Uncertainty Feature Optimization for the Airline Scheduling Problem

Niklaus Eggenberg
Dr. Matteo Salani

Funded by Swiss National Science Foundation (SNSF)

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Transport and Mobility Laboratory, EPFL, Switzerland

- Head: Prof. Michel Bierlaire
- http://transp-or.epfl.ch
- 17 members
- 8 PhD Students
- 3 Post-Docs

Research Activities: http://transp-or2.epfl.ch/projets.php

- Transportation Research
  - A Prototype Transportation Land-use Model for the Region of Lausanne, Switzerland

- Operations Research
  - Optimization of container terminal operations
  - Simulation-based optimization of the performance in hospital operating suites

- Discrete Choice Models
  - Behavioral modeling of human experts for scene analysis

- Miscellaneous
  - Invariant features in omnidirectional images

Outline

- Uncertainty Feature Optimization (UFO)
- Application to Airline Scheduling
- The ROADEF Challenge 2009
- Computational Results
- Future Research

Optimization with Noisy Data

- Real world problems are due to noisy data
- Noise should not be neglected
- Methods using explicit uncertainty sets:
  - Uncertainty sets are hard to model
  - Methods are computationally hard
  - Solutions are sensitive to errors in noise modeling

=> Uncertainty Features capture noise implicitly

Uncertainty Feature Optimization (UFO) Eggenberg, Salani and Bierlaire (2008)

Uncertainty Feature (UF): an implicit noise characterization

- ✓ No uncertainty set required
- ✓ Problem Complexity similar to original problem*
- ✓ Not sensitive to modification in noise's nature
- ✓ Models what practitioners do for uncertain problems

➡ Requires a posteriori validation

UFO Framework

Deterministic Problem

$$z^* = \min \quad f(x)$$

$$s.t. \quad a(x) \leq b$$

$$x \in X$$

UFO Formulation
with scalar UF $\mu : X \to \mathbb{R}$

$$\max \quad \mu(x)$$

$$s.t. \quad a(x) \leq b$$

$$f(x) \leq (1+\rho) z^*$$

$$x \in X$$

BUDGET CONSTRAINT

Remarks

- UFs should increase robustness or recoverability
- Using UFs based on uncertainty sets is possible

  $\Rightarrow$ Can express Stochastic Optimization and

  Robustness of Bertsimas and Sim (2004) as UFs
- Can extend any existing model with UFO
- Complexity is similar as long the UF is of same complexity than the deterministic problem

Application to Airline Scheduling

Desired Properties of a Schedule

- Absorb Delays
- Avoid disruption propagation effect
- Easier to recover in case of disruption

Methods used by Practitioners

- Increase idle time
- Increase number of plane crossings

Aircraft Scheduling Problem (ASP)

- A set of flights
- A set of aircrafts (fleets)
- A departure time and plane type for each flight (maximizing some potential revenue metric)

→

- One feasible route for each aircraft
- All flights are covered
- Aircraft assignment and departures as close as possible to input

ASP Model Eggenberg, Salani and Bierlaire (2008b)

$$\min \sum_{r \in \Omega} c_r x_r$$

$$s.t. \quad \sum_{r \in \Omega} b_r^f x_r = 1 \quad \forall f \in F$$

$$\sum_{r \in \Omega} b_r^s x_r = 1 \quad \forall s \in S$$

$$\sum_{r \in \Omega} b_r^p x_r \leq 1 \quad \forall p \in P$$

$$x_r \in \{0,1\}$$

Column Generation Algorithm

- Use Constraint Specific Networks for each aircraft

- Pricing is a Resource Constrained Elementary Shortest Path Problem (RCESPP) on the networks

See `Eggenberg, Salani and Bierlaire (2008b)`

ASP: Budget Allocation

Lowest possible deviation of departure times

c_r = total deviation from original schedule of route *r*

Optimum of deterministic problem = 0

Budget Constraint => $f(x) \leq (1+\rho)0 = 0 = z^*$

SOLUTION: Use a constant C for total deviation

$$\sum_{r \in \Omega} c_r x_r \leq C$$

General UFO Formulation

$$
\begin{aligned}
\max \quad & \mu(x) \\
s.t. \quad & \sum_{r \in \Omega} b_r^f x_r = 1 \quad \forall f \in F \\
& \sum_{r \in \Omega} b_r^s x_r = 1 \quad \forall s \in S \\
& \sum_{r \in \Omega} b_r^p x_r \leq 1 \quad \forall p \in P \\
& \sum_{r \in \Omega} c_r x_r \leq C \\
& x_r \in \{0,1\}
\end{aligned}
$$

