An algorithm for the recovery of disrupted airline schedules

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Airline Scheduling Approach

1. Route Choice
2. Fleet Assignment
3. Tail Assignment
4. Crew Pairing
5. Crew Roistering
6. Passenger Routing (catering)
Maintenances

Maintenances are forced by RESOURCE consumption (eg. flown hours)

Resources are renewed during maintenance
Disrupted Schedule and Recovery

- Initial Schedule
- Disrupted
- Back to normal

The Airplane Recovery Problem (ARP)

**Input**
- Planes’ Position
- Initial Schedule
- Maintenance
- Cancellation Costs
- Delay Cost

**Output**
- \( T \) (Recovery Period)
- New schedule up to \( T \)
- Recovery cost
Determine a Final State:
Solution to the ARP:

A recovery scheme for each plane:

Initial State → Flights and Maintenances → Expected Final State
Multi-objective optimization:

Minimize both $T$ and recovery costs

Strategy: for fixed $T$ find optimal recovery plan

Give several recovery plans for different values of $T$ (decision aid)
Solving the ARP (2)

Column Generation Approach

Find out optimal solution by combining individual recovery schemes \( r \in R' \) (master problem) on a subset \( R' \subseteq R \) of all feasible recovery schemes

Generate potentially improving recovery schemes \( r \in R-R' \) dynamically for each plane (pricing problem)
Variables:

• $x_r = 1$ if route $r$ is in the solution, 0 otherwise

• $y_f = 1$ if flight $f$ is cancelled, 0 otherwise

• $z_s = 1$ if final state $s$ is uncovered, 0 otherwise
What is a column?

- cost $c_r$
- vector $b_r = \left( b_r^f, b_r^s, b_r^p \right)^T$

Where

- $b_r^f = 1$ if flight $f$ is covered by column $r$
- $b_r^s = 1$ if final state $s$ is covered by $r$
- $b_r^p = 1$ if column $r$ is affected to plane $p$
**Master Problem: MIP formulation**

\[
\begin{align*}
\text{min } & \quad z_{MP} = \sum_{r \in R} c_r x_r + \sum_{f \in F} c_f y_f + \sum_{s \in S} c_s z_s \\
\text{s. c. } & \quad \sum_{r \in R} b_r^f x_r + y_f = 1 \quad \forall f \in F \quad (\lambda_f) \\
& \quad \sum_{r \in R} b_r^s x_r + z_s = 1 \quad \forall s \in S \quad (\eta_s) \\
& \quad \sum_{r \in R} b_r^p x_r \leq 1 \quad \forall p \in P \quad (\mu_p) \\
& \quad x_r \in \{0,1\} \quad \forall r \in R \\
& \quad y_f \in \{0,1\} \quad \forall f \in F \\
& \quad z_s \in \{0,1\} \quad \forall s \in S
\end{align*}
\]
The Pricing Problem

Find new columns minimizing the reduced cost $\tilde{c}_r^p$:

$$
\min_{r \in R} \tilde{c}_r^p = c_r^p - \sum_{f \in F} b_r^f \lambda_f - \sum_{s \in S} b_r^s \eta_s - b_r^p \mu_p \quad \forall \ p \in P
$$

Solve Pricing $\iff$

Solve Elementary Shortest Path Problem in Recovery Networks
Recovery Networks (Argüello et al. 97)

1. Generate a recovery network for each plane
2. Update arc costs according to dual variables
3. Solve Resource Constrained Elementary Shortest Path (RCESPP)
4. Add Columns to $R'$
5. Resolve restricted LP until optimality and branch
Contributions

Originality in this work

• Considering **Maintenances**

• Apply **Column Generation** technique

• Improved Acceleration Techniques:
  • 2 phase pricing (heuristic and exact)
  • Logarithmical discretization

• Network based pricing

• RCESPP algorithm (Righini & Salani, 2006)
Implementation issues

- Implemented in C++ with COIN-OR BCP framework
- Used interior point methods to solve the LP
- Used linear time and logarithmical resource discretisation
- 2 phase pricing:
  - generation (keep also non optimal columns, heuristic pricing)
  - proving optimality (optimal column only, exact pricing)
Implementation Issues (2)

