Modeling dominated choice alternatives using the constrained multinomial logit

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Outline

- Motivation
- Concept of cutoffs (Constrained logit model)
- Concept of dominance
- Using dominance in the Constrained Logit Model
- Preliminary results
- Perspectives

Motivation

- Discrete choice models.
- Concept of utility based on trade-offs.
- Attributes threshold generally not accounted for.
- Dominated alternatives may not even be considered in the choice set.
- How do we model that?

Motivation

- Manski (1977): individual-based choice-set based on deterministic constraints
- Swait and Ben-Akiva (1987): random constraints
- Swait (2001), Martinez et al. (2008): Attribute cutoffs
- Cascetta and Papola (2005), Cascetta et al. (2007): implicit perception, dominance values

Idea: combine cutoffs and dominance

Cutoffs

Optimization problem of rational consumer n:

$$\max_{\delta_{ni}} \sum_{i \in \mathcal{C}} \delta_{ni} U_{in}(X_i)$$

subject to

$$\sum_{i \in \mathcal{C}} \delta_{ni} = 1, \quad \delta_{ni} \in \{0, 1\}, \forall i \in \mathcal{C}$$

But attributes are meaningful only within some bounds

$$\ell_{nk} \leq X_{ik} \leq u_{nk} \; \forall i \in \mathcal{C}, \forall k$$

Cutoffs

Idea: relax the constraint in a probabilistic way
Example: constraint $\ell \leq X$

$$
\begin{array}{rcccc}
V_{\text{not considered}} & = & \ell & + & \varepsilon_1 \\
V_{\text{considered}} & = & X & + & \varepsilon_2
\end{array}
$$

$$
P(\text{considered}) = \frac{e^{\rho X}}{e^{\rho X} + e^{\rho \ell}} = \frac{1}{1 + e^{\rho(\ell - X)}}
$$

Example: constraint $X \leq u$

$$
P(\text{considered}) = \frac{e^{-\rho X}}{e^{-\rho X} + e^{-\rho u}} = \frac{1}{1 + e^{\rho(X - u)}}
$$