Used Uncertainty Features

Total Idle Time (IT) $\mu_{IT}(x) = \sum_{r \in \Omega} \delta_r x_r$

Sum of Minimum Idle Times (MIT) $\mu_{MIT}(x) = \sum_{r \in \Omega} \delta_r^{MIN} x_r$

Number of Plane Crossings (CROSS) $\mu_{CROSS}(x)$

The ROADEF Challenge 2009

- Solve the disrupted airline recovery problem
- Qualification: 10 instances A01 – A10
- 1012 flights, 85 aircrafts (A05 and A10)
- 608 flights, 85 aircrafts (A01-A04 and A06-A09)
- Provided solution and cost checkers

Tests Performed

- Compare a priori UF values for original schedule Or and schedules obtained by IT, MIT and CROSS
- Adapt disruption to schedule
- Compare a posteriori results of our recovery algorithm

A priori results (A01-A04, A06-A09)

MODEL	Or	IT 2500	IT 5000	IT 10000	MIT 2500	MIT 5000	MIT 10000	CROSS 2500	CROSS 5000	CROS 10000
IT [k min]	12	14.5	17	19.2	13.5	14.1	16.8	11.5	11.4	11.1
MIT [min]	790	1025	1110	1255	2280	2225	3330	570	550	515
CROSS	3430	3462	3501	3489	3448	3426	3418	3510	3508	3522
Loss of Revenue [%]	0.0	0.19	0.21	1.02	0.40	1.35	1.85	0.91	1.70	1.95

Max Cost: 169,539€ (Avg: 87,426€ i.e. 1.00%)

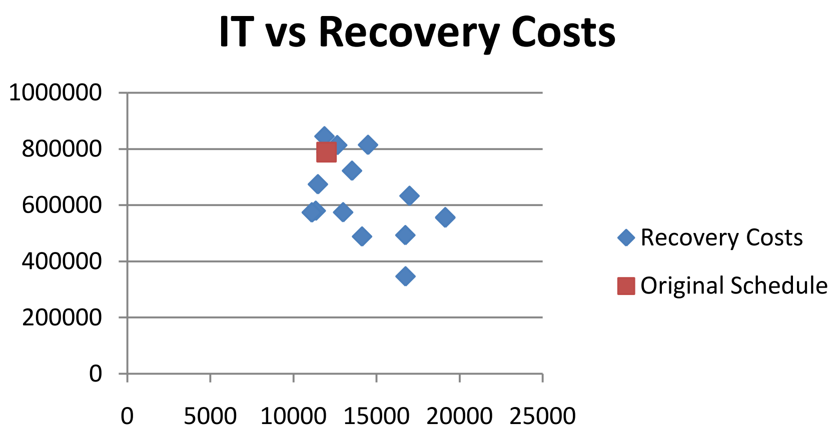
Max Passengers lost: 1.31% (Avg: 0.6%)

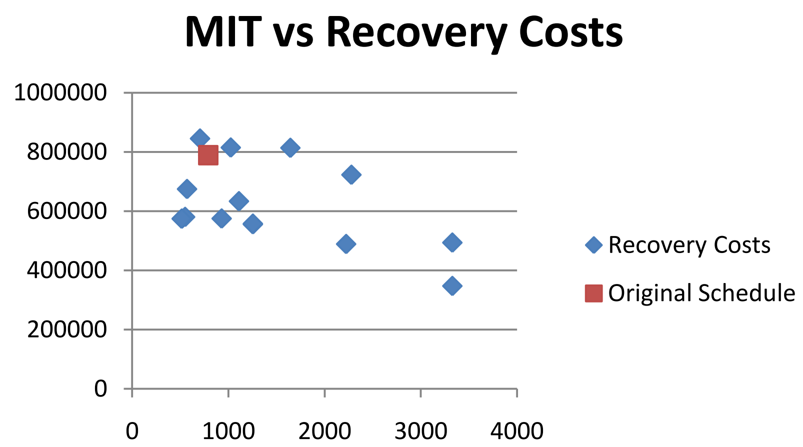
A posteriori results (A01-A04, A06-A09)

