Optimization of Uncertainty Features for Transportation Problems

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Outline

- Optimization under Uncertainty: Existing Methods
- Uncertainty Feature Optimization (UFO)
- UFO: generalized framework
- Example: Multi-Dimensional Knapsack Problem
- Simulation Results for MDPK
- Future Work and Conclusions

Optimization with Noisy Data

Typical Examples

- Portfolio Optimization
- Vehicle Routing (GPS, transport problems, …)
- Project Management
- Many others!

Four Approaches

1. Neglect and solve deterministic problem

   - Not realistic (Herroelen 2005, Sahinidis 2004)

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Optimization
   - Data-driven
   - Not feasible for some problems (e.g. airline schedules)

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Optimization
3. Characterize the Uncertainty and solve robust or stochastic problems
   - Need explicit Uncertainty characterization
   - Hard to characterize/model in general
   - Leads to difficult problems
   - Sensitive to uncertainty characterization
   - Solutions tend to “simple” properties

Examples from Airline Scheduling

- Increase plane's idle time (Al-Fawzana & Haouari 2005)
- Decrease plane rotation length (Rosenberger et al. 2004)
- Departure de-peaking (Jiang 2006, Frank et al. 2005)
- More plane crossings (Bian et al. 2004, Klabjan et al. 2002)
- ...

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Scheduling
3. Characterize the Uncertainty
4. Model Uncertainty Implicitly => Uncertainty Features

Uncertainty **F**eature **O**ptimization

I. Increase robustness/stability (e.g. idle time)

II. Increase recoverability (e.g. plane crossings)

UF: Definition

Given a problem with Decision Variables ***x***

UF: a function $\mu(\boldsymbol{x})$ measuring the "*quality*" of a solution ***x***

OBJECTIVE: MAX $\mu(\boldsymbol{x})$

s.t. ***x*** feasible solution to initial problem

General Optimization Problem

$$
\begin{aligned}
&MIN \quad f(x) \\
&s.t. \quad a(x) \le b \\
&\qquad\quad x \in X
\end{aligned}
$$

UF and Optimality Budget

$\mu(\boldsymbol{x})$	Uncertainty Feature
f^*	Original Optimum
ρ	Maximal Optimality Gap

UFO: Multi-Objective Problem

$$
\begin{aligned}
& OPT \;\; [f(x), \mu(x)] \\
& s.t. \qquad a(x) \le b \\
& \qquad\qquad x \in X
\end{aligned}
$$

UFO with Budget Relaxation

$$MAX\ \mu(x)$$

$$s.t. \qquad a(x) \leq b$$

$$f(x) \leq (1+\rho)f^*$$

$$x \in X$$

UFO Properties

I. Complexity not changed if $\mu(x)$ similar to $f(x)$
II. Implicit modeling of uncertainty
III. Differentiate solutions on optimal facet
IV. "Plug" tool for any existing method
V. Can use UF based on explicit uncertainty set
VI. Generalizes existing methods

Stochastic Problem as an **UFO**

Given an Uncertainty Set ***U*** with a probability measure on it

$$\begin{aligned} \min \quad & E_U\{f(\boldsymbol{x})\} \\ s.t. \quad & a(x) \le b \\ & x \in X \end{aligned}$$

Stochastic Problem as an **UFO**

$$MAX\ \mu(x) = -E_U\{f(x)\}$$

$$s.t. \qquad a(x) \leq b$$

$$f(x) \leq (1+\infty)f^*$$

$$x \in X$$

Robust Optimization

(Bertsimas & Sim 2004)

- Solving Linear Problems with noisy data
- Solution is feasible in the worst case
- Worst case parametrized and solution-dependent

BONUS

- Methodology to compute maximal values for the parameters to ensure a robust solution exists
- Similar to `Fischetti & Monaci, 2008` in this context

Multi-**D**imensional **K**napsack **P**roblem

$$max \quad \sum_{i=1}^{N} p_i x_i$$

$$s.t. \quad \sum_{j=1}^{N} a_{ij} x_i \leq b_i \quad \forall i = 1, \ldots, M$$

$$\boldsymbol{x} \in \mathbb{Z}_+^N$$

MDKP with Max Taken Object **UFO**

$$\min \left\{ \mu(\boldsymbol{x}) = \max_{i=1,\ldots,N} \{x_i\} \right\}$$

$$s.t. \quad A\boldsymbol{x} \leq \boldsymbol{b}$$

$$\boldsymbol{p}^T \boldsymbol{x} \geq (1 - \rho) p^*$$

$$\boldsymbol{x} \in \mathbb{Z}_+^N$$

Other derived **UF**

- Max Taken (MTk): $\mu(\boldsymbol{x}) = \max_{i=1,\ldots,N} \{x_i\}$
- Diversification (Div): $\mu(\boldsymbol{x}) = \sum(\min \{1, x_i\})$
- Impact Ratio (IR): $\mu(\boldsymbol{x}) = -\max_{i} \frac{a_{ij}x_j}{b_i}$
- 2Sum: $\mu(\boldsymbol{x}) = -\max_{i,j\neq k} \frac{a_{ij}x_j + a_{ik}x_k}{b_i}$