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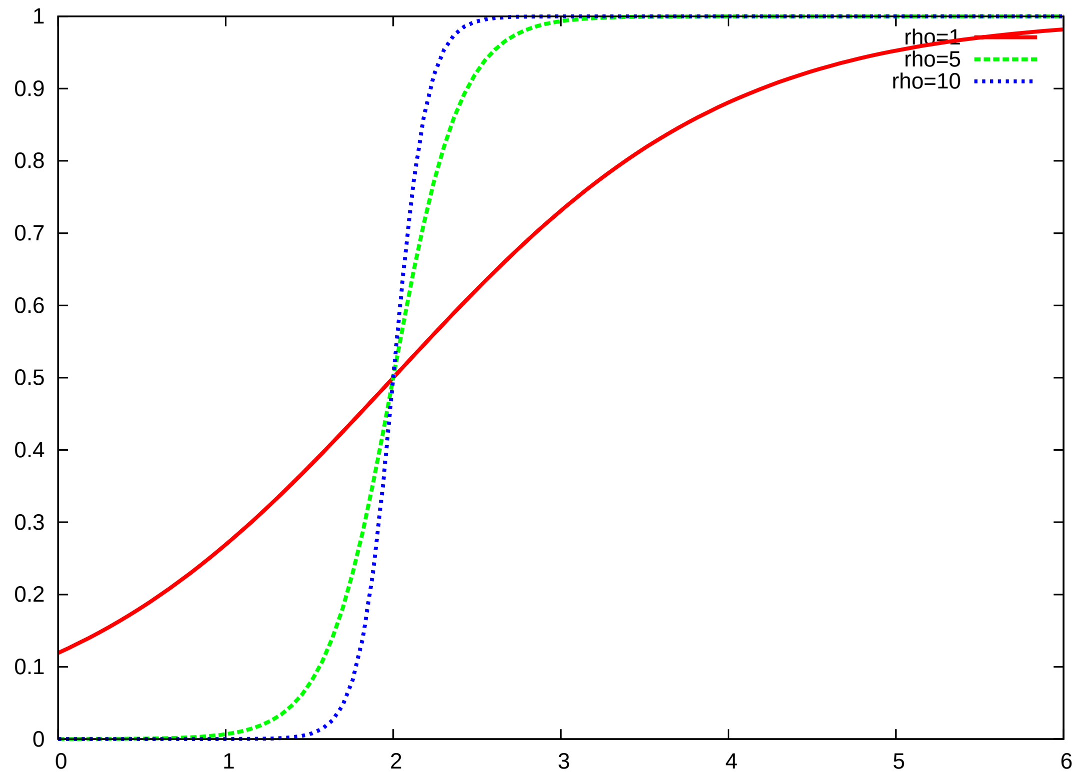
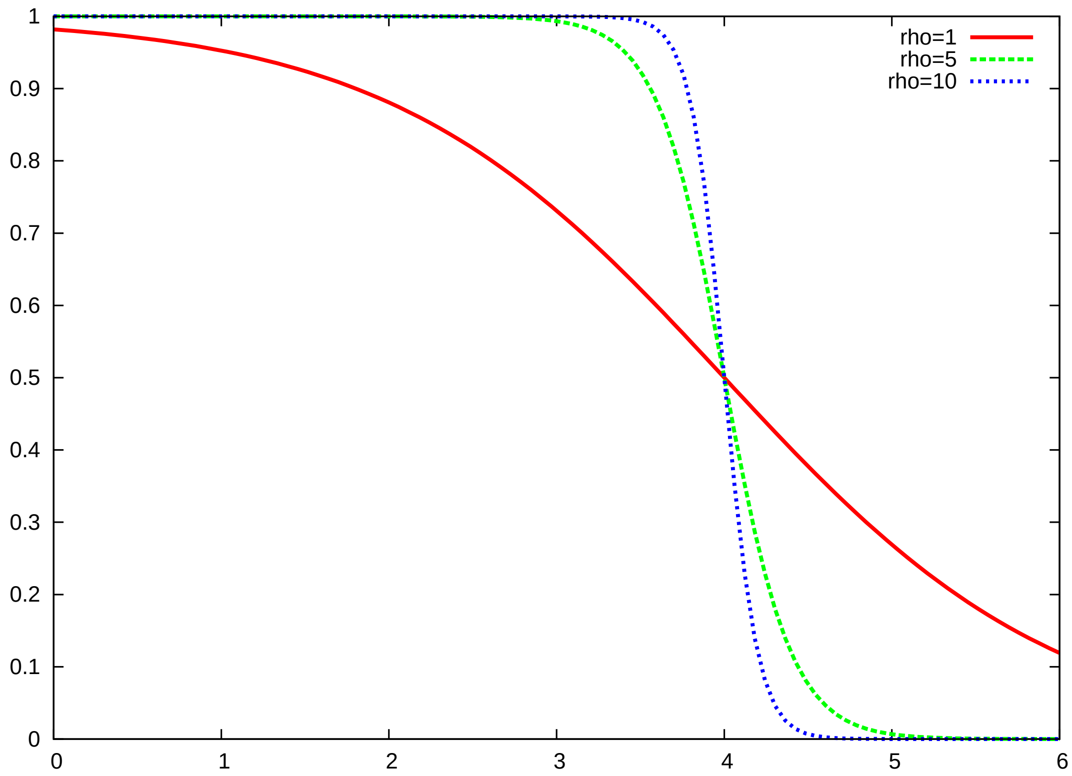
Cutoffs

Example: $2 \leq X$



Cutoffs

Example: $X \leq 4$



Cutoffs

Constraint $\ell \leq X \leq u$

$$P(\text{considered}) = \frac{1}{1+e^{\rho(\ell - X)}} \frac{1}{1+e^{\rho(X-u)}}$$

