Three challenges in route choice modeling

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Route choice modeling

Given a transportation network composed of nodes, links, origin and destinations.
For a given transportation mode and origin-destination pair, which is the chosen route?
Applications

- Intelligent transportation systems
- GPS navigation
- Transportation planning
Challenges

- Alternatives are often highly correlated due to overlapping paths
- Data collection
- Large size of the choice set
Dealing with correlation

Existing Approaches

- Few models explicitly capturing correlation have been used on large-scale route choice problems
  - C-Logit (Cascetta et al., 1996)
  - Path Size Logit (Ben-Akiva and Bierlaire, 1999)
  - Link-Nested Logit (Vovsha and Bekhor, 1998)
  - Logit Kernel model adapted to route choice situation (Bekhor et al., 2002)
- Probit model (Daganzo, 1977) permits an arbitrary covariance structure specification but cannot be applied in a large-scale route choice context
Existing Approaches

- Link based path-multilevel logit model (Marzano and Papola, 2005)
  - Illustrated on simple examples and not estimated on real data
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?
Subnetworks

*How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?*

- Which are the behaviorally important decisions?
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?

- Which are the behaviorally important decisions?
- Our hypothesis: choice of specific parts of the network (e.g. main roads, city center)
- Concept: subnetwork
Subnetworks

- Subnetwork approach designed to be behaviorally realistic and convenient for the analyst
- Subnetwork component is a set of links corresponding to a part of the network which can be easily labeled
- Paths sharing a subnetwork component are assumed to be correlated even if they are not physically overlapping
Subnetworks - Example

Path 1
Path 2
Path 3

O
S_a
S_b
D

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Subnetworks - Methodology

- Factor analytic specification of an error component model (based on model presented in Bekhor et al., 2002)

\[ U_n = \beta^T X_n + F_n T \zeta_n + \nu_n \]

- \( F_n (J \times Q) \): factor loadings matrix
- \( (f_n)_{iq} = \sqrt{l_{niq}} \)
- \( T_{(Q \times Q)} = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_Q) \)
- \( \zeta_n (Q \times 1) \): vector of i.i.d. N(0,1) variates
- \( \nu_{(J \times 1)} \): vector of i.i.d. Extreme Value distributed variates
Subnetworks - Example

\[
U_1 = \beta^T X_1 + \sqrt{l_{1a}} \sigma_a \zeta_a + \sqrt{l_{1b}} \sigma_b \zeta_b + \nu_1
\]

\[
U_2 = \beta^T X_2 + \sqrt{l_{2a}} \sigma_a \zeta_a + \nu_2
\]

\[
U_3 = \beta^T X_3 + \sqrt{l_{3b}} \sigma_b \zeta_b + \nu_3
\]

\[
\text{FTT}^T \text{FT} = \\
\begin{bmatrix}
    l_{1a} \sigma_a^2 + l_{1b} \sigma_b^2 & \sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & \sqrt{l_{1b}} \sqrt{l_{3b}} \sigma_b^2 \\
    \sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & l_{2a} \sigma_a^2 & 0 \\
    \sqrt{l_{3b}} \sqrt{l_{1b}} \sigma_b^2 & 0 & l_{3b} \sigma_b^2
\end{bmatrix}
\]
Empirical Results

- The approach has been tested on three datasets: Boston (Ramming, 2001), Switzerland, and Borlänge
- Deterministic choice set generation
  - Link elimination
- GPS data from 24 individuals
  - 2978 observations, 2179 origin-destination pairs
- Borlänge network
  - 3077 nodes and 7459 links
- BIOGEME (biogeme.epfl.ch, Bierlaire, 2003) has been used for all model estimations
Borlänge Road Network
Model Specifications

- Six different models: MNL, PSL, EC₁, EC₁', EC₂ and EC₂'
- EC₁ and EC₁' have a simplified correlation structure
- EC₁' and EC₂' do not include a Path Size attribute
- Deterministic part of the utility

\[ V_i = \beta_{PS} \ln(PS_i) + \beta_{EstimatedTime} \text{EstimatedTime}_i + \beta_{NbSpeedBumps} \text{NbSpeedBumps}_i + \beta_{NbLeft Turns} \text{NbLeft Turns}_i + \beta_{AvgLinkLength} \text{AvgLinkLength}_i \]
Estimation Results

- Parameter estimates for explanatory variables are stable across the different models.