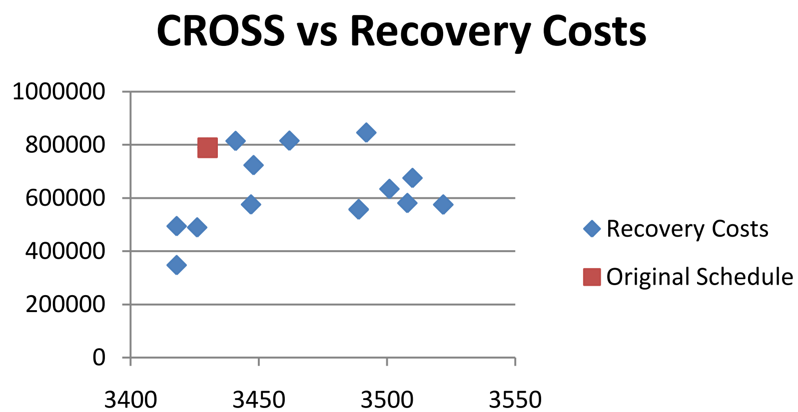
MODEL	Or	IT 2500	IT 5000	IT 10000	MIT 2500	MIT 5000	MIT 10000	CROSS 2500	CROSS 5000	CROSS 10000
Cost [k€]	788.8	814.9	633.4	555.4	722.8	488.7	493.5	674.6	580.3	574.4
Savings [%]	0.00	-3.19	19.70	29.59	8.37	38.05	37.44	14.48	26.43	27.18
Avg. Psg Delay [min]	34.6	35.1	38.7	24.6	30.0	29.5	29.8	27.9	29.5	20.8
# Psg Canceled	582.8	580	499.3	420.0	546.9	384.5	385.3	500.0	422.0	429.4

Maximum Savings: 905,739.3€ (82.7%)

UF vs Recovery Costs





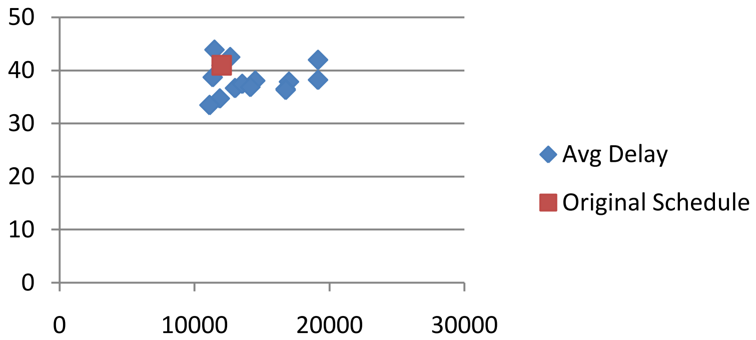


TRANSP-OR

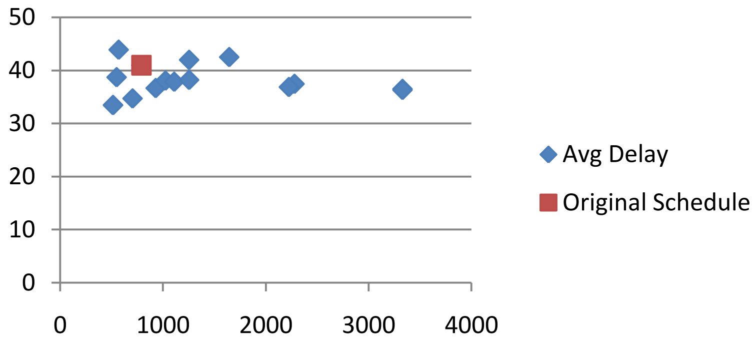
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

