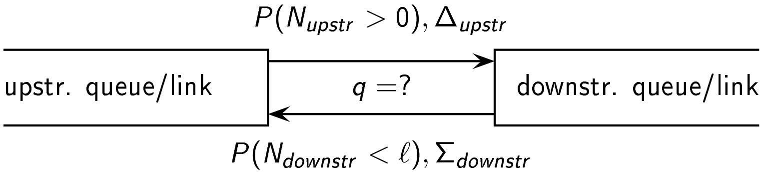
Queueing theory and LWR model – symmetries

queueing theory	LWR model (Lebacque, '96)
arrival rate λ	upstream demand Δ
service rate μ	downstream supply Σ
number of jobs N	traffic density ϱ
max. number of jobs ℓ	maximum density $\hat{\varrho}$

Boundary conditions – symmetries



queueing theory	LWR model
availability prob. $P(N_{upstr} > 0)$	upstr. demand Δ_{upstr}
non-blocking prob. $P(N_{downstr} < \ell)$	downstr. supply $\Sigma_{downstr}$
$q \propto P(N_{upstr} > 0,\ N_{downstr} < \ell)$	$q = \min\{\Delta_{upstr}, \Sigma_{downstr}\}$

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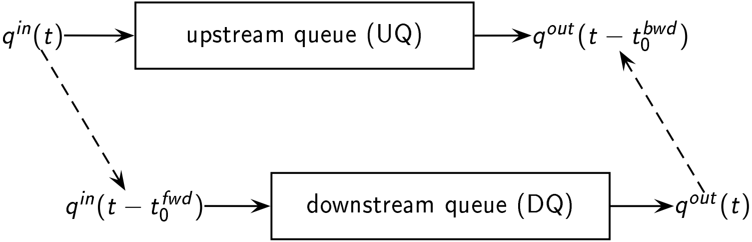
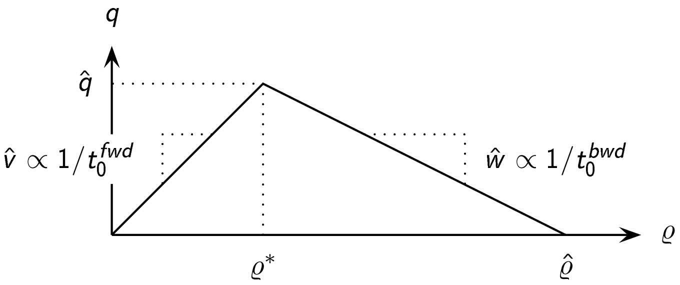
Link model: composed of two queues



(e.g., Yperman et al., '07; Charypar et al., '07)

- $q^{in}(t)$, $q^{out}(t)$ is inflow, outflow at continuous time t
- forward lag t_0^{fwd} generates lower bound on link travel time
- backward lag t_0^{bwd} generates slow queue dissipation

Link model: FD for *deterministic* queues



- *expected* FD in *distributed* model look qualitatively similar
  - main difference: stationary queue distribution depends on λ/μ
  - tentative discussion in paper

Node model: dynamics of adjacent queues

- state space of two consecutive queues (1 and 2)

$$\mathbf{s} = (N_1, N_2) \subset \mathbb{N}^2$$

- distribution dynamics follow linear differential equation

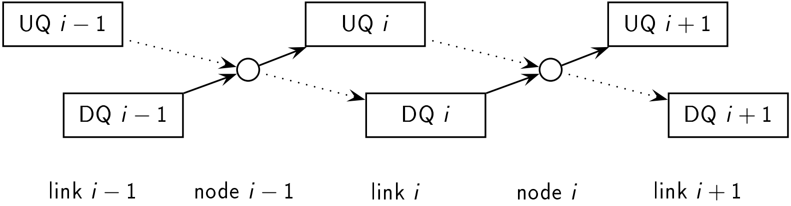
$$\dot{p}(\mathbf{s}(t)) = p(\mathbf{s}(t)) \cdot Q$$

  where Q is a square matrix of transition rates
- flows across a node results from

$$q_1^{\text{out}} = \mu_1 \Pr(N_1 > 0,\, N_2 < \ell_2) = \mu_1 \sum_{\mathbf{s}:\, N_1 > 0,\, N_2 < \ell_2} p(\mathbf{s})$$

  where μ_1 is the exogeneous node service rate

(Currently) realized correlation structure



- captures joint distribution of queues adjacent to nodes
- does not capture dependency of UQ & DQ within links

Discrete time simulation of (linear) networks

1. set initial queue distributions
2. repeat through discrete time:
   2.1 compute node boundaries $\Pr(N_i > 0,\ N_{i+1} < \ell_{i+1})$
   2.2 compute node flows $q_i^{\text{out}} = q_{i+1}^{\text{in}} = \mu_i \Pr(N_i > 0,\ N_{i+1} < \ell_{i+1})$
   2.3 compute corresponding queue service and arrival rates
   2.4 compute (integrate) next time step's queue distributions