Linear Time Discretization

Logarithmic Resource Discretization
Real Instances

- Got real schedules from **Thomas Cook Airlines** (APM’s main customer)
- Solved original schedules up to 250 flights (algorithm validation)
- Generated disruption scenarios
  - delayed planes (initial states)
  - grounded planes (initial states)
  - airport closures (activity slots)
  - forced maintenances (initial resource consumption)
### Solved Instances (1): Solvable Problem Sizes (with small disruptions)

<table>
<thead>
<tr>
<th>Instance</th>
<th>2D_5AC</th>
<th>2D_5AC_1del</th>
<th>2D_10AC</th>
<th>2D_10AC_1del</th>
<th>2D_10AC_2del</th>
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<tbody>
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<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10</td>
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<td># flights</td>
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<td>38</td>
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<td>75</td>
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<td>0</td>
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<tr>
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<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
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<tr>
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<td>4</td>
<td>0</td>
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<td>5</td>
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<td>total delay [min]</td>
<td>0</td>
<td>969</td>
<td>0</td>
<td>969</td>
<td>989</td>
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<td>max delay [min]</td>
<td>0</td>
<td>370</td>
<td>0</td>
<td>370</td>
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<td>cost</td>
<td>380(*)</td>
<td>21175(*)</td>
<td>750(*)</td>
<td>21545(*)</td>
<td>21745(*)</td>
</tr>
<tr>
<td>tree size</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>run time [s]</td>
<td>&lt; 0.1</td>
<td>&lt; 0.1</td>
<td>0.7</td>
<td>0.7</td>
<td>1.0</td>
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<table>
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<th>4D_10AC</th>
<th>5D_5AC</th>
<th>5D_10AC</th>
<th>7D_16AC</th>
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<tbody>
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<td>10</td>
<td>5</td>
<td>10</td>
<td>16</td>
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<td>147</td>
<td>93</td>
<td>184</td>
<td>242</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td># cancelled flts</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>11</td>
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<td>1470(*)</td>
<td>930(*)</td>
<td>1840(*)</td>
<td>5600(*)</td>
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<td>6.5</td>
<td>1.0</td>
<td>29.1</td>
<td>3603</td>
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</table>
The Denver Instance: Hub and Spoke

• 10 planes
• 36 flights
• Initial Cost (without any disruption) : 36000
• $n_{del}$ : number of delayed planes
• $n_{grd}$ : number of grounded planes (whole period)
• 3x100 and 1x300 : hub closure for time periods and time lengths
• Storm : four local spoke airports closed for 300 (1) and 500 (2) min
The Denver Instance

• Affected Planes = number of DIRECTLY affected planes
  (without considering propagation)

• Cancellation cost = 12000 to 22000 cost units

• Delay cost = 10 cost units per minute
Solved Instances (2): Behavior against disruptions

Initial Cost = 36000

<table>
<thead>
<tr>
<th>Instance</th>
<th>Den2del</th>
<th>Den2grd</th>
<th>Den4del</th>
<th>Den4grd</th>
<th>Den2del2grd</th>
<th>Den6del</th>
<th>Den6grd</th>
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<tbody>
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<td>2</td>
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<td>4</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0</td>
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<tr>
<td># grounded planes</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>6</td>
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<tr>
<td># affected flights</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>16</td>
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<tr>
<td># cancelled flights</td>
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<td>2</td>
<td>0</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>16</td>
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<tr>
<td># delayed flights</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>13</td>
<td>2</td>
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<tr>
<td>total delay</td>
<td>10</td>
<td>920</td>
<td>230</td>
<td>380</td>
<td>490</td>
<td>640</td>
<td>380</td>
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<td>max delayed flight</td>
<td>10</td>
<td>275</td>
<td>85</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>200</td>
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<tr>
<td>cost</td>
<td>36100(*)</td>
<td>83200(*)</td>
<td>38300(*)</td>
<td>163800(*)</td>
<td>84900(*)</td>
<td>42400(*)</td>
<td>251800(*)</td>
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<td>41</td>
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<td>run time</td>
<td>0.7</td>
<td>0.5</td>
<td>0.6</td>
<td>0.3</td>
<td>0.5</td>
<td>1.6</td>
<td>0.2</td>
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<table>
<thead>
<tr>
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<th>Den3del3grd</th>
<th>Den_3x100</th>
<th>Den_1x300</th>
<th>Den_Storm1</th>
<th>Den_Storm2</th>
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</thead>
<tbody>
<tr>
<td># delayed planes</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># grounded planes</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># affected flights</td>
<td>9</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>6</td>
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<tr>
<td># cancelled flights</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># delayed flights</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>total delay</td>
<td>950</td>
<td>675</td>
<td>2560</td>
<td>350</td>
<td>1550</td>
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<tr>
<td>max delayed flight</td>
<td>200</td>
<td>90</td>
<td>385</td>
<td>140</td>
<td>340</td>
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<tr>
<td>cost</td>
<td>127500(*)</td>
<td>42750(*)</td>
<td>125600(*)</td>
<td>39500(*)</td>
<td>51500(*)</td>
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<tr>
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<td>1</td>
<td>1</td>
<td>35</td>
<td>1</td>
<td>3</td>
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<tr>
<td>run time</td>
<td>0.4</td>
<td>0.3</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Considering Maintenances