UF vs Average Delay

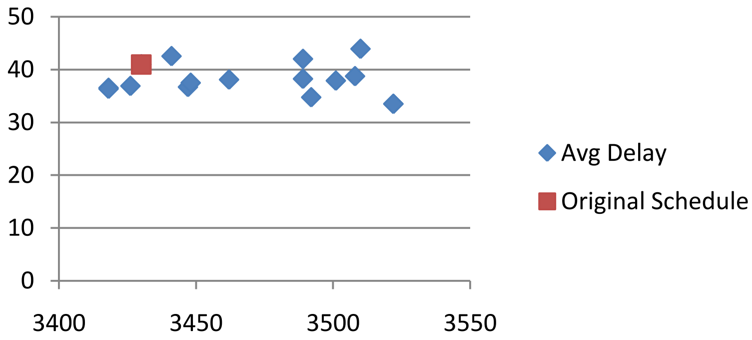
IT vs Avg Delay



MIT vs Avg Delay

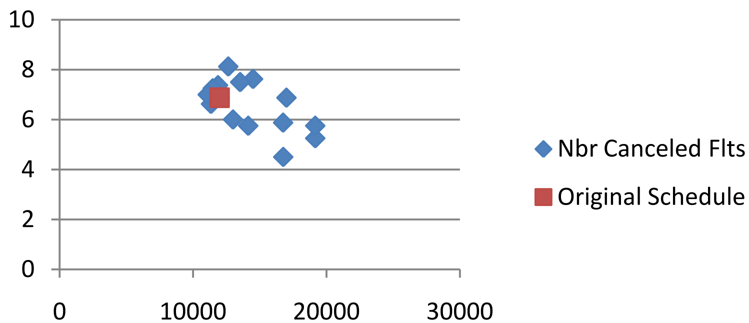


CROSS vs Avg Delay

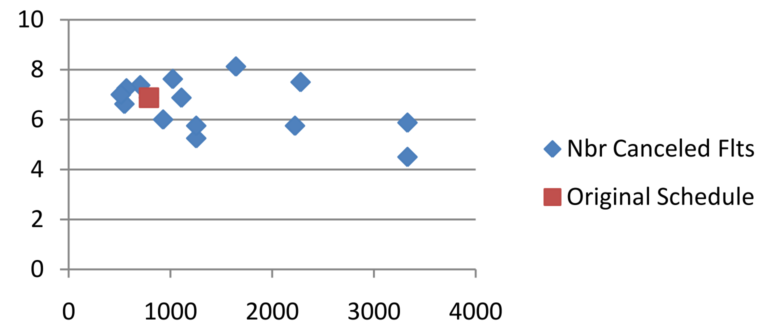


UF vs Canceled Flights

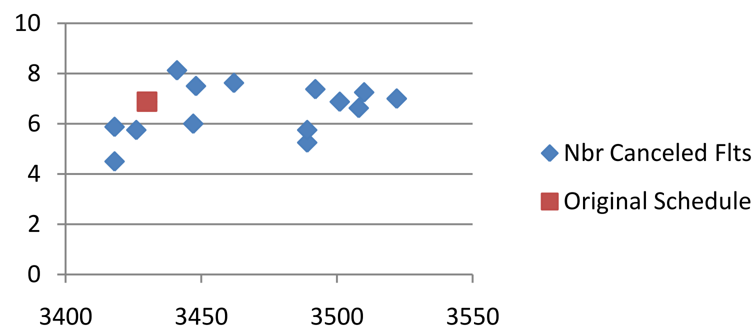
IT vs Nbr Canceled Flts



MIT vs Nbr Canceled Flts



CROSS vs Nbr Canceled Flts



TRANSP-OR

EPFL
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Bigger Instances (A05 & A10)

Results show same behavior, but there are convergence difficulties.

Conclusions

- UFO leads to *better* (more recoverable) solutions
- MIT 10000: Reduction of recovery costs by 37.4% in average
- Loss of revenue of 1.00% in average (87,426€)
- Number of passengers lost less than 0.6% in average

Future Work

- Improve convergence for bigger instances
- Try different UFs and recovery algorithms
- Model extensions:
    - Missed connections
    - Crew scheduling
- Application of UFO to other problems

THANKS for your attention! Any Questions?

References

http://transp-or2.epfl.ch/pubsPerPerson.php?Person=EGGENBERG

or contact me at

neg@mit.edu