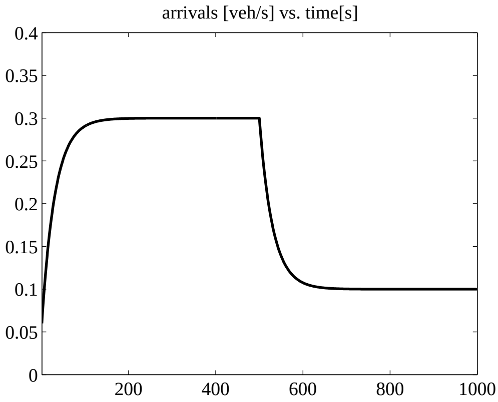
Outline

Settings

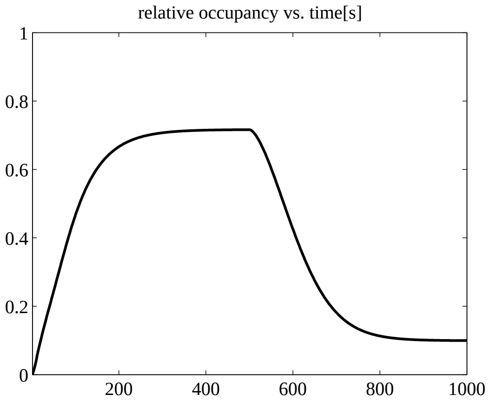
- a single, initially empty link
- parameters:

parameter	value	normalized
vehicle length	5 m	1 place
link length	100 m	20 places
max. density $\hat{\varrho}$	200 veh/km	1 veh/place
time step length	1 s	1 s
free flow velocity $\hat{v}$	36 km/h	2 places/s
backward wave speed $\hat{w}$	18 km/h	1 place/s
downstream bottleneck	720 veh/h	0.2 veh/s

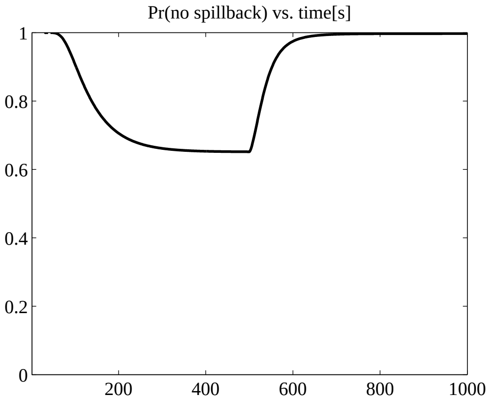
Arrivals – $\lambda(t)$



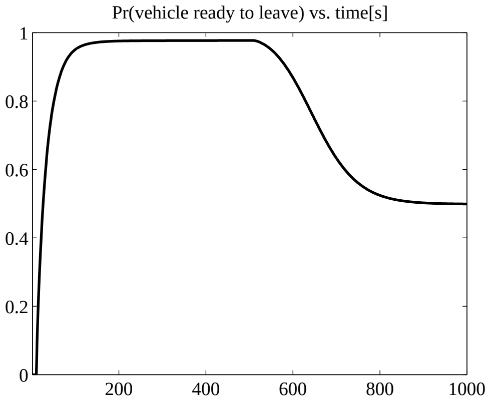
Relative occupancy – $\varrho(t)/\hat{\varrho}$



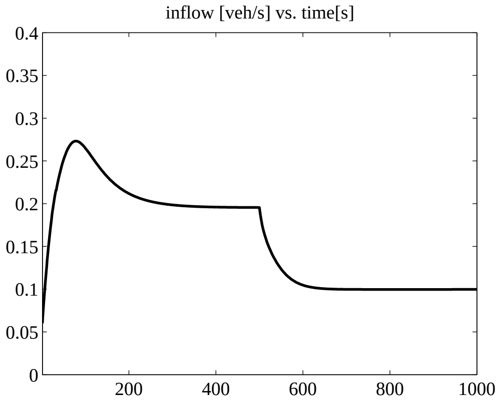
Non-blocking probability – $P(N_{UQ}(t) < \ell)$



Availability probability – $P(N_{DQ}(t) > 0)$

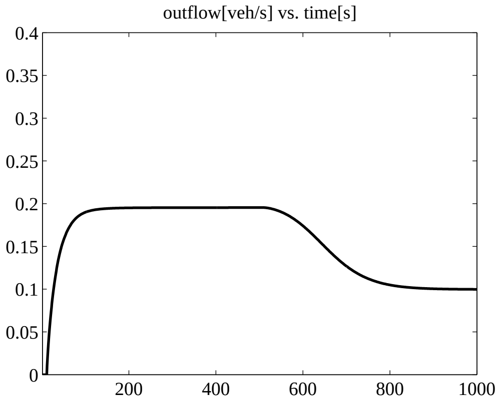


Inflow – $q^{in}(t)$



inflow [veh/s] vs. time[s]
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
200
400
600
800
1000

Outflow – $q^{out}(t)$



Discussion

- build-up, spillback, and dissipation of congestion is captured
  - bottleneck shifted upstream in supply regime
  - arrival rate shifted downstream in demand regime
- stochastic model captures
  - spillback probabilities
  - queueing in undersaturated conditions
  - (variances of queue lengths, travel times, ...)

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- dynamic network loading with finite capacity queueing theory
  - captures queue length distributions
  - accounts for spillback
  - describes well build-up and dissipation of queues
- differentiability good for estimation, optimization, assignment
- ongoing work:
  - more comprehensive correlations
  - general node models & network flows