We denote this quantity by $\phi_n(X)$

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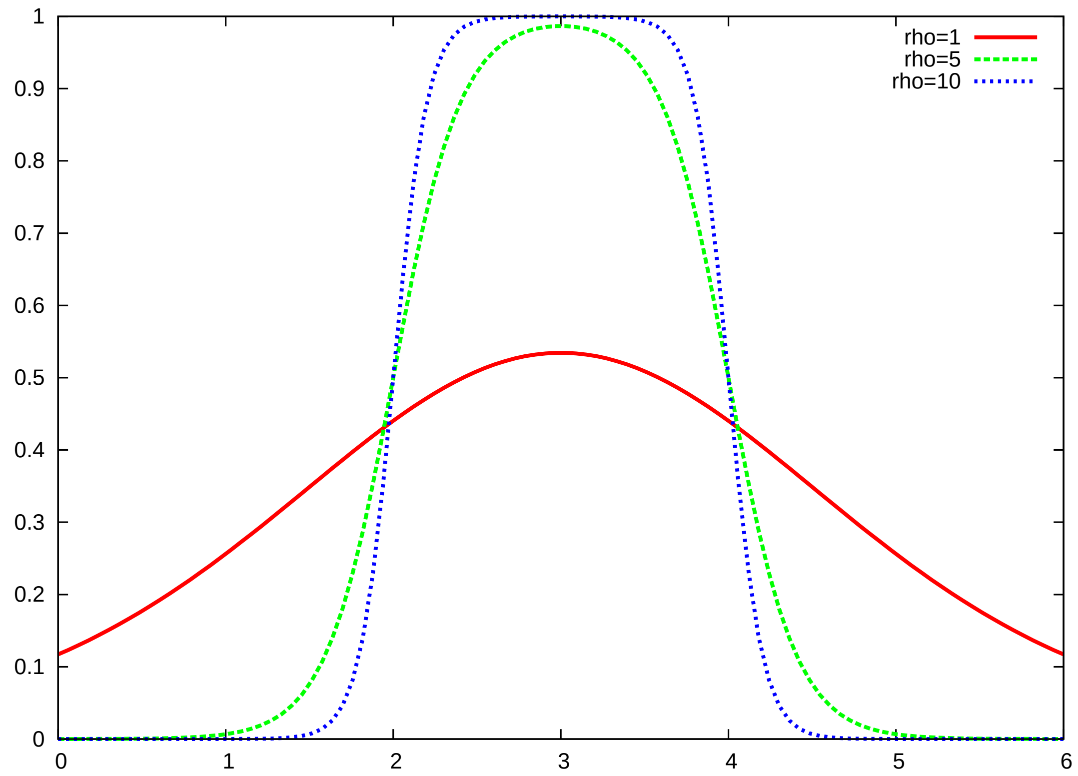
Cutoffs

Example: $2 \leq X \leq 4$



Cutoffs

The utility function now becomes

$$V_i = \sum_k \beta_k X_{ik} + \sum_{k^*} \frac{1}{\rho} \ln \phi_n(X_{ik^*})$$

where k^* ranges only on constrained attributes. Note that

$$\begin{aligned} \ln \phi(X) &= -\ln(1 + e^{\rho(\ell - X)}) - \ln(1 + e^{\rho(X-u)}) \\ &= -\ln(1 + e^{\rho\ell} e^{-\rho X}) - \ln(1 + e^{\rho X} e^{-\rho u}) \end{aligned}$$