- Path size parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>EC₁</th>
<th>EC₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Size</td>
<td>-0.28</td>
<td>-0.49</td>
<td>-0.53</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.33</td>
<td>-0.53</td>
<td>-0.56</td>
</tr>
<tr>
<td>Rob. T-test 0</td>
<td>-4.05</td>
<td>-5.61</td>
<td>-5.91</td>
</tr>
</tbody>
</table>

- All covariance parameters estimates in the different models are significant except the one associated with R.50 S.
## Estimation Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Nb. $\sigma$</th>
<th>Nb. Estimated Parameters</th>
<th>Final L-L</th>
<th>Adjusted Rho-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNL</td>
<td>-</td>
<td>12</td>
<td>-4186.07</td>
<td>0.152</td>
</tr>
<tr>
<td>PSL</td>
<td>-</td>
<td>13</td>
<td>-4174.72</td>
<td>0.154</td>
</tr>
<tr>
<td>$EC_1$ (with PS)</td>
<td>1</td>
<td>14</td>
<td>-4142.40</td>
<td>0.161</td>
</tr>
<tr>
<td>$EC'_1$</td>
<td>1</td>
<td>13</td>
<td>-4165.59</td>
<td>0.156</td>
</tr>
<tr>
<td>$EC_2$ (with PS)</td>
<td>5</td>
<td>18</td>
<td>-4136.92</td>
<td>0.161</td>
</tr>
<tr>
<td>$EC'_2$</td>
<td>5</td>
<td>17</td>
<td>-4162.74</td>
<td>0.156</td>
</tr>
</tbody>
</table>

1000 pseudo-random draws for Maximum Simulated Likelihood estimation
2978 observations
Null log likelihood: -4951.11
BIOGEME (biogeme.epfl.ch) has been used for all model estimations.
Forecasting Results

- Comparison of the different models in terms of their performance of predicting choice probabilities
- Five subsamples of the dataset
  - Observations corresponding to 80% of the origin destination pairs (randomly chosen) are used for estimating the models
  - The models are applied on the observations corresponding to the other 20% of the origin destination pairs
- Comparison of final log-likelihood values
Forecasting Results

- Same specification of deterministic utility function for all models
- Same interpretation of these models as for those estimated on the complete dataset
- Coefficient and covariance parameter values are stable across models
Forecasting Results

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1: MNL
2: PSL
3: EC₁
4: EC₂
5: EC₂
6: EC₂
Models based on subnetworks are designed for route choice modeling of realistic size.

Correlation on subnetwork is explicitly captured within a factor analytic specification of an Error Component model.

Estimation and prediction results clearly show the superiority of the Error Component models compared to PSL and MNL.

The subnetwork approach is flexible and the model complexity can be controlled by the analyst.
Network-free data


Data collection and processing

- Link-by-link descriptions of chosen routes necessary for route choice modeling but never directly available
- Data processing in order to obtain network compliant paths
  - Map matching of GPS points
  - Reconstruction of reported paths
- Difficult to verify and may introduce bias and errors
Modeling with network-free data

• An observation \( i \) is a sequence of individual pieces of data related to an itinerary. Examples: sequence of GPS points or reported locations

• For each piece of data we define a Domain of Data Relevance (DDR) that is the physical area where it is relevant

• The DDRs bridge the gap between the network-free data and the network model
Example - GPS data
Example - Reported trip

- Home
- Intersection: Main St and Cross St
- City center
- Mall

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Domain of Data Relevance

- For each piece of data $d$ we generate a list of relevant network elements $e$ (links and nodes)

We define an indicator function

$$
\delta(d, e) = \begin{cases} 
1 & \text{if } e \text{ is related to the DDR of } d \\
0 & \text{otherwise}
\end{cases}
$$
Model estimation

- We aim at estimating the parameters $\beta$ of route choice model $P(p|C_n(s); \beta)$
- We have a set $S_i$ of relevant od pairs
- The probability of reproducing observation $i$ of traveler $n$, given $S_i$ is defined as

$$P_n(i|S_i) = \sum_{s \in S_i} P_n(s|S_i) \sum_{p \in C_n(s)} P_n(i|p) P_n(p|C_n(s); \beta)$$
Model estimation

