Uncertainty Feature Optimization for the Airline Scheduling Problem

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- Head: Prof. Michel Bierlaire
- [http://transp-or.epfl.ch](http://transp-or.epfl.ch)
- 17 members
- 8 PhD Students
- 3 Post-Docs
Research Activities: [http://transp-or2.epfl.ch/projets.php](http://transp-or2.epfl.ch/projets.php)

- **Transportation Research**
  - A Prototype Transportation Land-use Model for the Region of Lausanne, Switzerland

- **Operations Research**
  - Optimization of container terminal operations
  - Simulation-based optimization of the performance in hospital operating suites

- **Discrete Choice Models**
  - Behavioral modeling of human experts for scene analysis

- **Miscellaneous**
  - Invariant features in omnidirectional images
Outline

- Uncertainty Feature Optimization (UFO)
- Application to Airline Scheduling
- The ROADEF Challenge 2009
- Computational Results
- Future Research
Optimization with Noisy Data

- Real world problems are due to noisy data
- Noise should not be neglected
- Methods using explicit uncertainty sets:
  - Uncertainty sets are hard to model
  - Methods are computationally hard
  - Solutions are sensitive to errors in noise modeling

=> Uncertainty Features capture noise implicitly
Uncertainty Feature Optimization (UFO) Eggenberg, Salani and Bierlaire (2008)

Uncertainty Feature (UF): an implicit noise characterization

- No uncertainty set required
- Problem Complexity similar to original problem*
- Not sensitive to modification in noise’s nature
- Models what practitioners do for uncertain problems

Requires a posteriori validation
UFO Framework

Deterministic Problem

\[ z^* = \min f(x) \]
\[ s.t. \quad a(x) \leq b \]
\[ x \in X \]

UFO Formulation with scalar UF \( \mu : X \to \mathbb{R} \)

\[ \max \mu(x) \]
\[ s.t. \quad a(x) \leq b \]
\[ f(x) \leq (1 + \rho)z^* \]
\[ x \in X \]

BUDGET CONSTRAINT
Remarks

• UFs should increase robustness or recoverability
• Using UFs based on uncertainty sets is possible
  ⇒ Can express Stochastic Optimization and Robustness of Bertsimas and Sim (2004) as UFs
• Can extend any existing model with UFO
• Complexity is similar as long the UF is of same complexity than the deterministic problem
Application to Airline Scheduling

Desired Properties of a Schedule

• Absorb Delays
• Avoid disruption propagation effect
• Easier to recover in case of disruption

Methods used by Practitioners

• Increase idle time
• Increase number of plane crossings
Aircraft Scheduling Problem (ASP)

• A set of flights
• A set of aircrafts (fleets)
• A departure time and plane type for each flight (maximizing some potential revenue metric)
• One feasible route for each aircraft
• All flights are covered
• Aircraft assignment and departures as close as possible to input
ASP Model \textsuperscript{Eggenberg, Salani and Bierlaire (2008b)}

\begin{align*}
\text{min} & \sum_{r \in \Omega} c_r x_r \\
\text{s.t.} & \sum_{r \in \Omega} b_r^f x_r = 1 \quad \forall f \in F \\
& \sum_{r \in \Omega} b_r^s x_r = 1 \quad \forall s \in S \\
& \sum_{r \in \Omega} b_r^p x_r \leq 1 \quad \forall p \in P \\
& x_r \in \{0,1\}
\end{align*}
Column Generation Algorithm

• Use Constraint Specific Networks for each aircraft

• Pricing is a Resource Constrained Elementary Shortest Path Problem (RCESPP) on the networks

See Eggenberg, Salani and Bierlaire (2008b)
ASP: Budget Allocation

Lowest possible deviation of departure times

\[ C_r = \text{total deviation from original schedule of route } r \]

Optimum of deterministic problem = 0

Budget Constraint \[ \Rightarrow f(x) \leq (1+\rho)0 = 0 = z^* \]

SOLUTION: Use a constant \( C \) for total deviation

\[ \sum_{r \in \Omega} c_r x_r \leq C \]
General UFO Formulation

\[
\begin{align*}
\text{max} & \quad \mu(x) \\
\text{s.t.} & \quad \sum_{r \in \Omega} b_r^f x_r = 1 \quad \forall f \in F \\
& \quad \sum_{r \in \Omega} b_r^s x_r = 1 \quad \forall s \in S \\
& \quad \sum_{r \in \Omega} b_r^p x_r \leq 1 \quad \forall p \in P \\
& \quad \sum_{r \in \Omega} c_r x_r \leq C \\
& \quad x_r \in \{0,1\}
\end{align*}
\]
Used Uncertainty Features

Total Idle Time (IT)

\[ \mu_{IT}(x) = \sum_{r \in \Omega} \delta_r x_r \]

Sum of Minimum Idle Times (MIT)

\[ \mu_{MIT}(x) = \sum_{r \in \Omega} \delta_r^{MIN} x_r \]