Can be estimated, although it is difficult

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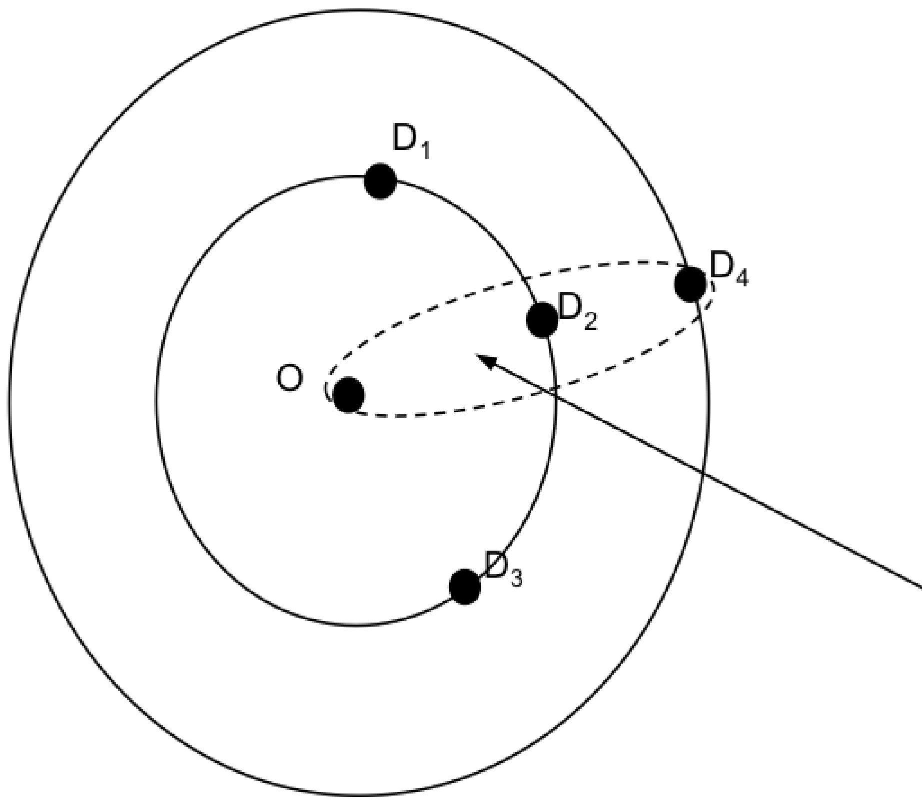
Dominance

- Destination choice (origin o)
- Dominance variables: reflect the spatial position and hierarchies of alternatives
- Dominance rules:
  - Weak dominance: Alternative d dominates alternative d^* if
    1. $A_d > A_{d^*}$ (attractivity attribute)
    2. $c_{od} < c_{od^*}$ (generalized transportation cost)
  - Strong dominance: d strongly dominates d^* if it weakly dominates it and is along the path to reach d^* from o

Dominance



$WP_1 = WP_2 = WP_3 = WP_4$

$c_{OD1} = c_{OD2} = c_{OD3} < c_{OD4}$

D_1, D_2, D_3 dominate **WEAKLY** D_4

D_2 **STRONGLY** dominates D_4

area of possible zones **STRONGLY** dominating D_4

Dominance

Examples of dominance variables for destination d.
Consider 3 conditions:

(a) d^* has average price lower than d

(b) dist$(o, d^*) <$ dist(o, d)

(c) Strong rule: dist(o, d^*) + dist$(d^*, d) <$ dist(o, d)

Strong global dominance variable nbr of d^* verifying (a), (b) and (c).

Weak global dominance variable nbr of d^* verifying (a) and (b)

Weak spatial dominance variable nbr of d^* verifying (b)

Strong spatial dominance variable nbr of d^* verifying (b) and (c).

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Dominance

Dominance variables are introduced directly in the utility function of an MNL model (Cascetta and Papola, 2005):

$$U_d = \sum_k \beta_k X_{dk} + \sum_j \gamma_j Y_{dj}$$

Dominance within CML

Idea: alternatives with a high dominance variable are not considered
Constraint:

$$Y_{dj} \leq u$$

Problem: what is a reasonable threshold u?
Let's use the cutoffs:

$$\ln \phi(Y_{dj}) = -\ln(1 + e^{\rho Y_{dj}} e^{-\rho u}) = -\ln(1 + \bar{u} e^{\rho Y_{dj}})$$