**Maintenance Scheduling:**

- 10 planes, 147 flights
- Compare 3 approaches:
  - **Neglect maintenances:** allow resource excess (5, 10 and 20 %)
  - **Dummy maintenance:** perform maintenance when at least 90% of the resource is consumed
  - **Optimize Maintenance** using the proposed algorithm

Average Results for 10 randomly generated instances
Solved Instances (3): Added value of maintenances

<table>
<thead>
<tr>
<th>Instance</th>
<th>No maint. + 5%</th>
<th>No maint. + 10%</th>
<th>No maint. + 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled flts</td>
<td>52.7</td>
<td>46.7</td>
<td>33.2</td>
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<tr>
<td># delayed flts</td>
<td>5</td>
<td>4.7</td>
<td>5.5</td>
</tr>
<tr>
<td># uncovered final states</td>
<td>1.2</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>total delay [min]</td>
<td>851.3</td>
<td>635.7</td>
<td>712.5</td>
</tr>
<tr>
<td>max delay [min]</td>
<td>271.3</td>
<td>251.5</td>
<td>218.2</td>
</tr>
<tr>
<td>cost</td>
<td>289462</td>
<td>272067</td>
<td>144388</td>
</tr>
<tr>
<td>optimality gap [%]</td>
<td>0.61</td>
<td>0.54</td>
<td>1.27</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Instance</th>
<th>Greedy maint.</th>
<th>Maint. Opt</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled flts</td>
<td>2.2</td>
<td>2</td>
</tr>
<tr>
<td># delayed flts</td>
<td>2.7</td>
<td>1.5</td>
</tr>
<tr>
<td># uncovered final states</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>total delay [min]</td>
<td>89.6</td>
<td>52.3</td>
</tr>
<tr>
<td>max delay [min]</td>
<td>37.7</td>
<td>37.1</td>
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<tr>
<td>cost</td>
<td>15881</td>
<td>14683</td>
</tr>
<tr>
<td>optimality gap [%]</td>
<td>0.73</td>
<td>0</td>
</tr>
</tbody>
</table>

Considering maintenances is crucial!!!
Pareto behavior for increasing $T$
Future Work

• Benchmark solutions against practitioners

• Allow repositioning flights and early departures

• Extend Pricing Solver for acceleration

• Include in APM solutions
Conclusions

• Developed a flexible and fast algorithm

• Solutions are very promising

• Maintenance planning is an added value
THANKS for your attention!

Any Questions?
Some References

• Argüello et al. (1997): recovery without maintenance
  up to 27 planes, 162 flights, 30 airports

• Desrosiers et al. (1997): daily scheduling NOT recovery
  up to 91 planes, 383 flights, 33 airports; max delay of 30 minutes

• Clarke (1997): maintenances requirements but no decision on them
  up to 177 planes, 612 flights, 37 airports; only 0 or 30 min delay

• Kohl et al. (2004): Descartes project, good survey of state of the art
  no instance size mentioned for DAR

• Barnhart and Bratu (2006): passenger oriented recovery algorithm
  up to 302 planes, 1032 flights, 74 airports