- Measurement equation $P_n(i|p)$

- Reported trips

\[
P_n(i|p) = \begin{cases} 
1 & \text{if } i \text{ corresponds to } p \\
0 & \text{otherwise}
\end{cases}
\]

- GPS data

$P_n(i|p) = 0$ if $i$ does not correspond to $p$

If $i$ corresponds to $p$ then $P_n(i|p)$ is a function of the distance between $i$ and $p$
Model estimation

- Measurement equation $P_n(i|p)$ for GPS data
- Distance between $i$ and a the closest point on a link $\ell$
  
  is $D(d, p) = \min_{\ell \in A_{pd}} \Delta(d, \ell)$
Model estimation

\[ P_n(i|\mathcal{S}_i) = \sum_{s \in \mathcal{S}_i} P_n(s|\mathcal{S}_i) \sum_{p \in \mathcal{C}_n(s)} P_n(i|p)P_n(p|\mathcal{C}_n(s); \beta) \]

\[ P(i|s) = P(i|p_1)P(p_1|\mathcal{C}(s); \beta) + P(i|p_2)P(p_2|\mathcal{C}(s); \beta) \]
Empirical Results

- Simplified Swiss network (39411 links and 14841 nodes)
- RP data collection through telephone interviews
- Long distance car travel
- The chosen routes are described with the origin and destination cities as well as 1 to 3 cities or locations that the route pass by
- 940 observations available after data cleaning and verification
Empirical Results
Empirical Results

• No information available on the exact origin destination pairs

\[ P(s|i) = \frac{1}{|S_i|} \forall s \in S_i \]

• \( P(r|i) \) is modeled with a binary variable

\[ \delta_{ri} = \begin{cases} 
1 & \text{if } r \text{ corresponds to } i \\
0 & \text{otherwise} 
\end{cases} \]
Empirical Results

- Two origin-destination pairs are randomly chosen for each observation.
- 46 routes per choice set are generated with a choice set generation algorithm.
- After choice set generation, 780 observations are available.
  - 160 observations were removed because either all or none of the generated routes crossed the observed zones.
Empirical Results

• Probability of an aggregate observation $i$

$$P(i) = \sum_{s \in S_i} \frac{1}{|S_i|} \sum_{r \in C_s} \delta_{ri} P(r|C_s)$$

• We estimate Path Size Logit (Ben-Akiva and Bierlaire, 1999) and Subnetwork (Frejinger and Bierlaire, 2007) models

• BIOGEME (biogeme.epfl.ch) used for all model estimations
Empirical Results - Subnetwork

- Subnetwork: main motorways in Switzerland
- Correlation among routes is explicitly modeled on the subnetwork
- Combined with a Path Size attribute
- Linear-in-parameters utility specifications
Empirical Results - Subnetwork
<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>In(path size) based on free-flow time</td>
<td>1.04 (0.134) 7.81</td>
<td>1.10 (0.141) 7.78</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Freeway free-flow time 0-30 min</td>
<td>-7.12 (0.877) -8.12</td>
<td>-7.45 (0.984) -7.57</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-7.12</td>
<td>-7.04</td>
</tr>
<tr>
<td>Freeway free-flow time 30min - 1 hour</td>
<td>-1.69 (0.875) -1.93</td>
<td>-2.26 (1.03) -2.19</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-1.69</td>
<td>-2.14</td>
</tr>
<tr>
<td>Freeway free-flow time 1 hour +</td>
<td>-4.98 (0.772) -6.45</td>
<td>-5.64 (1.00) -5.61</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-4.98</td>
<td>-5.33</td>
</tr>
<tr>
<td>CN free-flow time 0-30 min</td>
<td>-6.03 (0.882) -6.84</td>
<td>-6.25 (0.975) -6.41</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-6.03</td>
<td>-5.91</td>
</tr>
<tr>
<td>CN free-flow time 30 min +</td>
<td>-1.87 (0.331) -5.64</td>
<td>-2.16 (0.384) -5.63</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-1.87</td>
<td>-2.04</td>
</tr>
<tr>
<td>Main free-flow travel time 10 min +</td>
<td>-2.03 (0.502) -4.05</td>
<td>-2.46 (0.624) -3.95</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-2.03</td>
<td>-2.33</td>
</tr>
<tr>
<td>Small free-flow travel time</td>
<td>-2.16 (0.685) -3.16</td>
<td>-2.75 (0.804) -3.42</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-2.16</td>
<td>-2.60</td>
</tr>
<tr>
<td>Proportion of time on freeways</td>
<td>-2.2 (0.812) -2.71</td>
<td>-2.31 (0.865) -2.67</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-2.2</td>
<td>-2.18</td>
</tr>
<tr>
<td>Proportion of time on CN</td>
<td>0 fixed</td>
<td>0 fixed</td>
</tr>
<tr>
<td>Proportion of time on main</td>
<td>-4.43 (0.752) -5.88</td>
<td>-4.40 (0.800) -5.51</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-4.43</td>
<td>-4.16</td>
</tr>
<tr>
<td>Proportion of time on small</td>
<td>-6.23 (0.992) -6.28</td>
<td>-6.02 (1.03) -5.83</td>
</tr>
<tr>
<td><strong>Scaled Estimate</strong></td>
<td>-6.23</td>
<td>-5.69</td>
</tr>
<tr>
<td>Covariance parameter</td>
<td>0.217 (0.0543) 4.00</td>
<td>0.205</td>
</tr>
</tbody>
</table>
## Empirical Results