Number of Plane Crossings (CROSS)

\[ \mu_{CROSS}(x) \]
The ROADEF Challenge 2009

• Solve the disrupted airline recovery problem

• Qualification: 10 instances A01 – A10

• 1012 flights, 85 aircrafts (A05 and A10)

• 608 flights, 85 aircrafts (A01-A04 and A06-A09)

• Provided solution and cost checkers
Tests Performed

• Compare a priori UF values for original schedule Or and schedules obtained by IT, MIT and CROSS

• Adapt disruption to schedule

• Compare a posteriori results of our recovery algorithm
# A priori results (A01-A04, A06-A09)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Or</th>
<th>IT 2500</th>
<th>IT 5000</th>
<th>IT 10000</th>
<th>MIT 2500</th>
<th>MIT 5000</th>
<th>MIT 10000</th>
<th>CROSS 2500</th>
<th>CROSS 5000</th>
<th>CROSS 10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT [k min]</td>
<td>12</td>
<td>14.5</td>
<td>17</td>
<td>19.2</td>
<td>13.5</td>
<td>14.1</td>
<td>16.8</td>
<td>11.5</td>
<td>11.4</td>
<td>11.1</td>
</tr>
<tr>
<td>MIT [min]</td>
<td>790</td>
<td>1025</td>
<td>1110</td>
<td>1255</td>
<td>2280</td>
<td>2225</td>
<td>3330</td>
<td>570</td>
<td>550</td>
<td>515</td>
</tr>
<tr>
<td>CROSS</td>
<td>3430</td>
<td>3462</td>
<td>3501</td>
<td>3489</td>
<td>3448</td>
<td>3426</td>
<td>3418</td>
<td>3510</td>
<td>3508</td>
<td>3522</td>
</tr>
<tr>
<td>Loss of Revenue [%]</td>
<td>0.0</td>
<td>0.19</td>
<td>0.21</td>
<td>1.02</td>
<td>0.40</td>
<td>1.35</td>
<td>1.85</td>
<td>0.91</td>
<td>1.70</td>
<td>1.95</td>
</tr>
</tbody>
</table>

Max Cost: 169,539€ (Avg: 87,426€ i.e. 1.00%)
Max Passengers lost: 1.31% (Avg: 0.6%)
### A posteriori results (A01-A04, A06-A09)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Or</th>
<th>IT 2500</th>
<th>IT 5000</th>
<th>IT 10000</th>
<th>MIT 2500</th>
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<th>MIT 10000</th>
<th>CROSS 2500</th>
<th>CROSS 5000</th>
<th>CROSS 10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost [k€]</td>
<td>788.8</td>
<td>814.9</td>
<td>633.4</td>
<td>555.4</td>
<td>722.8</td>
<td>488.7</td>
<td>493.5</td>
<td>674.6</td>
<td>580.3</td>
<td>574.4</td>
</tr>
<tr>
<td>Savings [%]</td>
<td>0.00</td>
<td>-3.19</td>
<td>19.70</td>
<td>29.59</td>
<td>8.37</td>
<td>38.05</td>
<td>37.44</td>
<td>14.48</td>
<td>26.43</td>
<td>27.18</td>
</tr>
<tr>
<td>Avg. Psg Delay [min]</td>
<td>34.6</td>
<td>35.1</td>
<td>38.7</td>
<td>24.6</td>
<td>30.0</td>
<td>29.5</td>
<td>29.8</td>
<td>27.9</td>
<td>29.5</td>
<td>20.8</td>
</tr>
<tr>
<td># Psg Canceled</td>
<td>582.8</td>
<td>580</td>
<td>499.3</td>
<td>420.0</td>
<td>546.9</td>
<td>384.5</td>
<td>385.3</td>
<td>500.0</td>
<td>422.0</td>
<td>429.4</td>
</tr>
</tbody>
</table>

Maximum Savings: 905,739.3€ (82.7%)
UF vs Recovery Costs

IT vs Recovery Costs

MIT vs Recovery Costs

CROSS vs Recovery Costs
UF vs Average Delay

IT vs Avg Delay

MIT vs Avg Delay

CROSS vs Avg Delay
UF vs Canceled Flights

IT vs Nbr Canceled Flts

MIT vs Nbr Canceled Flts

CROSS vs Nbr Canceled Flts
Bigger Instances (A05 & A10)

Results show same behavior, but there are convergence difficulties.
Conclusions

• UFO leads to *better* (more recoverable) solutions

• MIT 10000: Reduction of recovery costs by **37.4%** in average

• Loss of revenue of **1.00%** in average (87,426€)

• Number of passengers lost less than **0.6%** in average
Future Work

- Improve convergence for bigger instances
- Try different UFs and recovery algorithms
- Model extensions:
  - Missed connections
  - Crew scheduling
- Application of UFO to other problems
THANKS for your attention!
Any Questions?

References

http://transp-or2.epfl.ch/pubsPerPerson.php?Person=EGGENBERG
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