Instances with 50 objects

- 1, 5 or 10 constraints
- Profit-weight correlation or not
- Marginal Profit Distribution: clustered, normal, wide
- Deviation Matrix $\hat{A}$ proportional to A (0.2, 0.5, 0.8)
- Maximal varying coefficients: 2 or 50

IN TOTAL: 3240 Instances

Described by $\boldsymbol{p}$, $\boldsymbol{b}$, A and $\hat{A}$

Simulation

A scenario is characterized by it's realized constraint matrix $\tilde{A}$:

- $\hat{A}$: $\tilde{A} \sim \rho\hat{A}$ matrix ($\rho = 0.75, 1.0$)
- A : $\tilde{A} \sim \rho A$ matrix ($\rho = 0.1, 0.2, 0.5$)
- R : $\tilde{A}$ randomly with average coefficient $\tilde{a}_{ij} = 10, 20, 30$

5 scenarios per instance => 129'600 scenarios

Comparison Criteria

- normalized UF value (max is always 1.0)
- # unfeasible scenarios (and percentage)
- Optimality gap to scenario's optimal solution
- Maximal number of violated constraints

MDKP Package

- Generation of problems
- Solve Models inc. Robust (combining possible)
- Simulation with user-defined parameters

Planned to be online soon.

TESTERS ARE WELKOME!!!

Different Simulations for clustered profit-correlated instances with 10 constraints

		Det	Rob_Â	Rob_A_0.1	MTk_0.2	Div_0.1	IR_0.3	2Sum_0.1
Â 75, 100 (1800 Scen.)	UF value	-	-	-	0.974	0.610	0.962	0.932
	# Infeas.	1642	166	1014	914	1199	85	1174
	Infeas [%]	91.22	9.22	56.33	50.78	66.61	4.72	65.22
	Avg Opt Gap [%]	0.56	20.93	4.72	10.53	5.29	31.04	5.56
	Max Opt Gap [%]	25.21	68.36	38.91	49.81	50.47	59.53	41.99
	Max # Violated	9	3	7	4	5	1	5
A 10, 25, 50 (2700 Scen.)	UF value	-	-	-	0.974	0.610	0.962	0.932
	# Infeas.	2404	489	1232	1141	1544	76	1566
	Infeas [%]	89.04	18.11	45.63	42.26	57.19	2.81	58.00
	Avg Opt Gap [%]	0.56	18.13	5.09	11.38	6.03	30.04	6.03
	Max Opt Gap [%]	34.03	57.07	40.47	47.12	40.39	52.16	40.19
	Max # Violated	8	6	7	3	5	2	5
R 10, 20, 30 (2700 Scen.)	UF value	-	-	-	0.974	0.610	0.962	0.932
	# Infeas.	2616	1079	2100	1506	1974	171	1962
	Infeas [%]	96.89	39.96	77.78	55.78	73.11	6.33	72.67
	Avg Opt Gap [%]	0.45	17.32	3.36	11.24	5.36	33.33	5.53
	Max Opt Gap [%]	46.67	62.71	51.87	57.25	51.81	61.33	51.65
	Max # Violated	8	7	8	4	6	2	6

Performance evolution for increasing budget ρ (same instances)

		2Sum_0.1	2Sum_0.2	2Sum_0.3	IR_0.1	IR_0.2	IR_0.3
Â 75, 100 (1800 Scen.)	UF value	0.932	0.958	0.962	0.932	0.958	0.962
	# Infeas.	1174	528	90	1220	528	85
	Infeas [%]	65.22	29.33	5.00	67.78	29.33	4.72
	Avg Opt Gap [%]	5.56	16.17	30.60	5.13	16.24	31.04
	Max Opt Gap [%]	41.99	52.36	59.53	42.18	48.58	59.53
	Max # Violated	5	3	1	5	3	1
A 10, 25, 50 (2700 Scen.)	UF value	0.932	0.958	0.962	0.932	0.958	0.962
	# Infeas.	1566	579	84	1671	592	76
	Infeas [%]	58.00	21.44	3.11	61.89	21.93	2.81
	Avg Opt Gap [%]	6.03	16.85	29.57	5.4	16.83	30.04
	Max Opt Gap [%]	40.19	46.95	46.75	40.39	46.99	52.16
	Max # Violated	5	3	2	5	3	2
R 10, 20, 30 (2700 Scen.)	UF value	0.932	0.958	0.962	0.932	0.958	0.962
	# Infeas.	1962	997	174	2001	996	171
	Infeas [%]	72.67	36.93	6.44	74.11	36.89	6.33
	Avg Opt Gap [%]	5.53	16.73	32.93	5.33	16.81	33.33
	Max Opt Gap [%]	51.65	57.11	60.71	51.81	57.15	61.33
	Max # Violated	6	3	2	5	4	2