<table>
<thead>
<tr>
<th></th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance parameter</td>
<td></td>
<td>0.217</td>
</tr>
<tr>
<td>(Rob. Std. Error) Rob. T-test</td>
<td>(0.0543)</td>
<td>4.00</td>
</tr>
<tr>
<td>Number of simulation draws</td>
<td>-</td>
<td>1000</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-1164.850</td>
<td>-1161.472</td>
</tr>
<tr>
<td>Adjusted rho square</td>
<td>0.145</td>
<td>0.147</td>
</tr>
<tr>
<td>Sample size: 780, Null log-likelihood:</td>
<td>-1375.851</td>
<td></td>
</tr>
</tbody>
</table>

Three challenges in route choice modeling – p.39/61
Empirical Results

- All parameters have their expected signs and are significantly different from zero
- The values and significance level are stable across the two models
- The subnetwork model is significantly better than the Path Size Logit (PSL) model
Concluding remarks

- Network-free data are more reliable
- Data processing may bias the result
- We prefer to model explicitly the relationship between the data and the model
Choice set generation

Introduction

- Choice sets need to be defined prior to the route choice modeling
- Path enumeration algorithms are used for this purpose, many heuristics have been proposed, for example:
  - Deterministic approaches: link elimination (Azevedo et al., 1993), labeled paths (Ben-Akiva et al., 1984)
  - Stochastic approaches: simulation (Ramming, 2001) and doubly stochastic (Bovy and Fiorenzo-Catalano, 2006)
Introduction

- Underlying assumption: the actual choice set is generated
- Empirical results suggest that this is not always true
- Our approach:
  - True choice set = universal set
  - Too large
  - Sampling of alternatives
Sampling of Alternatives

- Multinomial logit model (e.g. Ben-Akiva and Lerman, 1985):

\[
P(i|C_n) = \frac{q(C_n|i)P(i)}{\sum_{j \in C_n} q(C_n|j)P(j)} = \frac{e^{V_{in} + \ln q(C_n|i)}}{\sum_{j \in C_n} e^{V_{jn} + \ln q(C_n|j)}}
\]

- \(C_n\): set of sampled alternatives
- \(q(C_n|j)\): probability of sampling \(C_n\) given that \(j\) is the chosen alternative
Importance Sampling of Alternatives

- Attractive paths have higher probability of being sampled than unattractive paths
- Path utilities must be corrected in order to obtain unbiased estimation results
Stochastic Path Enumeration

- Flexible approach that can be combined with various algorithms, here a biased random walk approach
- The probability of a link $\ell$ with source node $v$ and sink node $w$ is modeled in a stochastic way based on its distance to the shortest path
- Kumaraswamy distribution, cumulative distribution function $F(x_\ell|a, b) = 1 - (1 - x_\ell^a)^b$ for $x_\ell \in [0, 1]$.

$$x_\ell = \frac{SP(v, d)}{C(\ell) + SP(w, d)}$$
Stochastic Path Enumeration

\[ F(x_\ell | a, b) \]

\( a = 1, 2, 5, 10, 30 \)

\( b = 1 \)
Stochastic Path Enumeration

- Probability for path $j$ to be sampled

\[ q(j) = \prod_{(v, w) \in \Gamma_j} q((v, w) | E_v) \]

- $\Gamma_j$: ordered set of all links in $j$
- $v$: source node of $j$
- $E_v$: set of all outgoing links from $v$

- Issue: in theory, the set of all paths $U$ is unbounded. We treat it as bounded with size $J$. 