We try to estimate $\bar{u}$

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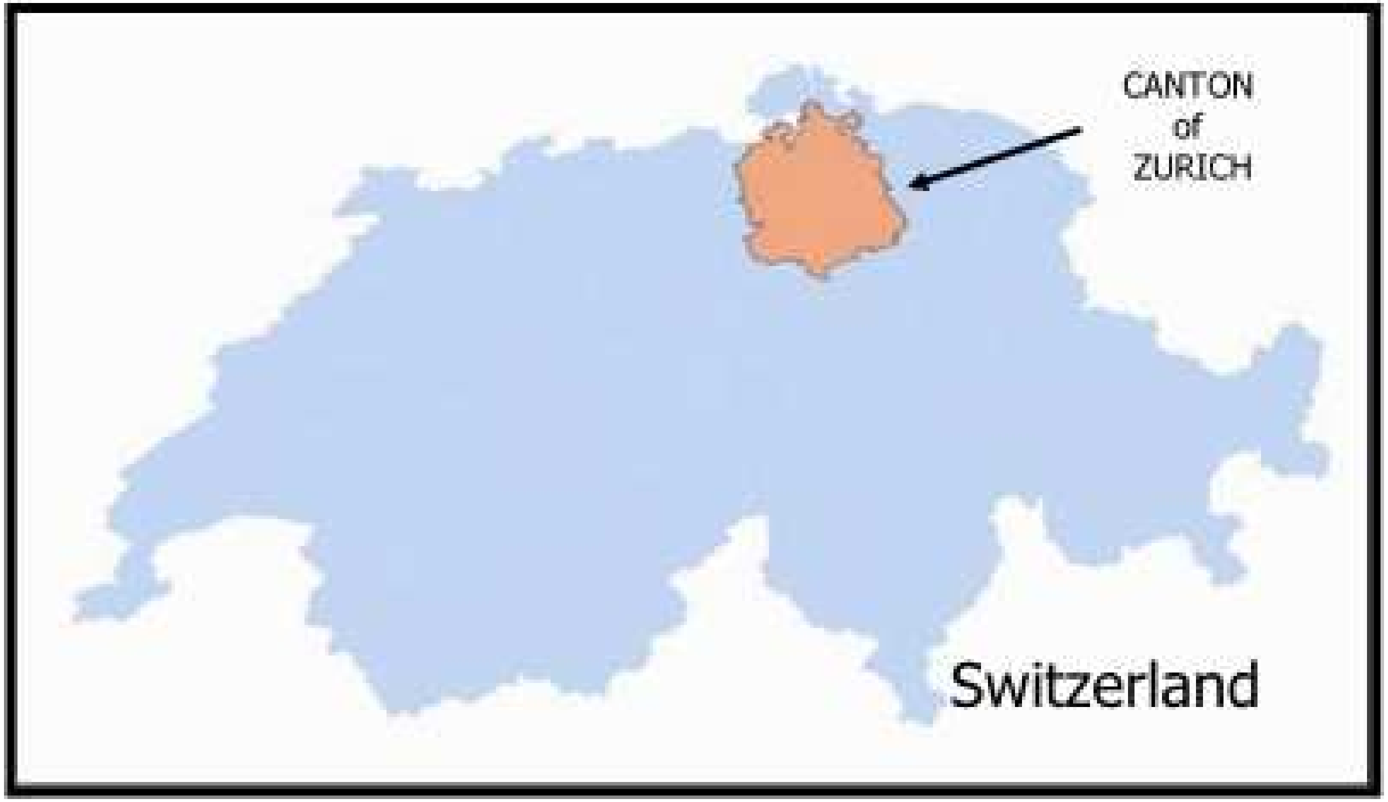
Case study: canton Zürich



Residential location choice

Model specification:

Price_d	average land price of zone d
LnStock_d	log of the housing stock in zone d
$\text{Logsum}_{od}^{\text{LM}}$	logsum of the mode choice model for work purpose (low-medium income)
$\text{Logsum}_{od}^{\text{H}}$	logsum of the mode choice model for work purpose (high income)
$\text{LnWorkPlacesServ}_d$	log of the workplaces in services (retail, leisure, services, incl. education and health) in d. Measure of quality of services.

MNL

Number of observations = 657

$$
\begin{aligned}
\mathcal{L}(0) &= -3419.032 \\
\mathcal{L}(\hat{\beta}) &= -53.971 \\
-2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6730.123 \\
\rho^2 &= 0.984 \\
\bar{\rho}^2 &= 0.983
\end{aligned}
$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	$\text{Logsum}^{\text{H}}_{od}$	15.3	2.85	5.36	0.00
2	$\text{Logsum}^{\text{LM}}_{od}$	16.6	2.97	5.58	0.00
3	Price_d	-0.00160	0.000221	-7.24	0.00
4	LnStock_d	1.12	0.102	10.92	0.00
5	$\text{LnWorkPlacesServ}_d$	0.187	0.180	1.04	0.30

MNL

- Very high ρ^2: 0.98
- Correct signs
- Significant parameters, except the level of services

Next model:

- Include the strong spatial dominance variable (based only on distance, not on price)
- Simple linear specification

$$V_d = \cdots + \beta\,\text{dom}_d$$

Linear dominance

Number of observations = 657

$$
\begin{aligned}
\mathcal{L}(0) &= -3419.032 \\
\mathcal{L}(\hat{\beta}) &= -47.055 \\
-2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6743.955 \\
\rho^2 &= 0.986 \\
\bar{\rho}^2 &= 0.984
\end{aligned}
$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	dom$_d$	-0.0859	0.0120	-7.17	0.00
2	Logsum$^{\mathrm{H}}_{od}$	16.1	2.62	6.16	0.00
3	Logsum$^{\mathrm{LM}}_{od}$	17.1	2.76	6.20	0.00
4	Price$_d$	-0.00245	0.000313	-7.82	0.00
5	LnStock$_d$	1.20	0.133	9.01	0.00
6	LnWorkPlacesServ$_d$	-0.172	0.198	-0.87	0.39

Linear dominance

- Significantly better fit: -2(-53.971 - 47.055) = 202.052
- Correct signs
- Significant parameters, except the level of services

Next model: cutoff

$$\begin{aligned} V_d &= \cdots - \ln(1 + \bar{u}\exp(\rho\,\mathsf{dom}_d)) \\ &= \cdots - \ln(1 + 1000\exp(\rho\,\mathsf{dom}_d)) \end{aligned}$$

Notes:

- the estimation of $\bar{u}$ failed; its value continuously increased
- in the final model, the value $\bar{u} = 1000$ was used.

Cutoff

Number of observations = 657

$$\begin{aligned}\mathcal{L}(0) &= -3419.032\\ \mathcal{L}(\hat\beta) &= -47.057\\ -2[\mathcal{L}(0)-\mathcal{L}(\hat\beta)] &= 6743.952\\ \rho^2 &= 0.986\\ \bar\rho^2 &= 0.984\end{aligned}$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	$\text{Logsum}^{\text{H}}_{od}$	16.1	2.62	6.16	0.00
2	$\text{Logsum}^{\text{LM}}_{od}$	17.1	2.76	6.20	0.00
3	Price_d	-0.00245	0.000313	-7.82	0.00
4	LnStock_d	1.20	0.133	9.01	0.00
5	$\text{LnWorkPlacesServ}_d$	-0.172	0.198	-0.87	0.39
6	ρ	0.0859	0.0120	7.17	0.00

Cutoff

- Same improvement than the linear specification
- Actually, the model is almost linear, due to the high value of $\bar{u}$
- Question: can we accept a linear specification?
- We test it using a Box-Cox transform.

$$V_d = \cdots + \beta \, \frac{\text{dom}_d^{\lambda} - 1}{\lambda}$$

Box-Cox test

Number of observations = 657

$$\begin{aligned}
\mathcal{L}(0) &= -3419.032 \\
\mathcal{L}(\hat{\beta}) &= -43.120 \\
-2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 6751.826 \\
\rho^2 &= 0.987 \\
\bar{\rho}^2 &= 0.985
\end{aligned}$$

Variable number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	dom_d	-0.579	0.0539	-10.74	0.00
2	$\text{Logsum}_{od}^{\text{H}}$	16.9	2.66	6.36	0.00
3	$\text{Logsum}_{od}^{\text{LM}}$	18.0	2.68	6.72	0.00
4	Price_d	-0.00292	0.000324	-9.00	0.00
5	LnStock_d	1.42	0.175	8.10	0.00
6	$\text{LnWorkPlacesServ}_d$	-0.328	0.257	-1.28	0.20
7	λ	0.434	0.0388	11.19	0.00

Box-Cox test

- λ is significantly different from 1.0 (t-test = 14.6)
- λ is significantly different from 0.0 (t-test = 11.2)
- The linear specification is rejected

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Conclusions

- Main idea: combination of two concepts: cutoffs and dominance
- First estimation results produces large values for the variance of the cutoff, so that it is basically equivalent to the linear model
- But... the linear specification is clearly rejected by a formal test.
- Next steps:
  - Consider new dominance rules, more consistent with the use of cutoffs
  - Investigate other data sets