Performance for combined (normalized) objectives

		MTK_2Sum_0.3	_Rob_A_0.1 Div_1.0	Rob_A_0.1	Rob_A_0.2
Â 75, 100 (1800 Scen.)	UF value	0.969	1.475	-	-
	# Infeas.	102	514	1014	643
	Infeas [%]	5.67	28.56	56.33	35.72
	Avg Opt Gap [%]	33.02	16.47	4.72	10.93
	Max Opt Gap [%]	80.03	53.41	38.91	44.15
	Max # Violated	2	5	7	5
A 10, 25, 50 (2700 Scen.)	UF value	0.969	1.475	-	-
	# Infeas.	111	542	1232	700
	Infeas [%]	4.11	20.07	45.63	25.93
	Avg Opt Gap [%]	31.90	16.93	5.09	11.59
	Max Opt Gap [%]	76.44	51.97	40.47	45.34
	Max # Violated	2	5	7	5
R 10, 20, 30 (2700 Scen.)	UF value	0.969	1.475	-	-
	# Infeas.	209	969	2100	1503
	Infeas [%]	7.74	35.89	77.78	55.67
	Avg Opt Gap [%]	34.94	17.16	3.36	9.18
	Max Opt Gap [%]	80.20	61.17	51.87	55.81
	Max # Violated	3	5	8	6

Aggregated Results

Number of constraints matters

Feasibility failure for the deterministic model

1 constraint	37%
5 constraints	84%
10 constraints	91%

Aggregated Results

Clustered M.P. Distribution works best for UFs

Feasibility failure for the IR_0.3 model

Clustered degeneration	29%
Normal degeneration	55%
Wide degeneration	63%

Robust less sensitive to degeneration & correlation

Aggregated Results

- UFO less sensitive to change in noise & number constraints
- Robust sensitive to noise change
- Budget is a decent optimality loss estimator

Future Work

- Application of UFO to Airline Transportation
- Find an UF generator ?

Conclusions

- UFO allows to cope with uncertainty IMPLICITLY
- Using explicit uncertainty model is still possible
- UFO can be combined with any already existing method
- It is not sensitive to erroneous noise characterization

THANKS for your attention

Any Questions?

Robust problem as an **UFO**

Original LP Problem

$$MAX\ \boldsymbol{c}^T\boldsymbol{x}$$

$$s.t.\quad A\boldsymbol{x} \leq \boldsymbol{b}$$

$$\boldsymbol{x} \in X$$

Robust problem as an **UFO**

Formulation of Bertsimas and Sim (2004)

$$MAX\ \mathbf{c}^T\mathbf{x}$$

$$s.t. \quad A\mathbf{x} + \boldsymbol{\beta}(\mathbf{x}, \boldsymbol{\Gamma}) \leq \boldsymbol{b}$$

$$\mathbf{x} \in X$$

$$\tilde{a}_{ij} \in [a_{ij} - \hat{a}_{ij}; a_{ij} + \hat{a}_{ij}] \quad \forall\ j \in J_i$$

$$\beta_i(x, \Gamma_i) = \max_{\{S_i \cup \{t_i\} | S_i \in J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}$$

$$\left\{ \sum_{j \in S_i} \hat{a}_{ij} \mid x_j \mid + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} \mid x_{t_i} \mid \right\}$$

Start with Feasibilty Problem

$$f^* = MIN \quad f(\boldsymbol{x})$$

$$= MIN \; [A\boldsymbol{x} \; + \; \boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J})] \; -\boldsymbol{b}$$

$$s.t. \qquad \boldsymbol{x} \in X$$

Define **UF** and budget

$$\mu(\boldsymbol{x}) = \boldsymbol{c}^T \boldsymbol{x} \qquad \rho = \max_i \left\{ \frac{\rho_i f_i(\boldsymbol{x}^*)}{f^*} - 1 \right\}$$

Where

$$\rho_i = \begin{cases} \dfrac{\bar{\beta}_i(\boldsymbol{x}, \Gamma_i)}{f_i(x^*)} \\ 0 \quad \text{if} \quad f_i(\boldsymbol{x}^*) = 0 \end{cases}$$

and

$$\boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J}) = \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{\Gamma}) + \bar{\boldsymbol{\beta}}(\boldsymbol{x}, \boldsymbol{\Gamma})$$

UFO formulation

$$MAX\ \mu(x) = \boldsymbol{c}^T \boldsymbol{x}$$

$$s.t. \quad [A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J})] - \boldsymbol{b} \le (1+\rho) f^*$$

$$\boldsymbol{x} \in X$$

Replace Elements in Constraint

$$[A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J})] - \boldsymbol{b} \leq (1+\rho) f^{*}$$

=

$$[A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{J})] - \boldsymbol{b} \leq \overline{\boldsymbol{\beta}}(\boldsymbol{x}, \Gamma)$$

Which is equivalent to

$$A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{J}) - \overline{\boldsymbol{\beta}}(\boldsymbol{x}, \Gamma) \leq \boldsymbol{b}$$

=

$$A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \Gamma) \leq \boldsymbol{b}$$

Retrieve Robust Formulation

$$MAX\ \mu(x) = \boldsymbol{c}^T \boldsymbol{x}$$

$$s.t. \quad A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{\Gamma}) \leq \boldsymbol{b}$$

$$\boldsymbol{x} \in X$$

Q.E.D.