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Sampling of Alternatives

- Following Ben-Akiva (1993)
- Sampling protocol
  1. A set $\tilde{C}_n$ is generated by drawing $R$ paths with replacement from the universal set of paths $U$
  2. Add chosen path to $\tilde{C}_n$
- Outcome of sampling: $(\tilde{k}_1, \tilde{k}_2, \ldots, \tilde{k}_J)$ and $\sum_{j=1}^{J} \tilde{k}_j = R$

$$P(\tilde{k}_1, \tilde{k}_2, \ldots, \tilde{k}_J) = \frac{R!}{\prod_{j \in U} \tilde{k}_j!} \prod_{j \in U} q(j)^{\tilde{k}_j}$$

- Alternative $j$ appears $k_j = \tilde{k}_j + \delta_{cj}$ in $\tilde{C}_n$
Sampling of Alternatives

Let \( C_n = \{ j \in \mathcal{U} | k_j > 0 \} \)

\[
q(C_n|i) = q(\tilde{C}_n|i) = \frac{R!}{(k_i - 1)!} \prod_{\substack{j \in C_n \setminus \{i\}}} k_j! \prod_{j \in C_n} q(j)^{k_j} = K_{C_n} \frac{k_i}{q(i)}
\]

\[
K_{C_n} = \frac{R!}{\prod_{j \in C_n} k_j!} \prod_{j \in C_n} q(j)^{k_j}
\]

\[
P(i|C_n) = \frac{e^{V_{in} + \ln \left( \frac{k_i}{q(i)} \right)}}{\sum_{j \in C_n} e^{V_{jn} + \ln \left( \frac{k_j}{q(j)} \right)}}
\]
Preliminary Numerical Results

- Estimation of models based on synthetic data generated with postulated models
  - Non-correlated paths
    Postulated model same as estimated model (multinomial logit)
  - Correlated paths in a “grid-like” network
    Postulated model is probit and estimated models are multinomial logit and path size logit
- True parameter values are compared to estimates
Preliminary Numerical Results

SB_1 = 3, L_1 = 2
SB_2 = 4, L_2 = 4
SB_3 = 1, L_3 = 6
SB_{j}, L_{j}
SB_{39} = 0, L_{39} = 78
SB_{40} = 0, L_{40} = 80
Preliminary Numerical Results

- True model: multinomial logit
  \[ U_j = \beta_L \text{length}_j + \beta_{SB} \text{nbspeedbumps}_j + \varepsilon_j \]
  \[ \beta_L = -0.6 \text{ and } \beta_{SB} = -0.3 \]
  \( \varepsilon_j \) is distributed Extreme Value with location parameter 0 and scale 1

- 500 observations, therefore 500 choice sets are sampled

- Biased random walk using 40 draws with \( a = 2 \) and \( b = 1 \)
  Generated choice sets include at least 7, maximum 18 and on average 11.9 paths
### Preliminary Numerical Results

<table>
<thead>
<tr>
<th>Sampling correction</th>
<th>MNL without</th>
<th>MNL with</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta}_L (-0.6) )</td>
<td>-0.203</td>
<td>-0.286</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.600</td>
<td>-0.600</td>
</tr>
<tr>
<td>Robust std.</td>
<td>0.0193</td>
<td>0.019</td>
</tr>
<tr>
<td>Robust t-test</td>
<td>-10.53</td>
<td>-15.01</td>
</tr>
<tr>
<td>( \hat{\beta}_{SB} (-0.3) )</td>
<td>-0.0194</td>
<td>-0.143</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.0573</td>
<td>-0.300</td>
</tr>
<tr>
<td>Robust std.</td>
<td>0.0662</td>
<td>0.0661</td>
</tr>
<tr>
<td>Robust t-test</td>
<td>-0.29</td>
<td>-2.17</td>
</tr>
<tr>
<td>Null log-likelihood</td>
<td>-1069.453</td>
<td>-1633.501</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-788.42</td>
<td>-759.848</td>
</tr>
<tr>
<td>Adjusted ( \bar{\rho}^2 )</td>
<td>0.261</td>
<td>0.288</td>
</tr>
</tbody>
</table>

BIOGEME has been used for all model estimations.
Preliminary Numerical Results

Three challenges in route choice modeling – p.56/61
Preliminary Numerical Results

- True model: probit (Burrell, 1968)
  \[ U_\ell = \beta_L \text{length}_\ell + \beta_{SB} \text{nbspeedbumps}_\ell + \sigma \sqrt{L_\ell} \nu_\ell \]
  \[ \beta_L = -0.6 \quad \text{and} \quad \beta_{SB} = -0.4 \]
  \( \nu_\ell \) is distributed standard Normal
  Link utility variance assumed proportional to length
  with parameter \( \sigma = 0.8 \)

- Path utilities are link additive

- 382 observations are generated after 500 realizations of
  the link utilities
Preliminary Numerical Results

- Biased random walk using 30 draws with $a = 2$ and $b = 1$ (382 choice sets)
  Generated choice sets include at least 7, maximum 19 and on average 13.5 paths
## Preliminary Numerical Results

<table>
<thead>
<tr>
<th>Sampling correction</th>
<th>MNL without</th>
<th>MNL with</th>
<th>PSL without</th>
<th>PSL with</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_L (-0.6)$</td>
<td>-0.627</td>
<td>-0.978</td>
<td>-0.619</td>
<td>-0.969</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.600</td>
<td>-0.600</td>
<td>-0.600</td>
<td>-0.600</td>
</tr>
<tr>
<td>Robust std.</td>
<td>0.0397</td>
<td>0.032</td>
<td>0.0407</td>
<td>0.0358</td>
</tr>
<tr>
<td>Robust t-test</td>
<td>-15.79</td>
<td>-30.57</td>
<td>-15.22</td>
<td>-27.04</td>
</tr>
<tr>
<td>$\hat{\beta}_{SB} (-0.4)$</td>
<td>-0.0822</td>
<td>-0.0801</td>
<td>-0.347</td>
<td>-0.461</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.0787</td>
<td>-0.0491</td>
<td>-0.336</td>
<td>-0.285</td>
</tr>
<tr>
<td>Robust std.</td>
<td>0.052</td>
<td>0.0559</td>
<td>0.182</td>
<td>0.158</td>
</tr>
<tr>
<td>Robust t-test</td>
<td>-1.58</td>
<td>-1.43</td>
<td>-1.90</td>
<td>-2.92</td>
</tr>
<tr>
<td>$\hat{\beta}_{PS}$</td>
<td></td>
<td></td>
<td>1.17</td>
<td>1.74</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td></td>
<td></td>
<td>1.13</td>
<td>1.08</td>
</tr>
<tr>
<td>Robust std.</td>
<td></td>
<td></td>
<td>0.788</td>
<td>0.705</td>
</tr>
<tr>
<td>Robust t-test</td>
<td></td>
<td></td>
<td>1.49</td>
<td>2.47</td>
</tr>
</tbody>
</table>
## Preliminary Numerical Results

<table>
<thead>
<tr>
<th>Sampling correction</th>
<th>MNL without</th>
<th>MNL with</th>
<th>PSL without</th>
<th>PSL with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null log-likelihood</td>
<td>-988.63</td>
<td>-2769.959</td>
<td>-988.63</td>
<td>-2769.959</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-676.111</td>
<td>-653.396</td>
<td>-674.481</td>
<td>-649.268</td>
</tr>
<tr>
<td>Adjusted $\bar{\rho}^2$</td>
<td>0.314</td>
<td>0.337</td>
<td>0.315</td>
<td>0.340</td>
</tr>
</tbody>
</table>

BIOGEME has been used for all model estimations.
Conclusions and Future Work

- Stochastic path enumeration algorithms are viewed as an approach for importance sampling of alternatives.
- We propose an algorithm that allows for computation of path selection probabilities and correction for sampling.
- Ongoing research, further work will be dedicated, for example, to empirical results on real data and correction in prediction.